Propellant Balancing Considering the Propellant Remaining Difference in Formation Maintenance

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Formation maintenance is the process of multiple satellites to keep them in the desired formation. Propellant balancing which equalizes the propellant consumption of each satellite is important to achieve a longer mission duration. In previous researches on propellant balancing, they were performed for propellant consumption up to a certain period to equalize the remaining propellant. However, in actual missions, the remaining propellant difference between each satellite might be occurred before the initial time of formation maintenance due to formation reconfiguration, maintenance, collision avoidance, and troubles including failures. To extend the mission period, it is necessary to take into account this difference in initial propellant difference. In this study, we developed a method to solve it and discussed the results of the method. Furthermore, this propellant balancing method flexibly changes the orbit of the virtual Chief satellite so no additional maneuvers are required.

フォーメーション維持における推薬残量差を考慮した推薬バランシング

フォーメーション維持とは、複数の衛星を望ましいフォーメーションで維持するために制御を行うことである。複数の衛星を扱うという性質上、各衛星の推進消費量を均等化する推薬バランシングは、ミッションをより長期間で実現するために重要である。従来の推薬バランシングの研究では，ある一定期間までの推薬消費量についてバランシングを行い推薬残量の均一化を計っていた．しかし，実際のミッションではフォーメーション再構成や維持，衝突回避，故障を含めたトラブル等により，フォーメーション維持を行う初期時刻で既に衛星間の推薬残量が生じている可能性がある．より長期間でのミッションを実現するにはこの初期推薬残量差も考慮する必要があり，本研究ではその手法と構築と，シミュレーション結果の考察を行った．さらにこの手法のメリットとして，仮想Chief衛星の軌道を柔軟に変化させることで推薬のバランシングを行っているため，追加のマヌーバは必要ない．

Key Words: *Formation Flying, Propellant Balancing, a Virtual Chief, Orbital Perturbation*

1. Introduction

Formation flight is expected to further expand mission possibilities because it enables simultaneous multi-point and long-baseline observations, which were not possible with a single satellite. In Japan, DECIGO and B-DECIGO have been considered, and SILVIA (Space Interferometer Laboratory Voyaging towards Innovative Applications) [1] is being considered as a technology demonstrator to realize them. SILVIA will consist of three satellites in an equilateral triangular formation orbiting at an altitude of 550 km in a low orbit. Previous research has focused on minimizing the amount of propellant consumed and on propellant balancing to keep the amount of propellant consumed constantly over a certain period. This is an important issue not only for SILVIA but also for other FF satellites.

Formation maintenance is the process of maintaining a satellite's formation in its desired shape under disturbances, including perturbations. Previous research studied control laws for formation maintenance [2] and methods for balancing the difference in the amount of control depending on the initial angle of each satellite [3]. However, previous researches have focused on the propellant amount consumed between a certain initial time and a certain end of time. In other words, they did not take into account the propellant remaining amount at the initial time. If there is a difference in the initial amount of propellant remaining, the lifetime of each satellite will vary and the mission period will not be optimal unless this difference is taken into account. In this study, we focus on this problem and propose a new method that takes into account the difference in the initial propellant remaining during formation maintenance. In this method, the virtual chief orbit is changed as the optimal orbit according to the propellant remaining difference and it balances each propellant remaining and extends the mission duration. A unique feature of this method is that no additional maneuvers are required.

2. Formation flight dynamics

2.1. Reference orbit

The HCW (Hill Clohessy Wiltshire) equation is an approximate derivation of the motion between two satellites orbiting each other at a close distance. This equation deals only with two-body problems. In the LVLH (Local Vertical Local Horizontal) coordinate system, when $\boldsymbol{x} := [x, y, z, \dot{x}, \dot{y}, \dot{z}]$, the relative motion of the deputy is defined by

$$\dot{\boldsymbol{x}}_{\boldsymbol{HCW}} = \boldsymbol{A}_{\boldsymbol{HCW}}\boldsymbol{x} \tag{2.1}$$

$$\boldsymbol{A}_{\boldsymbol{HCW}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 3n^2 & 0 & 0 & 0 & 2n & 0 \\ 0 & 0 & 0 & -2n & 0 & 0 \\ 0 & 0 & -n^2 & 0 & 0 & 0 \end{bmatrix} \tag{2.2}$$

This equation also has a general solution (HCW solution)
when we use the state transition matrix e^{At}. The solution is described by

$$\boldsymbol{x}_{HCW}(\mathrm{t}) = \mathrm{e}^{\mathbf{At}}\boldsymbol{x}_{HCW}(0) \tag{2.3}$$

$$e^{\boldsymbol{At}} = \begin{bmatrix} 4-3c_{nt} & 0 & 0 & \dfrac{s}{n} & \dfrac{2(1-c_{nt})}{n} & 0 \\ 6(s_{nt}-nt) & 1 & 0 & -\dfrac{2(1-c_{nt})}{n} & \dfrac{4s_{nt}-3nt}{n} & 0 \\ 0 & 0 & c_{nt} & 0 & 0 & \dfrac{s_{nt}}{n} \\ 3ns_{nt} & 0 & 0 & c_{nt} & 2s_{nt} & 0 \\ -6n(1-c_{nt}) & 0 & 0 & -2s_{nt} & -3+4c_{nt} & 0 \\ 0 & 0 & -ns_{nt} & 0 & 0 & c_{nt} \end{bmatrix} \tag{2.4}$$

and $c_{nt} = \cos nt$, $s_{nt} = \sin nt$. However, we add the constraint $\dot{y}(0) = -2nx(0)$ to reduce the time evolution term and make equation periodic function, the relative motion is written as

$$\begin{cases} x(t) = \rho_x \sin(nt + \alpha_x) \\ y(t) = \rho_y + 2\rho_x \cos(\mathrm{nt} + \alpha_\mathrm{x}) \\ z(t) = \rho_z \sin(nt + \alpha_z) \end{cases} \tag{2.5}$$

where ρ_x, ρ_y, ρ_z, α_x *and* α_z are parameters that can be designed depending on how the initial condition $\boldsymbol{x}(0)$ is taken. Furthermore, you can get the GCO orbit centered at the origin can be obtained by designing $\boldsymbol{x}(0)$ as $\rho_z = \sqrt{3}\rho_x$, $\rho_y = 0$ and $\alpha_x = \alpha_z$.

2.2. $\boldsymbol{J_2}$ Perturbation

When a satellite orbits the Earth, it is subject to various perturbations and disturbances that cause it to take a different orbit from the reference orbit. In the case of SILVIA, which is considered as JAXA's future mission, it orbits attitude 550 km. At this attitude, the $\boldsymbol{J_2}$ perturbation is more than 10^2 times larger than the other perturbation sources[4]. $\boldsymbol{J_2}$ is the second-order term of the gravitational potential due to the flatness of the Earth. Therefore, we consider only $\boldsymbol{J_2}$ perturbation in this study. In addition, we use the relative equation [5] that takes into account all the secular, long-period, and short-period terms since we are considering an FF that requires precise relative position control. The relative motion equation is expressed as follows

$$\dot{\boldsymbol{x}}_{J_2} = \boldsymbol{A}_{J_2}\boldsymbol{x} \tag{2.6}$$

$$\boldsymbol{A}_{J_2} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & 0 & 2\omega_z & 0 \\ a_{51} & a_{52} & a_{53} & -2\omega_z & 0 & 2\omega_x \\ a_{61} & a_{62} & a_{63} & 0 & -2\omega_x & 0 \end{bmatrix} \tag{2.7}$$

where the following substitutions are used for clarity

$$a_{41} = \omega_z^2 + \frac{2\mu}{r_0^3} + \varUpsilon(1 - 3\sin^2\overline{\iota_0}\sin^2\overline{u_0})$$

$$a_{42} = \dot{\omega}_z + \varUpsilon(\sin^2\overline{\iota_0}\sin 2\overline{u_0})$$

$$a_{43} = -\omega_x\omega_z + \varUpsilon(\sin 2\overline{\iota_0}\sin\overline{u_0})$$

$$a_{51} = -\dot{\omega}_z + \varUpsilon(sin^2\overline{\iota_0}\sin 2\overline{u_0})$$

$$a_{52} = \omega_x^2 + \omega_z^2 - \frac{\mu}{r_0^3} + \varUpsilon\left[-\frac{1}{4} + \sin^2\overline{\iota_0}\left(\frac{7}{4}\sin^2\overline{u_0} - \frac{1}{2}\right)\right]$$

$$a_{53} = \dot{\omega}_x + \varUpsilon\left(-\frac{1}{4}\sin 2\overline{\iota_0}\cos\overline{u_0}\right)$$

$$a_{61} = -\omega_x\omega_z + \varUpsilon(\sin 2\overline{\iota_0}\sin\overline{u_0}) \tag{2.8}$$

$$a_{62} = -\dot{\omega}_x + \Upsilon\left(-\frac{1}{4}\sin 2\,\bar{\iota}_0 \cos \overline{u_0}\right)$$
$$a_{63} = \omega_x^2 - \frac{\mu}{r_0^3} + \Upsilon\left[-\frac{3}{4} + \sin^2 \bar{\iota}_0 \left(\frac{5}{4}\sin^2 \overline{u_0} + \frac{1}{2}\right)\right]$$
$$\omega_x = 2\dot{\bar{\Omega}}_0 \sin \bar{\iota}_0 \sin \overline{u_0}$$
$$\omega_y = 0$$
$$\omega_z = \dot{\bar{\Omega}}_0 \cos \bar{\iota}_0 + \dot{\overline{u_0}} + \frac{1}{4} J n_0 \cos 2\,\overline{u_0} \sin^2 \bar{\iota}_0$$
$$\dot{\omega}_x = 2\dot{\bar{\Omega}}_0 \dot{\bar{u}}_0 \sin \bar{\iota}_0 \cos \overline{u_0}$$
$$\dot{\omega}_y = 0$$
$$\dot{\omega}_z = -\frac{1}{2} J n_0 \dot{\bar{u}}_0 \sin 2\,\overline{u_0} \sin^2 \bar{\iota}_0$$
$$\overline{u_0} = \overline{u_0}(0) + \dot{\bar{u}}_0 t$$
$$\dot{\bar{u}}_0 = n_0\left[1 - \frac{3}{2} J (1 - 4\cos^2 \bar{\iota}_0)\right] \tag{2.9}$$
$$\dot{\bar{\Omega}}_0 = -\frac{3}{2} J n_0 \cos \bar{\iota}_0$$
$$r_0 = \overline{a_0}\left[1 + J\left\{\frac{3}{4}(1 - 3\cos^2 \bar{\iota}_0) + \frac{1}{4} sin^2\, \bar{\iota}_0 \cos 2\,\overline{u_0}\right\}\right]$$
$$\Upsilon = 6J_2 \frac{\mu r_e^2}{r_0^5}$$
$$J = J_2 \frac{r_e^2}{\overline{a_0^2} \eta^4}$$
$$\eta = \sqrt{1 - e^2}$$

Where t is the only variable. We can calculate the orbital propagation around the earth by varying the argument of latitude $\bar{u}$.

Furthermore, the relative acceleration error can be reduced by generating a reference trajectory that takes into account the perturbation of the $\boldsymbol{J_2}$ perturbation. This can be done by replacing the mean motion *n* in the HCW equation and HCW solution with the following angular velocity of latitude velocity $\dot{u_0}$, and the phase *nt* with the argument of latitude $u_0$respectively.

$$\begin{aligned} u_0 &= u_0(0) + \dot{\bar{u}}_0 t + \frac{J}{8}(1 - 7\cos^2 \bar{\iota}_0) \sin 2\overline{u_0} \\ \dot{u_0} &= \overline{u_0}\left[1 + \frac{J}{4}(1 - 7\cos^2 \bar{\iota}_0)\cos 2\overline{u_0}\right] \end{aligned} \tag{2.10}$$

2.3. Control amount for formation maintenance

To fly along the reference trajectory (HCW equation), we can compensate for the acceleration component for continuous control. The equation described by

$$\dot{\boldsymbol{x}} = \boldsymbol{A_{J_2}} \boldsymbol{x} + \boldsymbol{B} \boldsymbol{u_{ff}} = \boldsymbol{A_{HCW}} \boldsymbol{x} \tag{2.11}$$

where $\boldsymbol{A_{J_2}}$ is (2.6), $\boldsymbol{A_{HCW}}$ is (2.1), $\boldsymbol{u_{ff}}$ is feedforward control term. From this equation, it can be confirmed that the acceleration is proportional to the x that is the distance from the origin. However, this control amount is changed by the initial angle of formation. The initial angle is defined in Fig 1.

3. The proposed propellant balancing method

3.1. Overview of the method

In this study, we consider an FF where three satellites orbit GCO in an equilateral triangle, such as the JAXA future mission SILVIA. The amount of control required to maintain the formation in each satellite is proportional to the distance from the origin (the virtual Chief) of the relative coordinate system LVLH. Therefore, if the origin is moved from the conventional GCO orbit shown by the dotted line in Fig. 2 as shown by the red arrow, the GCO radius of each satellite changes and the control amount of each satellite changes. This means that the orbit of the virtual chief changes the control amount of each satellite. Our balancing method is performed by using the two GCO orbits shown in the solid lines as reference orbits for changing the control amount. The GCO radius of SC1 is ρ_{in}, and the GCO radius of SC2 is ρ_{out}. At this time, we assume that the initial amount of remaining propellant at SC1 is lower than that at the other two satellites, in the order SC1<SC2<SC3.

In the proposed propellant balancing method, the reference orbit for each satellite can be described as

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} \frac{\rho_{in}}{2}\sin(u_0 + \alpha_0) \\ \rho_{in}\cos(u_0 + \alpha_0) \\ \frac{\sqrt{3}\rho_{in}}{2}\sin(u_0 + \alpha_0) \end{bmatrix} \tag{3.1}$$

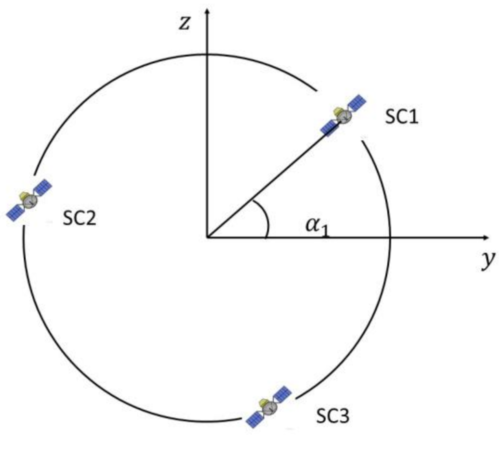

Fig.1 Initial angle

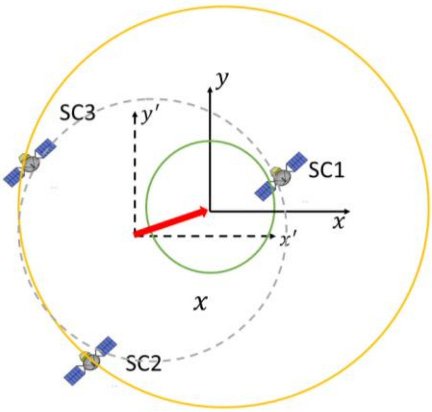

Fig.2 Overview of the balancing method

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} \frac{\rho_{out}}{2}\sin(u_0+\alpha_0+\theta_2) \\ \rho_{out}\cos(u_0+\alpha_0+\theta_2) \\ \frac{\sqrt{3}\rho_{out}}{2}\sin(u_0+\alpha_0+\theta_2) \end{bmatrix} \tag{3.2}$$

$$\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} = \begin{bmatrix} \frac{\rho_{out}}{2}\sin(u_0+\alpha_0+\theta_3) \\ \rho_{out}\cos(u_0+\alpha_0+\theta_3) \\ \frac{\sqrt{3}\rho_{out}}{2}\sin(u_0+\alpha_0+\theta_3) \end{bmatrix} \tag{3.3}$$

where the following substitutions are used for clarity

$$\theta_2 = \pi - \tan^{-1}\left(\frac{B}{\sqrt{3}B - 2\rho_{in}}\right) \tag{3.4}$$

$$\theta_3 = \pi + \tan^{-1}\left(\frac{B}{\sqrt{3}B - 2\rho_{in}}\right) \tag{3.5}$$

$$\rho_{out} = \sqrt{B^2 - \sqrt{3}B\rho_{in} + \rho_{in}^2} \tag{3.6}$$

3.2. The formulation as an optimization problem

Our propellant balancing method optimizes the virtual chief orbit since the GCO radius for each satellite changes. Assuming that there is some initial propellant remaining, the equation relating propellant remaining and time is defined as

$$V_i - \frac{\Delta V_{Ti}}{T} t_{fi} = 0 \tag{3.7}$$

$$\Delta V_{Ti} = \int_0^T \boldsymbol{u_{ff}}\, dt \tag{3.8}$$

where t_{fi} is the time when the remaining propellant for each satellite is zero, V_i is initial propellant remaining, T is a period, and ΔV_{Ti} is control amount per one period.

Next, we formulate the problem as an optimization problem. Since the objective of this optimization is to extend the mission duration, it is sufficient to maximize the lifetime of a satellite which propellant remaining reaches zero at the first among all satellites. It means the optimization is required

$$\text{maximize: minimum} \quad t_{fi}$$

the objective function and constraints are described by

$$\begin{aligned} &\text{minimize} \quad -z \\ &\text{subject to} \quad z \le t_{fi} \\ &\rho_{out} = \sqrt{\mathrm{B}^2 - \sqrt{3}\mathrm{B}\rho_{in} + \rho_{in}^2} \\ &0 \le \rho_{in} \end{aligned} \tag{3.9}$$

In this optimization, the design variable is only ρ_{in}.

3.3. Numerical Results for the Formulated Optimization Problem

The simulation was performed assuming that one of the satellites was in trouble and the propellant remaining was significantly reduced. The reference orbit of a virtual chief is baseline length of 100 m, altitude is 550 km, mean orbital inclination is 97.59 deg, eccentricity is 0, and initial angle of SC1 is 0 deg. The initial remaining propellant for each satellite was set to SC1 = 1 m/s, SC2 = 10 m/s, and SC3 = 11 m/s in terms of ΔV.

Numerical calculations showed that the optimal solution was $\rho_{in} = 9.04$ m and $\rho_{out} = 92.3$ m. The change in the lifetime of each satellite is shown in Table 1, and the decrease in the propellant remaining is shown in Fig. 3. The results show that the lifetime of the satellite increased 6.4 times when the formation maintenance was constantly performed. It indicates that the mission can be performed for a longer period than before the proposed method was applied.

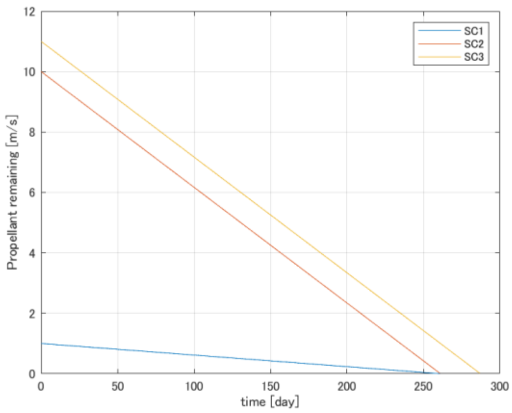

Fig. 3 Propellant remaining decrease of each satellite

Table.1 Numerical results of optimization problem

satellite	Conventional method[day]	Proposed method [day]
SC1	40.9	261.1
SC2	421.7	261.1
SC3	463.8	287.2

3.4. Derivation of analytical solution of the optimization problem

As shown in Fig. 3, it can be confirmed that the optimal solution is the intersection of SC1 and SC2 at the time when their propellant remaining reaches zero. The reason for this is that the control amount required for each satellite changes by the GCO radius and the slope of each line in Fig. 3. Since our method uses this change in slope to search for the optimal solution, it is thought that the intersection of the two satellites with the least initial propellant remaining converged to the optimal solution. In other words, we can derivate an analytical solution of the GCO radius at this point.

To obtain the analytical solution, we note that the amount of control required to compensate for the J_2 perturbation is proportional to the distance from the GCO origin. First, the equation relating the propellant remaining to the end time t_{fi} is described by

$$t_{fi} = \frac{V_i\,T}{\Delta V_{Ti}} \tag{3.10}$$

The ΔV_{Ti} cannot be solved analytically at this time because it varies depending on an initial angle as shown in Fig.4. To obtain the analytical solution ΔV_{Ti} approximated using the GCO radius ρ_B at the baseline length B and the control amount ΔV_{TBi} required for one revolution. ΔV_{TBi} is changed by the initial angle so we approximate it by taking the average value. In this case, the ΔV_{TBi} of each satellite is described by

$$\Delta V_{TB1} = \Delta V_{TB2} = \Delta V_{TB3} = \frac{1}{2\pi}\int_0^{2\pi}\int_0^{T} u_{ff}\Big|_{\rho_{in},\rho_{out}=\rho_B} dt\,d\alpha \tag{3.11}$$

Using ΔV_{TBi}, the equation relating the propellant remaining to the end time t_{fi} can be transformed as follows

$$t_{fi} = \frac{V_i\,T\,\rho_B}{\Delta V_{TBi}\,\rho_{in}(or\ \rho_{out})} \tag{3.12}$$

Since the optimal solution is the intersection of SC1 and SC2, the equation of intersection is described by

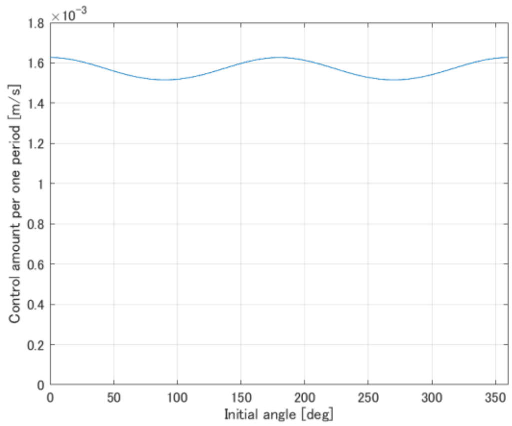

Fig.4 Control amount per one period as a function of the initial angle

$$t_{f1} = t_{f2} \tag{3.13}$$

$$\frac{V_1 T \rho_B}{\Delta V_{TB1}\rho_{in}} = \frac{V_2 T \rho_B}{\Delta V_{TB2}\rho_{out}} \tag{3.14}$$

The analytical solution can be obtained by solving this for ρ_{in}.

$$\rho_{in} = \frac{B\left(\frac{V_1}{V_2}\right)\cdot\left\{\sqrt{3}\left(\frac{V_1}{V_2}\right) - \sqrt{4-\left(\frac{V_1}{V_2}\right)^2}\right\}}{2\left\{\left(\frac{V_1}{V_2}\right)^2 - 1\right\}}, \quad (V_1 < V_2)$$

$$\rho_{in} = \frac{B}{\sqrt{3}}, (V_1 = V_2) \tag{3.15}$$

3.5. Evaluation of approximation error of analytical solution

In order to evaluate the error due to the approximation of the analytical solution, we will clarify the error of the analytical solution against the optimum value obtained from the numerical results. Fig.5 shows the error when the initial angle is variable. The maximum error in absolute value is about 3.7 %. It was confirmed that the error was sufficiently small even if the initial angle was changed. In other words, the analytical solution obtained in this study has a sufficiently small error compared to the optimal solution by numerical calculation. Therefore, the analytical solution is practical enough.

4. Conclusion

In this study, we proposed a propellant balancing method that takes into account the initial propellant remaining in formation maintenance, and we were able to extend the mission duration. We also derived an analytical solution to the optimization problem and

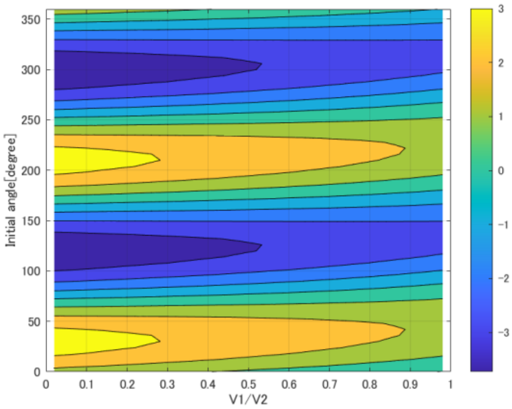

Fig. 5 Approximation error as functions of initial angle and V1/V2

compared it with numerical results, which showed that the error was sufficiently small.

In the future, we plan to study a method to eliminate the restriction imposed by the proposed propellant balancing method that one propellant remaining is lower than the others, and to perform balancing with three apparent GCO orbits. We are also considering analyzing the effects of changing the virtual chief orbit and considering perturbations and disturbances other than the $\boldsymbol{J_2}$ perturbation.

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