Realization of Bicycles for Senior Ladies

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Abstract: Last year, a proposal for bicycles for senior ladies was made. Such bicycles must be able to keep upright position at zero speed, while it must tilt to proper angle at turn. For such control, the control principle of the inverted pendulum designed by the coefficient Diagram Method (CDM) is used. This year, practical design for the realization of such bicycles is presented. For this purpose, a model with 1/160 in mass and 1/2 in size is made to rove the validity of the design. Various problems, which may arise in real bicycles, are clarified. The controller consists of two force sensors, a rate sensor, and an unstable controller. In order to adjust the unstable controller efficiently, a special control device (Aproc-1) is used. The real bicycle will consists of a commercial motor-assisted bicycle (about 100,000 Yen or 1000 dollars) and side wheels with intelligent arms (about 30,000 Yen or 300 dollars), whose length will be adjusted properly by the controller.

Keywords: Control system design, Control theory, Inverted pendulum, Polynomial approach.

老婦人用自転車の実現

内容梗概：昨年度は、ゼロ速度では垂直に自立していても、曲がるとこには適切に傾くことができる、老婦人用自転車の原理を発表した。今年度はこれを実現する具体的な方法について述べる。そのためにまず質量 1/160、大きさ 1/2 の模型を作り、要求される性能がえられるかを検証する。また実自転車に適用する場合の問題点を明らかにする。制御系は力センサ、レートセンサ及び不安定制御器で構成される。不安定制御器を効率的に調整するための特殊な制御装置（Aproc-1）を使用している。市販の電動アシスト自転車（略 10 万円）に腕が適切に伸び縮みする補助輪（略 3 万円）を取り付ける形を構想している。

1. INTRODUCTION

With age advance, people feel difficulty in riding bicycle, because of the loss of reflex in keeping proper upright posture. They cannot drive cars, because of latent danger. This is more prominent in senior ladies. They feel difficulty in daily shopping with the result of loss of proper social connection, which is detrimental to their physical and mental health.

Senior people can ride bicycles, even they feel difficulty in walking. For walking, both legs have to be sufficiently strong, while in riding bicycles, one leg can be weak. Especially the handlebar is very helpful in keeping proper posture with weak waist muscle.

It would be very helpful to senior people and as well as to the society as a whole, if bicycles, which do not fall down even at zero or very slow speed, and also tilt to the proper direction with proper amount of angle at the turn with some speed, are invented.

Last year, the intelligent side-wheels with the capacity that the arms will extend or detract as needed are proposed. When such side-wheels are attached to motor-assisted bicycles, stable and maneuverable light-weight bicycles will be realized. It is very helpful for shopping of senior ladies. It will contribute to their health, too.

Such Intelligent side-wheels must be simple and of low price, about 30,000 Yen or 300 dollars in consideration that the price of motor-assisted bicycles are about 100,000 Yen or 1000 dollars.

From the previous study, it was found that there are two difficulties in the development of such side-wheels. First difficulty is the controller, which has the capacity for an unstable controller to be implemented and for the debugging of the program to be done efficiently. The second difficulty is the development of the force actuator with bilateral action, where the actuator easily detracts, if the external force is greater than actuator force. It is similar to human muscle.

The first difficulty is overcome by the use of a special controller called Aproc-1, developed by Arima-Dennshi-Kizai (有馬電子機材). The company was founded by Inoue Masao (井上正夫), who worked in the control of spacecraft under Prof. Tsuchiya Kazuo.

The second difficulty is not solved at present, and it is the aim of the present research. However, several proposals are made to overcome this difficulty. One proposal is the improvement of gear system with small friction. Other proposal is the use of force sensor and rate sensor, such that the closed-loop system works like frictionless mechanical system.

In order to prove that the theoretical concepts are valid, a model is being built, where the size is 1/2, and the mass is 1/160. It is an inverted pendulum supported by two intelligent arms. The pendulum keeps upright position even the base is tilted. Ordinary road is not flat; the center is higher than both sides. The pendulum tilts to the left side, when the force directed to the right is exerted. This is the case, when the bicycle turns to the left. The centrifugal force to the right is balanced by the left-tilt component of gravity force.

This paper is organized as follows. After introduction, in section 2, the controller design for real bicycle, which was in the previous paper [5], is repeated in compact form. In section 3, the motor and gear ratio selection problem, which plays a vital role in the realization of bicycle, is discussed. In section 3, the bicycle model is introduced, and the controller for the model is designed. The validity of the control design concept will be verified by the experiment of the model. The size is 1/2 and mass is 1/160. In section 5, the motor and gear selection is made for the model. In section 6, implementation of controller is discussed. In section 7, important results are summarized as conclusions.

2. CONTROLLER DESIGN FOR THE REAL BICYCLE

The bicycle dynamics is well-explained by Åströ m [1]. It was modified and simplified to serve for the present purpose [5]. The main points are repeated in the following.

General terms are defined as follows.

The point P_1 is the contact point of the rear wheel.
The point P_2 is the contact point of the front wheel.
The point P_3 is the intersection of the steer axis with the horizontal plain.

The coordinates are defined as follows.

The x-axis is the direction of motion of bicycle.
The y-axis is the left hand direction.
The z-axis is the upward direction.
The origin is at the point P_1.

Variables and parameters are defined as follows.

φ = Tilt angle, right tilt plus,
δ = Handlebar deflection, left turn plus,
h = Height of center of gravity,
V = Forward speed of bicycle,
a = Location of center of gravity at x-axis,
b = Contact point of front wheel at x-axis,
c = Trail, the distance between P_3 and P_2,
λ = Head angle of steer axis, upright 90 deg.

The simplified torque balance equation in x-axis is shown below.

$$(s^2 - g/h)\varphi = d + u,$$
$$d = [aVs + (V^2 - acg/h)]\frac{\sin\lambda}{bh}\delta, \qquad (1)$$
$$u = T_{xsw}/(mh^2).$$

Explanation and values of terms are as follows.

T_{xsw} = Side-wheel torque around x axis, CW plus,
u = Normalized side-weheel torque around x axis,
V = Speed of bicycle, $m/\sec$,
$m = 80kg, \quad g = 9.8m/\sec^2, \quad h = 1m,$
$a = 0.35m, \quad b = 1.15m, \quad c = 0.05m,$
$\lambda = 70\deg, \quad \sin\lambda = 0.940, \quad \cos\lambda = 0.342.$

The value of d is about 1, for $V = 3\, m/\sec$ and $\delta = 8$ deg.

$$d = [3^2 - 0.35*0.05*9.8/1]*\frac{0.94}{1.15*1}*8*\pi/180$$
$$= 1.0076. \qquad (2)$$

For the side wheel controller, the result of the inverted pendulum [2] is used.

$$u = T_{xsw}/(mh^2) = -G_c(s)\dot{\varphi}_s = -\frac{k_2 s^2 + k_1 s}{s^2 - l_1 s - l_0}\dot{\varphi}_s, \qquad (3)$$
$$\dot{\varphi}_s = \dot{\varphi} + n.$$

In this controller, the input is $\dot{\varphi}_s$, which is the angular velocity of the tilt angle with the constant error n. This type of controller eliminates the effect of constant error n.

The design is made by assigning stability indexes as $\gamma_i = [4 \ 4 \ 2.5]$ and the coefficient of characteristic polynomial as $a_2 = 2\times 9.8 = 19.6$. From these data the equivalent time constant τ is obtained as follows.

$$a_3 = \sqrt{a_4 a_2 \gamma_3} = \sqrt{1\times 19.6\times 4} = 8.8544,$$
$$\tau = \gamma_3\gamma_2\gamma_1 a_4 / a_3 = 4\times 4\times 2.5\times 1/8.8544 = 4.5175.$$

CAD command for CDM design [4] is as follows.

```
ap=[1  0  -9.8];bp=[1  0  0];gr=[4  4  2.5];nc=2;mc=1;
t=4.5175; ba=0,
[bc,ac,aa,g,tau,gs,rr]=g2c(ac,bp,nc,mc,gr,t,2);tm=0.5;cc
```

Design results are obtained as follows.

$$G(s) = G_c(s)G_p(s) = \frac{9.9612s^2 + 29.645s}{s^2 - 1.1068s - 0.24500}\,\frac{s}{s^2 - 9.8},$$
$$P(s) = s^4 + 8.8544s^3 + 19.6s^2 + 10.847s + 2.401,$$
$$\gamma_i = [4\ \ 4\ 2.5], \quad \tau = 4.5175,$$
$$s_i = -5.5716, \quad -2.4027, \quad -0.34005 \pm j0.2398,$$
$$\phi_m = -38.72 \text{ deg}, \quad g_m = 0.46826. \qquad (4)$$

The block diagram is given in Fig. 1. The coefficient diagram is shown in Fig. 2. The step responses for $d = 1$ are given in Fig. 3.

Explanations of the responses are as follows.

(1) The step disturbance for $d = 1$ corresponds to plus x-axis torque or right tilt of $mh^2 d = 80\ m\text{-}N$. When the bicycle speed is $V = 3\ m/\text{sec}$ or $10.8\ km/hr$ (about 3 times of walking speed), the left turn handlebar of about 8 degrees corresponds to $d = 1$.

(2) The response of u by d is given as follows.

$$u = -\frac{9.9612s^3 + 29.645s^2}{s^4 + 8.8544s^3 + 19.6s^2 + 10.847s + 2.401}\,d.$$

Or approximately,

$$u \simeq -(29.645s^2/19.6s^2)d \simeq -1.5d.$$

This is observed in Fig. 3(c).

(3) This strong negative torque tilts the bicycle to the left. This action continues even the tilt returns to zero due to the unstable nature of controller.

(4) Due to the left tilt, the strong left tilt torque of the gravity is produced.

(5) The torque gradually replaces the controller output u which finally settles to zero.

(6) The maximum torque of the controller is

$$T_{xsw-\max} = mh^2 u_{\max} = 80 \times 1.5 = 120\ m\text{-}N.$$

(7) The maximum angular rate is

$$\dot{\varphi}_{\max} = 0.07\ rad/\text{sec} = 4.0107\ \text{deg/sec}.$$

(8) The final tilt angle is

$$\varphi = -d/(g/l) = -1/9.8 = -0.10204\ rad$$
$$= -5.8456\ \text{deg} \simeq -6\ \text{deg}.$$

(9) The maximum motor power is observed in Fig. 3 (d) at around 1.2 sec.

$$Power_{\max} = mh^2(\dot{\varphi} \times u)_{\max} = 80 \times 0.07 = 5.6\ W.$$

(10) The maximum torque multiplied by the maximum angular rate is

$$T_{xsw-\max} \times \dot{\varphi}_{\max} = 120 \times 0.07 = 8.4\ W.$$

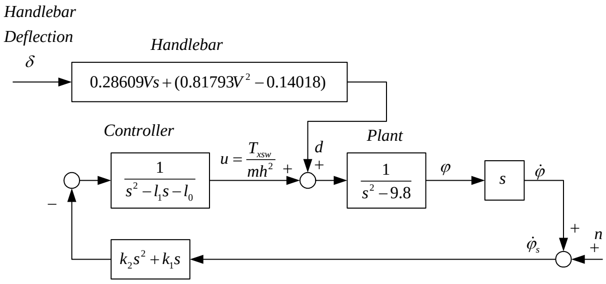


Fig.1. Block diagram of the control system

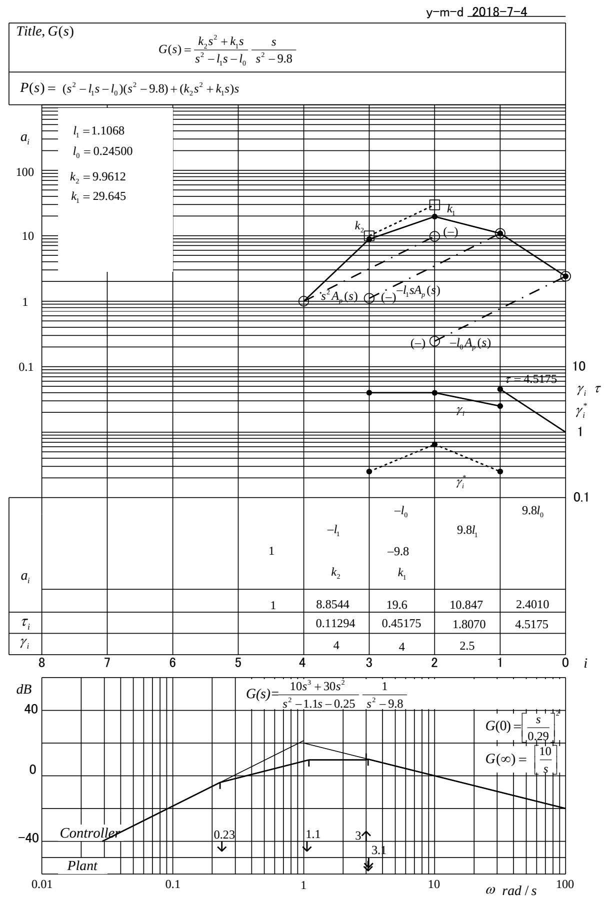
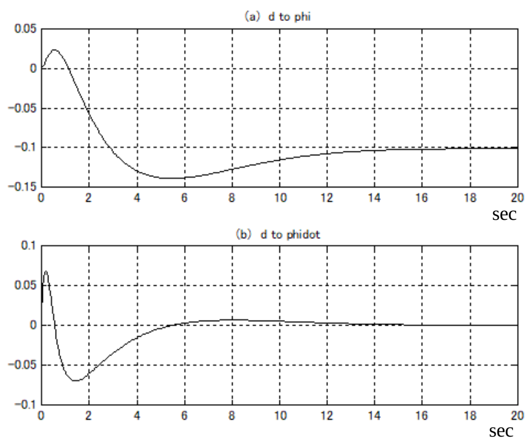


Fig. 2. Coefficient diagram



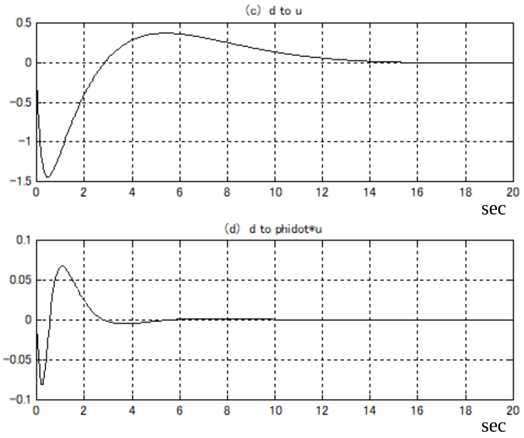


Fig. 3. Various responses to disturbance

3. MOTOR AND GEAR RATIO (GR) SELECTION

For the bilateral action of the side-wheel-arm actuator, the motor and gear ratio selection is very important. The condition for proper selection will be discussed.

Definitions of terms are as follows.

$i =$ Motor current ,

$V =$ Terminal voltage ,

$E =$ Counter electro-motive force ,

$Power =$ Motor power ,

$k_T =$ Torque coefficient ,

$k_V =$ Voltage coefficient, $k_T = k_V$,

$GR_1 =$ Gear ratio of geared motor ,

$GR_2 =$ Gear ratio of mechanical part, like rack-pinion ,

$GR = GR_1 GR_2 =$ Total gear ratio ,

$l_W =$ Arm-length of side-wheel ,

$r =$ Radius of pinion

The motor current and terminal voltage is respectively restricted by the following formula.

$$i > \frac{1}{k_T GR_1 GR_2} \left| T_{xsw} \right|_{\max} = \frac{120}{k_T GR_1 GR_2}, \tag{5}$$
$$V > k_V GR_1 GR_2 \left| \dot{\varphi} \right|_{\max} = 0.07 k_V GR_1 GR_2 .$$

The maximum values $\left| T_{xsw} \right|_{\max}$ and $\left| \dot{\varphi} \right|_{\max}$ were obtained from the response curves. This relations lead to the formula for gear ratio restriction.

$$\frac{120}{k_T i} < GR_1 GR_2 < \frac{V}{0.07 k_V}. \tag{6}$$

Temporarily motors are selected as Tamiya motor 380 series. The main reason is that, in this series, bilateral action is possible, or the geared motor can be rotated from the load side shaft. Their main characteristics are as follows.

$$V = 7.2\,V, \quad i = 12\,A, \quad Power = 17.5\,W,$$
$$k_T = k_V = 0.00372.$$

Then the gear ratio selection formula becomes as

$$2,688.2 < GR_1 GR_2 < 27,650. \tag{7}$$

Gear ratios are selected temporarily as follows.

$$GR = GR_1 GR_2 = 3,000,$$
$$GR_1 = 75, \quad GR_2 = l_W / r = 40, \tag{8}$$
$$l_W = 0.15\,m, \quad r = 0.00345\,m.$$

The selected gear ratios are very high compared with the ordinary system, and further research is needed for the realization of the system.

4. BICYCLE MODEL

In order to prove the validity of the design concept, a bicycle model is being built, where size is 1/2 and mass is 1/160. It is an inverted pendulum supported by two intelligent arms. It must keep upright position even the base is tiled. It must tilt to the left direction, when force is exerted to the right direction. In this model, the final tilt angle is the same as the real bicycle or $0.1\,radian$.

Selected dimensions of the model are as follows.

$$m = 0.5\,kg, \quad h = 0.5\,m, \quad d = T_d /(mh^2) = 2,$$
$$l_W = 0.1\,m, \quad r = 0.015\,m.$$

The design is made in the same manner as the real bicycle. The block diagram is shown in Fig. 4. The coefficient diagram is shown in Fig. 5. The step responses for $d = 2$ are shown in Fig. 6.

Design results are as follows.

$$G(s) = G_c(s) G_p(s)$$
$$= \frac{14.087 s^2 + 59.290 s}{s^2 - 1.5653 s - 0.49000} \frac{s}{s^2 - 19.6},$$
$$P(s) = s^4 + 12.522 s^3 + 39.2 s^2 + 30.679 s + 9.6040,$$
$$\gamma_i = [4 \;\; 4 \;\; 2.5], \quad \tau = 3.1944,$$
$$s_i = -8.1622, \quad -3.3980, \quad -0.48091 \pm j0.33913,$$
$$\phi_m = -38.72 \text{ deg}, \quad g_m = 0.46826. \tag{9}$$

Reponses are similar to the one of the real bicycle.

(1) The step disturbance of $d = 2$ is given. This gives the same final tilt angle as the real bicycle, where $d = 1$.

(2) The maximum normalized torque is $\left| u \right|_{\max} = 3$. The maximum torque of the controller is
$$T_{xsw-\max} = mh^2 u_{\max} = 0.5 \times 0.5^2 \times 3 = 0.375\ N\text{-}m .$$

(3) The maximum angular rate is
$$\dot{\varphi}_{\max} = 0.1\,rad / \sec = 5.73 \text{ deg}/\sec .$$

(4) The final tilt angle is
$$\varphi = -d /(g / l) = -2 / 19.6 = -0.10204\ rad$$
$$= -5.8456 \text{ deg} \simeq -6 \text{ deg}.$$

(5) The maximum motor power is observed in Fig. 6 (d) at around 0.8 sec.
$$Power_{\max} = mh^2 (\dot{\varphi} \times u)_{\max} = 0.5 \times 0.5^2 \times 0.2 = 0.025\,W.$$

(6) The maximum torque multiplied by the maximum angular rate is
$T_{xsw-\max} \times \dot{\varphi}_{\max} = 0.375 \times 0.1 = 0.0375\, W.$

5. MOTOR AND GEAR RATIO SELECTION FOR BICYCLE MODEL

The maximum values to be used for the determination of the gear ratios and the selection of motors were obtained from the response curves.

$$|u|_{\max} = 3, \quad |T_{xsw}|_{\max} = 0.375 N\text{-}m,$$
$$|\dot{\varphi}|_{\max} = 0.1\, rad/\sec, \quad Power_{\max} = 0.025\, W.$$

The restrictions for motor current and terminal voltage are as follows.

$$i > \frac{1}{k_T GR_1 GR_2} |T_{xsw}|_{\max} = \frac{0.375}{k_T GR_1 GR_2}, \qquad (10)$$
$$V > k_V GR_1 GR_2 |\dot{\varphi}|_{\max} = 0.1 k_V GR_1 GR_2.$$

Then the restriction for gear ratios is obtained as follows.

$$\frac{0.375}{k_T i} > GR_1 GR_2 < \frac{V}{0.1 k_V}. \qquad (11)$$

For the geared motor, Tamiya Model (High-power gear box) was selected. The motor is Mabuchi motor RE-260. The main characteristics are as follows.

$$V = 3V, \quad i = 0.973\, A, \quad k_T = k_V = 0.00218,$$
$$Power = 1.591\, W, \quad GR_1 = 64.8.$$

The gear ratio selection formula becomes as

$$176.79 < GR_1 GR_2 < 13761. \qquad (12)$$

Gear ratios and rack-pinion are selected as follows.

$$l_W = 0.1m, \quad r = 0.015m,$$
$$GR_1 = 64.8, GR_2 = l_W / r = 0.1/0.015, \qquad (13)$$
$$GR_1 GR_2 = 432.$$

The motors in the bicycle model are much larger compared with that of the real bicycle. The ratio of the required power for control to the motor power is calculated as follows.

Real bicyclel. $\dfrac{Power_{\max}}{Power_{motor}} = 5.6/17.5 = 0.32,$

Bicycle model. $= 0.025/1.591 = 0.015713.$

From these values, the control of the real bicycle will be more difficult than the bicycle model. This problem must be carefully considered in future.

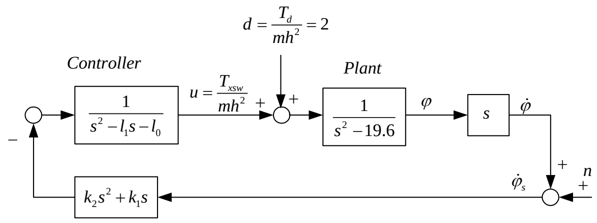


Fig. 4. Block diagram for the bicycle model

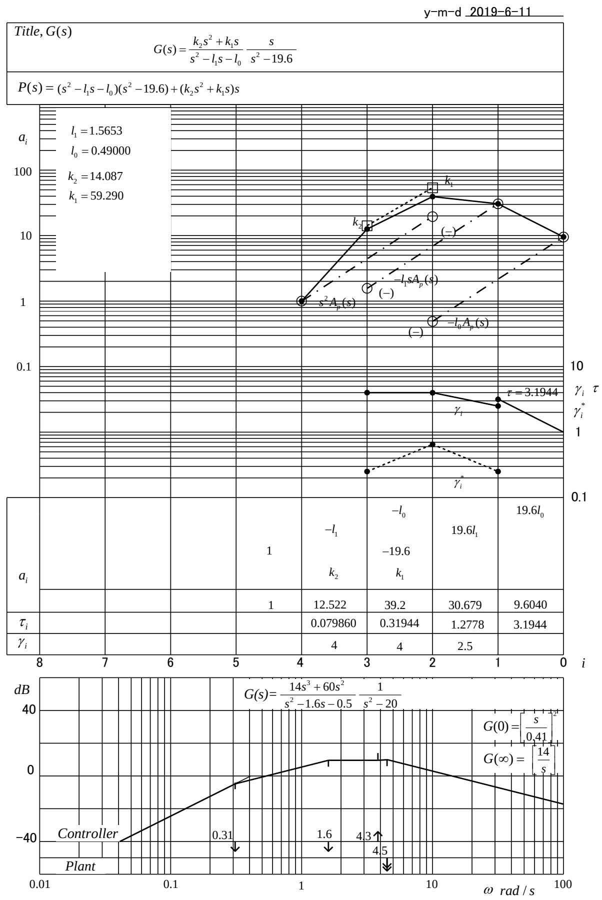
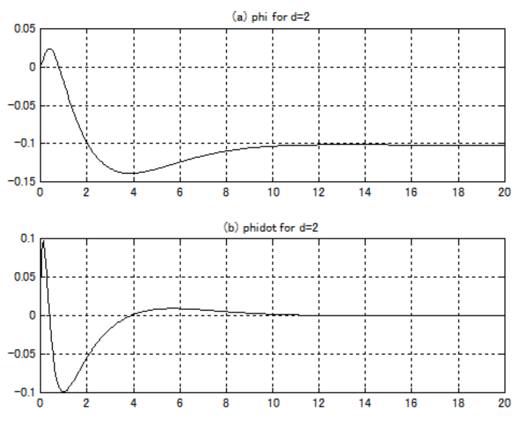


Fig. 5. Coefficient diagram of the bicycle model



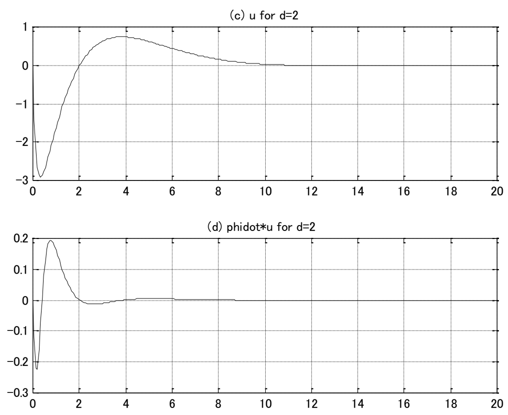


Fig. 6. Various responses for the bicycle model

6. CONTROLLER INPLEMENTATION

In implementing such unstable controller, the saturation is very important to prevent controller run-away. For this reason, careful studies are made. The temporary conclusions are shown below.

(1) The controller is reorganized as follows, so that instability is confined to a single unit, and saturation is properly placed.

$$G_c(s)=\frac{s^2k_2+k_1s}{s^2-l_1s-l_0}=\left[\frac{s}{s+\beta}\right]\left[\frac{k_2s+k_1}{s-\alpha}\right]$$
$$=\left[\frac{s}{s+\beta}\right]\left[\frac{k_2(s-\alpha)+(k_1+k_2\alpha)}{s-\alpha}\right]$$
$$=\left[\frac{s}{s+\beta}\right](k_1+k_2\alpha)\left[\frac{k_2}{k_1+k_2\alpha}+\frac{1}{s-\alpha}\right]$$
$$=\left[1-\frac{\beta}{s+\beta}\right](k_1+k_2\alpha)\left[\frac{k_2}{k_1+k_2\alpha}+\frac{1}{s-\alpha}\right],$$

$$s^2-l_1s-l_0=(s-\alpha)(s+\beta).$$

$$G_c(s)=\left[1-\frac{0.26737}{s+0.26737}\right]\times 85.107$$
$$\times\left[\frac{14.087}{85.107}+\frac{1}{s-1.8327}\right],$$

$$s^2-1.5653s-0.49000=(s-1.8327)(s+0.26737).$$

(2) Proper saturation units are placed to prevent excessive noise n, ($\pm L_1$), and prevent the run-away of the unstable unit, ($\pm L_2$).

(3) The intelligent side-wheels consist of the left and right wheels with arms which can extend and detract.

(4) Each arm consists of a geared-motor and a rack-pinion. By these mechanisms, the force acting on the side wheel is controlled. Eventually the torque around x-axis, T_{xsw}, is controlled.

(5) For the sensor-less system, the torque T_{xsw} is assumed to be computed from the motor torque. The angular rate $\dot{\varphi}$ is assumed to be observed from the motor current and the terminal voltage by the angular rate observer.

(6) When the force sensors are equipped, the torque T_{xsw} is directly controlled by adjusting each motor current in the closed-loop. When the angular rate sensor is equipped, the angular rate is directly measured.

Experiments are underway. But it was found that sensor-less system is not satisfactory due to the friction of the mechanical system.

7. CONCLUSIONS

The important conclusions are summarized as follows.

(1) A bicycle model is built, and experiment is underway. It is found that the sensor-less system is not satisfactory due to friction.

(2) The experiments will be continued for the system with force sensor and the angular rate sensor.

(3) The experiments of the bicycle model will be considered successful, when the inverted pendulum keeps upright position for the tilted base, and when the inverted pendulum tilts to the left at the force directed to the right.

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