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# ON THE ATTITUDE CONTROL BY THRUSTER OF A SPINNING SOLAR SAIL AND BENDING MOMENT'S EFFECT ANALYSIS

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**Abstract**

The membrane dynamics of spinning solar sails have a special relevance when considering attitude control of the spacecraft. So as to model the system accurately, the bending stiffness of the membrane has been included in the numerical approach, as it is believed to have strong effects on the behavior of the sail. First, this study shows that the influence of the bending moment on the attitude of the sail during its spin axis reorientation should not be neglected. Given the difficulty of measuring the actual bending stiffness of the membrane, finding a control system capable to perform the same regardless of its value is necessary. Therefore, this paper presents a new control system and its corresponding logic to lower the influence of the bending parameter on this attitude maneuver performance. Finally, a frequency analysis on the vibrations arising in the membrane when considering different bending stiffness values is done. This last analysis shows the shift in the natural frequencies obtained, remarking the importance of the bending stiffness when considering the dynamics of a mast-free sail.
**Keywords:** Solar Sail, Attitude Control, Bending Stiffness, Flexible Structure, Multi-Particle Model, Vibration

## 1. Introduction

Solar sails take advantage of the photons' momenta to be driven by SRP. As a means of fuel-free propulsion, these spacecrafts are actively studied at several institutes around the world.

IKAROS, launched by JAXA in 2010, was the world's first successful solar sail mission. Following its success, OKEANOS [1], in which this document focuses, was proposed as the next solar sail mission, its concept consisting on a landing and sample return of Jupiter Trojan asteroids by means of a solar sail-craft.

During IKAROS' operation, the membrane deformation and spacecraft attitude was observed to differ from the simulated one. Such difference is believed to be caused by the bending stiffness present in the membrane, often overlooked in the literature [2] [3]. Considering the importance of membrane dynamics and deformation when performing attitude control in a flexible body, a model including such rigidity has to be used. Besides, due to the fact that OKEANOS has more elements than IKAROS on its membrane, it is expected to have an increased rigidity, so further study on the bending dynamics of the sail is required for the better implementation of attitude control.

In this document, a study of the effects of the bending moment on the sail's motion will be done. Such analysis will take place during a simulated spin-axis reorientation maneuver for the aforementioned OKEANOS. The attitude control system that is to guide the maneuver will be based on thruster application. Control by means of an input in the membrane has already been studied in [4] by means of RCD. In our present case of study, however, considering that the surface of OKEANOS is set to be 2000 $m^2$, that is, 10 times larger than that of IKAROS [5], and that its operation takes place in a low SRP environment, thrusting has been chosen above the former control. Also, since the bending moment is a non measurable parameter of the membrane that plays an important role on its dynamics, a control method and its logic such that the motion of the sail does not get affected up to a great measure by the bending stiffness need to, and will, be proposed.

Finally, a frequency analysis on the out-of-plane motion of the particles of the sail will also be performed so as to compare the vibration modes arising in the membrane for different bending stiffness cases.

## 2. Modeling

As mentioned above, the target of analysis is the OKEANOS solar sail-craft, a 40m span solar sail. Given its dimensions and the impossibility of using scaled models due to the thickness of the membrane, the study of its dynamics has been done by means of the MPM.

### *2.1 MPM*

The MPM consists on substituting the elements of the membrane by particles connected by springs and dampers. Compared with other dynamic models such as Finite Element Method, the MPM offers a simplified construction and a lower computational cost. The model has been validated via numerous vibration experiments in vacuum chambers [6]- [8].

In this approach, the mass of each particle is determined based on the membrane configuration, obtaining a nonuniform mass distribution per each of the four petals. The spring constants are determined by applying the principle of virtual work on an element [9]. The inter-particle force is described in Equation 1 as presented in [10]:

$$F = \begin{cases} k_t(L-L_0)+\beta k_t\dot{L} & \text{if } L \geq L_0 \\ \alpha k_t(L-L_0)+\beta \alpha k_t\dot{L} & \text{if } L < L_0 \end{cases} \quad (1)$$

#### *2.1.1Bending Stiffness Model*

Bending stiffness is modeled my means of torsion springs, placed so as to connect two adjacent surface elements. The force acting in the outermost particles of one element, those being $C$ and $D$ are obtained from Equation 2:

$$F_i = k_{\theta_i} \cdot \frac{L_{AB}(\chi-\chi_0)}{{l_i}^2} \quad (2)$$

According to [11], a system of three consecutive particles can be regarded as a beam structure. From the cantilever equation, the deflection is:

$$w_i = \frac{F_i {l_i}^3}{3EI} \quad (3)$$

if $w_i$ is small enough, $\chi_i = w_i/l_i$ and $k_{\theta_i}$ is then derived as:

$$k_{\theta_i} = \frac{3EI}{l_i} \equiv k_{\theta_0} \quad (4)$$

Despite the cantilever model allows us to approximate the effect of the bending moment on the sail, according to [12] the deformation observed in IKAROS during low-spin operation did not match the simulations, but was that expected from a more rigid membrane model, that is, presenting a greater bending stiffness.

Both in IKAROS and OKEANOS cases, before its deployment the membrane was folded. The creases formed on the membrane during the folding process are believed to present a higher bending stiffness, which might have led to the divergence between the model and the real deformation. Therefore, the surface elements placed in a crease have to be treated accordingly.

For a system of surface elements in which the particle A exists over a crease, the torsion spring constant is obtained by a correction factor:

$$k^*_{\theta_i} = \kappa_\theta k_{\theta_0} \quad (5)$$

The parameter $\kappa_\theta$ is difficult to be obtained in ground, so in this paper several values will be considered so as to simulate different possible situations of membrane deformation. The forces acting on particles $C$ and $D$ as shown in Figure 1 are obtained by substituting $k_i$ in Equation 2 for its corresponding value.

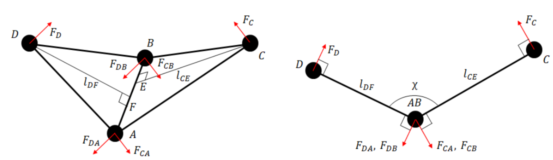

Fig. 1: Bending force model scheme.

Finally, forces acting on $A$ and $B$ are set to cancel the rigid-body motion of the triangular elements.

## 3. Spin-Axis Reorientation

This report will focus on the membrane dynamics when applying attitude control for the reorientation of the sail. The attitude control of the sail is done mainly by the impulsive torque originated from the thrusters present in the spacecraft. OKEANOS has a total number of eight thrusters on the main body to perform spin up, spin down and reorientation.

As only a spin-axis reorientation will be performed, only the Z-axial thrusters will be considered. Such thrusters provide a force of around 20 N and are displayed symmetrically along the main body, at a radial distance of 1.8 m from the center of the probe.

The Rhumb-Line maneuver will be used to perform the attitude change of the sail. It is one of the most effective and yet easier ways to change the orientation of the spinning axis up to a certain value. The maneuver simply consists on giving an angular impulse on detection of a certain inertial reference, in this case the Sun, approximately once every revolution around the major spinning axis.

## 4. In-Body Thrust Simulations

### *4.1 Results*

The spin-axis reorientation maneuver has been performed for several $\kappa_\theta$ so as to obtain a mean effective $K$ in creases going from zero (no bending stiffness considered) up to $1 \times 10^{-2}$, increasing its value by a factor of 10 from $K_\theta = 1 \times 10^{-8}$ onwards. $K = K_{beam}$ is the one obtained by a $\kappa_\theta = 1$, and its value is approximately $1 \times 10^{-5}$. The value indicated is just to give the order of magnitude in most part of the sail, since it changes accordingly with the distribution of the membrane devices.

Since the membrane was observed to start differing in behavior at $K = 1 \times 10^{-2}$, in this document the results for only the $K = K_{beam}$ and $K = 1 \times 10^{-2}$ are presented to avoid repetition.

In Figure 2 (L), the pitch angle for both cases of interest is presented, as well as the relative difference among its values. The difference between both cases grows as high as 20% in the last stages of the simulation, and keeps itself over that value during most part of it. For the roll angle (C), no significant differences are observed, the same as it happens with the angular velocity in the hub, that in both cases presents strong oscillations.

## 5. Tip-mass Thrust Simulations

In the previous section, the response of the sail was found to vary depending on the bending stiffness. Therefore, a new control system lowering the influence of the parameter is desired.

Besides, the current control method has a low performance and presents a strong nutation motion on the hub that tends to diverge. In [3], it was found that the disturbance introduced by the flexibility of the membrane into the equations of the system makes its stability depend on the dumping of the sail-hub connection mechanism, usually low and difficult to increase.

Here, a new control setting is proposed. The thruster input is to take place at the tip-masses, that is, four thrusters placed respectively in each of the tip masses of the spacecraft are considered. The thrusting force considered is 5 N, and the Rhumb Line Maneuver will still be used.

### *5.1 Results*

As it is shown in Figure 3 (L), when comparing the two cases of interest they both present a similar behavior, with the pitch angles overlapping. The difference among them is kept below 10% during great part of the simulation.

As for the roll angle, ideally, should have been kept zero. However, an undesired attitude appears. Besides, differences arises between the bending cases compared, like it is observed in Figure 3 (C). That difference is generated by the actual direction of the thrusting force in every one of the cases studied. Different bending moments affect the layout and particle position of the sail, translated into different thrusting vector orientations and, therefore, different values of the undesired roll angle in detriment of the pitch.

In Figure 3 (R), the values are far lower the ones got from the in-body thrusting.

The differences observed between the pitch angles are believed to be caused by the difference in roll angles commented above.

### *5.2 Roll Control*

In order to counteract the apparition of the undesired roll angle, a simple roll control is proposed to see whether the correction of the roll angle reduces the difference when having different bending stiffnesses.

The control system is still the Rhumb Line Maneuver, but the attitude of the hub is fed to the system allowing for different activation times for the thrusters.

As it can be seen in the Figure 4 (C) plot, with roll control being active, the roll angle is kept between 1 and $-1^\circ$ in the most unfavorable case ($K = 1 \times 10^{-2}$). Without significant differences in this angle, the divergence between pitch angles is found to be below 5% during the most part of the maneuver.

The nutation motion of the hub also disappears for both cases with the newly implemented control logic.

Therefore, with the roll control, the attitude does not depend anymore on the bending of the sail when a thrusting input of 5 N is applied over the membrane.

## 6. Frequency Analysis

From the results obtained in the previous sections, it is believed that the differences in the spin-axis reorientation maneuver performance might be caused by higher vibration modes excitation in the flexible-most membrane. Therefore, in this section, the vibrations arising in the membrane will be studied.

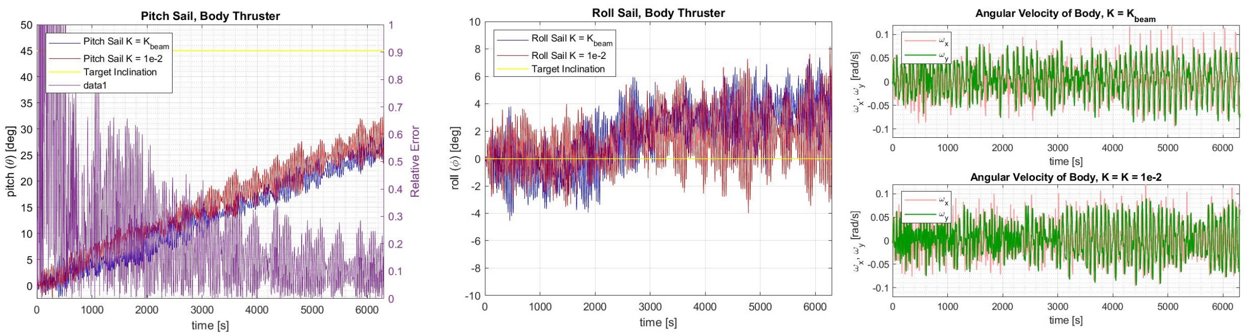


Fig. 2: Pitch (R), roll (C) and nutation of the probe (R) for $K = K_{beam}$ and $K = 1 \times 10^{-2}$ for in-body thrusting.

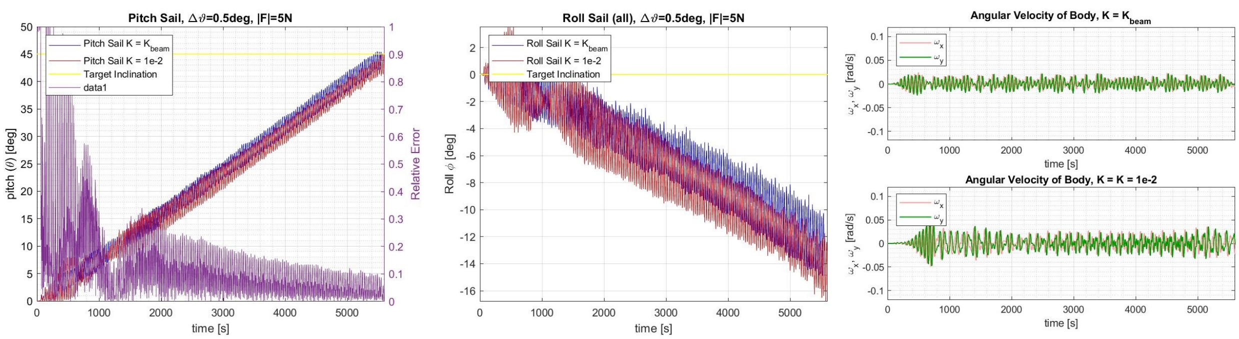


Fig. 3: Pitch (L), roll (C) and nutation of the probe (R) for $K = K_{beam}$ and $K = 1 \times 10^{-2}$ for tip-mass thrusting.

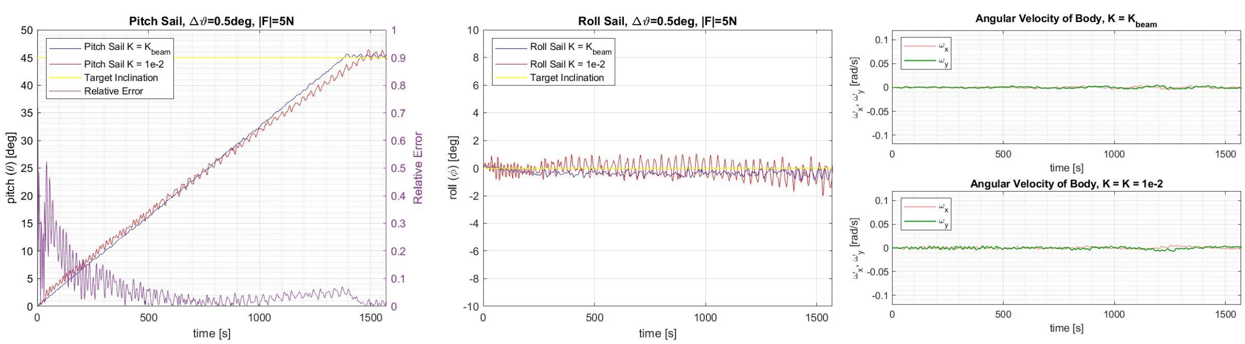


Fig. 4: Pitch (L), roll (C) and nutation of the probe (R) for $K = K_{beam}$ and $K = 1 \times 10^{-2}$ for tip-mass thrusting with Roll Control.

### *6.1 Equation of Motion*

Considering a uniform and circular membrane in a linear region, where in-plane and out-of-plane vibrations can be treated independently, the latter will be considered under the plane stress assumption.

Assuming $\Omega$ to be constant and the inner radius of the membrane to be far smaller than the external one, the force in the out-of-plane ($z$) direction acting on a membrane infinitesimal element $dM = \rho hrdrd\theta$ can be derived using the principles on the von Karman thin plate theory [14]:

$$dF_z = \left( \frac{\partial}{\partial r}\left( r\sigma_{rr}\frac{\partial w}{\partial r}\right) + \frac{1}{r}\frac{\partial}{\partial \theta}\left(\sigma_{\theta\theta}\frac{\partial w}{\partial \theta}\right) - \frac{Eh^2 r}{3(1-\nu^2)}\nabla^4 w \right) hdrd\theta \qquad (6)$$

Considering

$$dF_z = \frac{\partial^2 w}{\partial t^2} dM \qquad (7)$$

the equation of motion is finally obtained:

$$\rho \frac{\partial^2 w}{\partial t^2} = \frac{1}{r}\frac{\partial}{\partial r}\left( r\sigma_{rr}\frac{\partial w}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\sigma_{\theta\theta}\frac{\partial w}{\partial \theta}\right) - \frac{Eh^2}{3(1-\nu^2)}\nabla^4 w \qquad (8)$$

Ignoring the bending stiffness contribution in the equation of motion, Equation 8 can be solved analytically, from which the derivation of the natural frequencies can be obtained [2]. The natural frequency of the vibrations in a flexible membrane made dimensionless by the spin rate $\hat{\omega} = \omega/\Omega$ obtained in [2] is presented below:

$$\hat{\omega}_{n\nu'} = \sqrt{\frac{3+\nu}{8}(\nu'+n-1)(\nu'+n+1) - \frac{1+3\nu}{8}\nu'^2} \tag{9}$$

In our case of study, the force due to the thruster input should be added into the equation, as well as the coupling between the hub and membrane. Thus, the analytical approach to be solved gets its complication increased. Therefore, a numerical approach is chosen. The out-of-plane vibration of the particles along the simulations presented in the previous section will be analyzed in the frequency domain by using a FFT to search for the excited frequencies as well as the differences in front of a force input depending on the bending parameter $K$.

### *6.2 FFT Analysis*

Apart from the case of a forced input $|F| = 5$ N and $\Delta\theta = 0.5°$, the results for a higher input case, that is, $|F| = 10$ N and $\Delta\theta = 1.5°$, are presented to show the membrane response when increasing the magnitude of the external force.

Only the results for the $K = K_{beam}$ and $K = 1 \times 10^{-2}$ bending stiffness parameter values will be shown, being the first representative for the flexible case, that being from $K = 1 \times 10^{-8}$ to $K = 1 \times 10^{-3}$. They have been represented in Figure 5.

It can be seen that regardless of the input force, in terms of frequency all figures present the same behavior depending solely on the bending stiffness considered. As expected, the amplitude of the vibrations increases with the magnitude of the external force.

For a bending constant given from the cantilever approximation, $K = K_{beam}$, the excited frequencies are those corresponding to $\hat{\omega} = 1$, 2 and 3 which, considering $n = 1$ in Equation 9, are those modes excited for a circumferential order $\nu' = 1, 3, 5$, respectively. The dominant peak out of these three is the one at $\hat{\omega} = 2$, with $\hat{\omega} = 1$ gaining relevance as the input force increases.

As for $K = 1 \times 10^{-2}$, the excited frequencies are $\hat{\omega} = 1.2$, 2.9-3.1, and the peak at 2 does not appear. As the input force becomes larger, higher order frequencies are excited, and a peak at $\hat{\omega} = 5$ makes its apparition. Besides, as expected when observing Figure 4 (L) and (C), the amplitude of the vibrations for the higher bending case is major. Also, it is important to point out that, while the frequencies observed in Figure 5 (L) correspond up to a great extend with the theoretically predicted ones when considering the equation of motion without taking into account the bending stiffness, the ones in Figure 5 (R) present a certain deviation from the values stated in Equation 9. Therefore, a more extensive analysis on the vibration modes regarding the complete equation of motion should be done to understand the change of behavior observed.

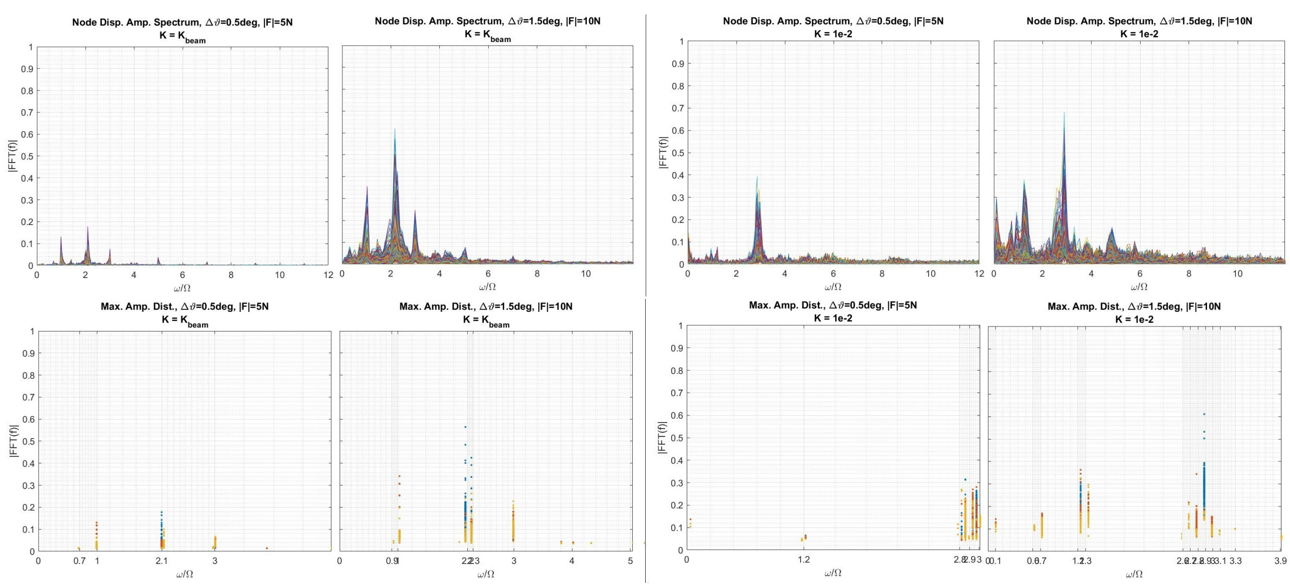


Fig. 5: Frequency Analysis for $K = K_{beam}$ (L) and $K = 1 \times 10^{-2}$ (R).

## 7. Conclusions

In this document, the effects of the bending stiffness on the solar sail to be used for the mission OKEANOS during an attitude control method for changing its spin axis have been studied.

In Section 4, the results for the reorientation by Rhumb-Line Maneuver using the in-body thrusting system of the sail where presented. The pitch angle of the whole sail was indeed changed effectively after the simulation, although its performance was lower than what was expected theoretically. Regarding the bending stiffness, when varying the rigidity of the sail, attitude changes followed.

So as to avoid the instability introduced by the connection between the body of the spacecraft and the sail itself, a new control system is introduced in Section 5: tip-mass thrusting. The tip-mass permitted undergoing an attitude maneuver with the same response from the sail regardless of it presenting a higher rigidity. However, because of the apparition of an undesired roll angle its performance was far from perfect.

In Subsection 5.2, the control algorithm is improved by feeding the roll of the hub to the policy. The roll angle and the nutation that in previous cases arose in the hub are kept close to zero. More importantly, with the newly implemented control the attitude of the sail does not vary with the rigidity. The control presented, although simple, offers an axis reorientation system the performance of which does not depend anymore on the stiffness of the sail.

As for the frequency analysis, a difference on the modes excited was found depending on the bending stiffness. As future work, a study on the natural frequencies and oscillation modes parting from the complete equation of motion of the sail should be developed thoroughly.

## References

[1] J. Kawaguchi et al., Science and Exploration in the Solar Power Sail OKEANOS Mission to a Jupiter Trojan Asteroid, pp. 910, 2018.

[2] Y. Takao, Active Shape Control of Spinning Membrane Space Structures and its Application to Solar Sailing, 2018.

[3] T. Nakano, O. Mori and J. Kawaguchi, Stability of Spinning Solar Sail-craft Containing a Huge Membrane, AIAA Guid., Navig. Control Conference, 2005.

[4] Y. Tsuda, An Attitude Control Strategy for Spinning Solar Sails, 2004.

[5] O. Mori, J. Matsumoto, T. Chujo, H. Kato, and T. Saiki, Navigation to Jovian Trojan Asteroid using Solar Power Sail-craft and Underground Sampling using Lander, 2017.

[6] N. Okuizumi and T. Yamamoto, Centrifugal Deployment of Membrane with Spiral Folding: Experiment and Simulation, Journal of Space Engineering, Vol.2, No.1, pp.41-50, 2009.

[7] A. Muta, S. Matunaga and N. Okuizumi, High Vacuum Experiment of Spinning Deployment using Scaled-down Model for Solar Sail, IAC-10-C2.5.4, 61st International Astronautical Congress, Prague, Czech Republic, 2010

[8] O. Mori et al. Adjusting the Membrane Model using the Multi-Particle Method via Vibration Experiments in a Vacuum Chamber, vol. 11, pp. 2731, 2013.

[9] Y. Tsuda, Improved Multi-Particle Model for Dynamics Analysis of Arbitrary-Shaped Thin Flexible Structure, 2004.

[10] O. Mori and Y. Shirasawa, Attitude Dynamics of Spinning Solar Sail IKAROS Considering Thruster Plume, Trans. JSASS Aerosp. Tech. Japan, vol. 10, pp. 2732, 2012.

[11] M. Hasome, O. Mori et al. Refining Membrane Model via Multi-Particle Method combined with Vibration Experiment in Vacuum Chamber, 2013.

[12] Shirasawa, Y., Mori, O., Miyazaki, Y., Sakamoto, H., Hasome, M., Okuizumi, N., Sawada, H., Matunaga, S., Furuya, H. and Kawaguchi, J., Evaluation of Membrane Dynamics of IKAROS Based on Flight Result and Simulation Using Multi-Particle Model, Aerospace Technology Japan, 2012.

[13] J. C. Van Der Ha, Models for Rhumb-Line Attitude Maneuvers and Error Propagation Effects, J. Guid. Control. Dyn., vol. 29, no. 6, pp. 13841394, 2006.

[14] N. Baddour, A revisit of spinning disk models. Linear transverse vibrations, 2000.