Estimation of angular momentum of a non cooperative object by using images and artificial markers for capturing space debris

Tetsuya Kusumoto* Osamu Mori† Yuki Takao* Yuichiro Nada* Junichiro Kawaguchi†

For a secure capturing of a space debris, approaching it from the direction of its angular momentum is effective. A method of estimating the angular momentum vector of a non-cooperative object by using images and markers is proposed. By putting markers on the target beforehand and tracking them in sequential images, the three-dimensional motion of the object was calculated, resulting in estimation of the angular momentum vector. Considering a broken satellite, the inertia tensor of the target object is assumed to be unknown.

1 Introduction

The amount of space debris has been increasing as human begings launch more and more spacecraft. Currently it is estimated that the number of space debris which are larger than 10 cm on the Low Earth Orbit and larger than 1 cm on the Geostationary Transfer Orbit is more than 20000 [1]. Space debris are rotating around earth with high speed, collision of spacecraft and them would give destructive damages to the space crafts. This means space debris might give a critical damage to a mission. Also the number of space debris is increasing. For these reasons,there is necessity to remove space debris and lots of countries have been considering it.

One of the ways of estimating the motion of space debris is to use images taken by chaser satellite like Fig.1. In the area of computer vision, the reconstruction of three-dimensional model from two-dimensonal images has been studied. This can be done by tracking feature points and it can be solved by nonlinear optimization. By tracking the feature points on the debris, thier motion can be obtained [2, 3].

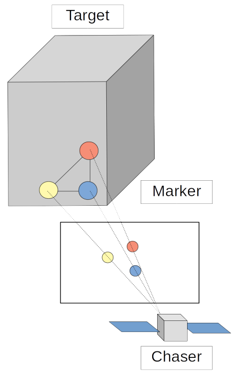

Fig.1 Image of a spacecraft that takes pictures of a target

When we think about approaching space debris, the way of approaching direction is important for safe approach. This is because the direction of angular momentum is the least likely way to go off course when capturing. Considering that a space debris is tumbling, there might be a destruction when chaser approaches the target. To avoid this, we assume that the safest way of approaching is to approach from the direction of angular momentum of space debris . To achieve this, accurate estimation of the angular momentum of the target is needed.

In this study, we assume artificial markers are put on the sattelite and it can be used as feature points. We also assume that a satellite is physically breken and its inertia tensor is unknown. This study focuses on the capturing non cooperative object. To tackle the problem we conduct nonlinear optimization and estimate the direction of angular momentum.

* The University of Tokyo, Tokyo, Japan
† Japan Aerospace Exploration Agency, Sagamihara, Japan

2 Proposed Method

2.1 Setup

The following is the explanations for each parameters as shown in Fig.2.

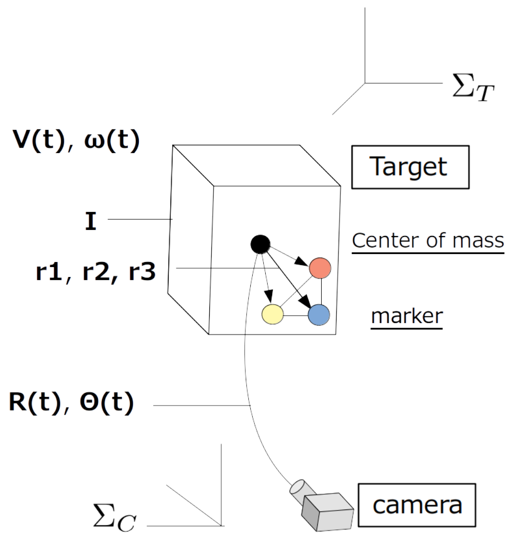

Fig.2 Image of each Parameters

Σ_T represents the body-fixed frame and Σ_C represents the camera-fixed frame and sight direction is set as z axis. $\boldsymbol{R(t)}$ [m] represents the distance between a target and a camera in Σ_C. $\boldsymbol{\Theta(t)}$ [rad] represents the attitude of a target in relative to a camera from Σ_C to Σ_T by Euler angle. $\boldsymbol{V(t)}$ [m/s] and $\boldsymbol{\omega(t)}$ [rad/s] represents the relative velocity and relative angular velocity of a target from Σ_C to Σ_T. $\boldsymbol{r1, r2, r3}$ [m] represents the relative distance of the marker to the center of the mass in Σ_T. $\boldsymbol{I}$ [kgm^2] represents the moment of inertia of a target.

2.2 Models

Translational movement of the target is assumed to be inertia movement. Attitude dynamics of the target are described by Euler's equation. In this case, we assume thers is no disturbance so the torque is assumed to be zero.

$$\boldsymbol{I}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \boldsymbol{I}\boldsymbol{\omega} = 0 \tag{1}$$

where $\boldsymbol{\omega}$ is the angular velocity of Σ_T relative to Σ_C.and $\boldsymbol{I}$ is the inertia tensor of the target, respectively.
As a model of an image, we adopt a simple pinhole camera model

$$\begin{pmatrix} x_{camera,i} \\ y_{camera,i} \\ 1 \end{pmatrix} = P \begin{pmatrix} X(t,i) \\ Y(t,i) \\ Z(t,i) \\ 1 \end{pmatrix} \tag{2}$$

where $x_{camera,i}, y_{camera,i}$ is the coordinate values on the camera coordinate and $X(t,i),\ Y(t,i),\ Z(t,i)$ is the coordinate value on Σ_C. Note that t means time and i means the index of each markers. Matrix P is called projection matrix which consists of internal parameters K and external parameters$[\boldsymbol{R} \mid \boldsymbol{t}]$. $\boldsymbol{R}$ represents retational matrix and $\boldsymbol{t}$ represents translational vector. K is represented by next equation.

$$K = \begin{pmatrix} f & 0 & x_0 \\ 0 & f & y_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where f is focal length, which is assumed to be 1 in the simulation and $(x_0,\ y_0)$ are image center, which is assumed to be (0, 0).

By following equation (1), motion of the target is integrated by Runge-Kutta method with an interval of 0.1 s and markers are projected with an interval of 1 s. Initial conditions are expressed by Table 1. To conduct non linear optimization, 12 images are used. Initial conditions of each parameters are expressed in the table below. To generate observations of the markers, the target body is rotated with the given angular velocity and markers are observed by a camera that remains still. This camera is supposed to be ideal, so images can be taken with no noises. Also we make an assumption that we can see through target body and the markers are always seen from the camera.

Table 1 Initial conditions

$\boldsymbol{R}$	$\boldsymbol{r1}$	$\boldsymbol{r2}$	$\boldsymbol{r3}$
0.0 0.0 20.0	4.0 4.0 2.0	4.0 0.0 2.0	0.0 2.0 2.0
$\boldsymbol{\Theta}$	$\boldsymbol{V}$	$\boldsymbol{\omega}$	
0.0 $\pi/3$ $\pi/3$	0.0 0.0 0.0	0.0 0.1 0.1	

2.3 Procedures of the method

In this study, we use artificial markers as feature points. We assume that markers are put before its launch, and we can utilize them after the satellite became a debris. And we assume a space debris is a physically broken satellite, so the moment of inertia of the target is unknown. This study focuses on the estimation when moment of inertia is unkown.

This method conducts nonlinear optimization of the reconstruction of three-dimensional geometry including the dynamics of the target. First thing to do is to give initial guess to all the parameters and by following equation (1), calculate the three dimensional position of each markers at each time. Secondly, by following equation (2), we calculate the position on the image by using three dimensional position. These process can be described by using function $f(\boldsymbol{x})$, which gives position on the image from initial value $\boldsymbol{x}$. Fig.3 shows one example of the estimated position of markers on the image. After calculating the guessed position on the image, we compare guessed postion $f(\boldsymbol{x})$ with the observed position y like a picture shown in Fig.4. We calculate the sum of each error as e. To get a maximum likelihood estimation, we minimize this squared observation error. By expressing gussed position as $f(\boldsymbol{x})$, observed position as y, error can be written as below. To optimize this equation, the proposed estimation method consists of a least-square method (levenverg Marquart method).

$$e = \sum \{y - f(\boldsymbol{x})\}^2 \tag{4}$$

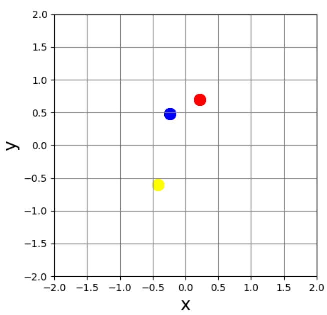

Fig.3 example of the guessed postion of the marker

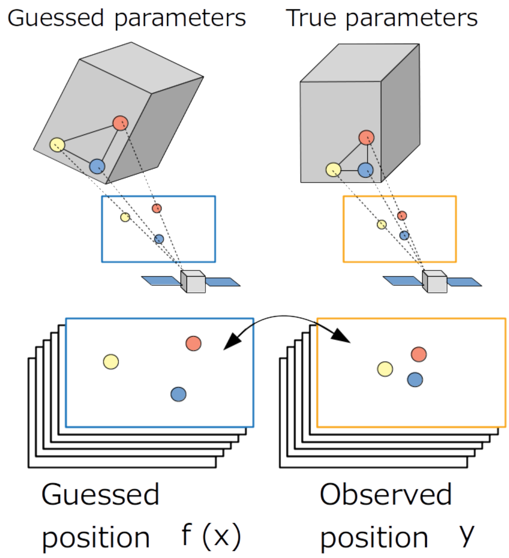

Fig.4 comparing the guessed position botained from guessed parameter $\boldsymbol{x}$ and observed position

3 Simulation

This section presents results of an simulation we conducted with artificially-generated data. This data is generated by using $f(\boldsymbol{x})$ and can be used as observed position.

3.1 Results

3.1.1 Result 1: Started with appropriate initial guess

Table 2 shows the result when initial guess started with parameters that are close enough to real answers.

Table 3 shows that in this case the direction of the angular momentum is accurately estimated but its scale is

Table 2 Result1

	Initial guess	Estimated value	Truth
$\boldsymbol{R}$	0.0 0.0 30.0	0.0 0.0 36.6	0.0 0.0 20.0
$\boldsymbol{r1}$	5.0 2.0 2.0	7.3 3.6 3.6	4.0 2.0 2.0
$\boldsymbol{r2}$	3.0 0.0 2.0	7.3 0.0 5.0	4.0 0.0 2.0
$\boldsymbol{r3}$	0.0 1.0 2.0	0.0 3.6 3.6	0.0 2.0 2.0
$\boldsymbol{\Theta}$	0 $\pi/3$ $\pi/3$	0 $\pi/3$ $\pi/3$	0 $\pi/3$ $\pi/3$
$\boldsymbol{\omega}$	0 0.2 0.2	0 0.1 0.1	0 0.1 0.1
$\boldsymbol{I}$	50 200 220	56 192 226	67 227 267

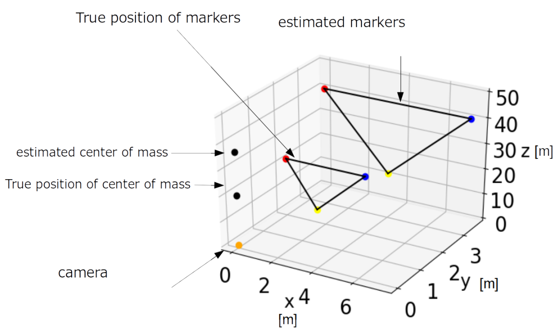

Fig.5 Estimated position and actual position of the markers

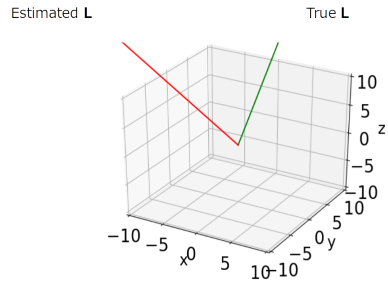

Fig.6 Estimated angular momentum and true angular momentum

different. Fig.5 shows the actual position of the markers and the estimated position of them. Black points represent the position of the center of mass and red, yellow and blue points represent the position of the markers. The position of the camera is represented by an orange point. This suggests that this answer is the same motion of the model with the different scale.

3.1.2 Result 2: Started with wrong initial guess

Table 4 shows the result when initial guess started with parameters that is not close enough to real answers and obtained answers were completely wrong. Fig.6 shows the true direction of angular momentum (green) and estimated one (red). Its values are expressed in Table 5. This clearly shows the estimated angular momentum does not correspond to the actual value.

3.1.3 When velocity is known

Table 6 shows the result when assuming that velocity of the target is known and obtained the exact scale of length. Still, norm of the I is not correct and norm of the angular momentum is not correct. This is because according to equation (1), norm of the I does not count and ratios of each I_x, I_y, I_z count.

Table 3 Angular momentum $\boldsymbol{L}$

	$\boldsymbol{L}$	Normalized $\boldsymbol{L}$
estimated	-6.86 26.6 11.3	-0.231 0.895 0.381
true	-8.08 31.3 13.3	-0.231 0.895 0.381

Table 4 Result2

	Initial guess	Estimated value	Truth
$\boldsymbol{R}$	0.0 0.0 35.0	4.5 -6.7 -0.79	0.0 0.0 20.0
$\boldsymbol{r1}$	0.0 4.0 2.0	0.0 -9.0 13	4.0 2.0 2.0
$\boldsymbol{r2}$	6.0 0.0 4.0	0.79 -4.5 6.6	4.0 0.0 2.0
$\boldsymbol{r3}$	0.0 2.0 2.0	4.0 -4.5 6.8	0.0 2.0 2.0
$\boldsymbol{\Theta}$	0 $\pi/3$ $\pi/3$	0 $\pi/3$ $\pi/3$	0 $\pi/3$ $\pi/3$
$\boldsymbol{\omega}$	0 0.4 0.4	-0.5 0.7 0.0	0 0.1 0.1
$\boldsymbol{I}$	100 200 220	158 159 159	67 227 267

Table 5 Angular momentum $\boldsymbol{L}$

	$\boldsymbol{L}$	Normalized $\boldsymbol{L}$
estimated	-89.8 17.9 51.4	-0.855 0.171 0.490
true	-8.08 31.3 13.3	-0.231 0.895 0.381

Table 6 simulation result when the velocity was known

	Initial guess	Estimated value	Truth
$\boldsymbol{R}$	0.0 0.0 30	0.0 0.0 20.0	0.0 0.0 20.0
$\boldsymbol{r1}$	5.0 2.0 2.0	4.0 2.0 2.0	4.0 2.0 2.0
$\boldsymbol{r2}$	3.0 0.0 2.0	4.0 0.0 2.0	4.0 0.0 2.0
$\boldsymbol{r3}$	0.0 1.0 2.0	0.0 2.0 2.0	0.0 2.0 2.0
$\boldsymbol{\Theta}$	0 $\pi/3$ $\pi/3$	0 $\pi/3$ $\pi/3$	0 $\pi/3$ $\pi/3$
$\boldsymbol{\omega}$	0 0.2 0.2	0 0.1 0.1	0 0.1 0.1
$\boldsymbol{I}$	50 200 220	110 375 442	67 227 267

3.2 Discussion

Result 1 shows that when initial guess is close enough to answers, exact direction of angular momentum is estimated, even though its scale is different form the answers. Result 2 shows that exact answers for direction of angular momentum is not obtained when initial guess is not close enough. This suggests that this least square method is sensitive to the initial guesses and it needs to start with appropriate value. Therefore we need rough estimation before this proposed method.

4 Conclusion

In this study, we presented a method for estimation of angular momentum of space debris. The proposed method successfully estimated the direction of anugular momentum of the target when initial guess started with appropriate values. However the scales of the distance and angular momentum is not decided. Besides the problem of uncertainty of scales, there is a problem of deciding the appropriate initial guess. Further research has to be done to cope with these problems.

References

[1] Satomi Komoto, Overview of Active Debris Removal,Proceedings of 56th Space Sciences and Technology Conference(2012).

[2] Youhei Shirasaka, Hideyuki Tanaka, Takehisa Yairi, Kazuo Machida,Attitude Motion Estimation of Tumbling Objects Using RFID, Journal of the Japan Society for Aeronautical and Space Sciences, Vol.54, No.632(2006), pp.375-383.

[3] Keigo Yoshida, Youhei Shirasaka, Takehisa Yairi, Kazuo Machida, Robust Attitude Motion Estimation for Uncontrolled Satellites Using the Unscented Kalman Filter,Journal of the Japan Society for Aeronautical and Space Sciences, Vol.56, No.649(2008), pp.65-71.