Study on Control System of Optical Aberration in Space Telescope by Mechanical Vibrations

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Abstract: Space telescopes are required to be larger and lighter. Therefore, space telescopes are subjected to mechanical vibrations that are caused by refrigerators and so on, which may affect the observation accuracy. In addition, it is not desirable to use multiple sensors because of thermal noise of the circuits for sensing system and reduction of the reliability due to complex system. This paper proposes a method to reduce the aberration using an image sensor instead of sensors to measure the deformation and vibration. Furthermore, since the proposed method requires a model of the space telescope, the modeling error is reduced by using an Unscented Kalman Filter. The effectiveness of the proposed method is confirmed by numerical analysis.

Key Words: Space Telescope, Ray Tracing, Finite Element Method, Control

宇宙望遠鏡における機械的振動に起因する光学収差の制御に関する研究

摘要: 宇宙望遠鏡には大型化と軽量化が要求される．そのため，宇宙望遠鏡は冷却機などに起因する機械的振動の影響を受け，観測精度が低下する可能性がある．一方，宇宙望遠鏡に複数のセンサを搭載することは，センサの電気回路から発生する熱雑音の影響や，システムの複雑化による信頼性の低下を招くため，好ましくない．そこで本研究では，振動や変形を計測するセンサを用いる代わりに撮像素子上の像の変化から振動を計測し，制御することで観測精度の低下を抑える手法を提案する．さらに，提案手法では宇宙望遠鏡の数学モデルを使用することから，数学モデルのモデル化誤差をアンセンティドカルマンフィルタによって低減することを提案する．提案手法の有効性を数値解析によって評価することを本研究の目的とする．

1. Introduction

In recent years, telescopes are increasing in size due to advancement of observation missions in space development. This is because the telescopes can collect more light when the primary mirror is large. In addition, space telescopes that are not affected by Earth's atmosphere are preferred to realize advanced observation missions. Therefore, space telescopes are required to be lightweight due to the limitation of the rocket payload. Space telescopes may have structures that deploy in space or flexible structures in order to achieve lager size and lighter weight. Hence, the structures of space telescopes are considered to be affected easily by mechanical vibration. The observation accuracy of the space telescopes is considered to be affected by vibration because the space telescopes have causes of vibration such as refrigerators, control systems, solar radiation pressure, etc. In fact Hubble Space Telescope's pointing accuracy deteriorated due to the vibration of the solar array(1). In addition to this, high-precision control is required because the diffraction limit becomes smaller due to the larger size of the primary mirror. In this study, the observation accuracy is improved by correcting the change in the position relationship between the primary and secondary mirrors due to mechanical vibration.

In a previous study, potion sensors are used to detect changes in the positions of the primary and secondary mirrors due to thermal deformation, etc., and the position of the secondary mirror is controlled to reduce aberrations(2). However, there is concern reduction of the system's reliability because multiple sensors are required to realize the control system. In addition, using multiple sensors is not desirable because it is necessary to reduce the thermal noise caused by power supplied to the sensors when performing observation in the infrared region. In a previous study, the effect of mechanical vibration on the primary mirror is analyzed using multibody dynamics and ray tracing(3). In that study, the effect of vibration on the space telescopes is analyzed. However, the control in the study is only for a lens that oscillates in one axis direction, and there is not enough study on feedback control for aberrations caused by mechanical vibration in space telescopes.

In this study, the relationship between the vibration and the aberration is clarified by numerically analyzing the mechanical vibration generated in the structure of a space telescopes. Based on the result, this paper proposes to estimate the vibration of the telescope from the change of the image on the image sensor and to control in order to reduce the aberration without using sensors to measure the deformation and vibration. The purpose of this study is to verify the effectiveness of the proposal method.

2. Telescope

2.1. Selecting a telescope

Space telescopes are generally reflecting telescopes. Reflecting telescopes include the Newtonian, Gregorian, and Cassegrain types. The Cassegrain telescope is generally used as space telescopes because it can be small and observe from behind the primary mirror(4). Therefore, this study chooses the Cassegrain telescope. The mechanism of the Cassegrain telescope is shown in Figure 1. The red lines show the optical path. The Cassegrain telescope uses a parabolic concave primary mirror and a hyperbolic convex secondary mirror to avoid spherical aberration.

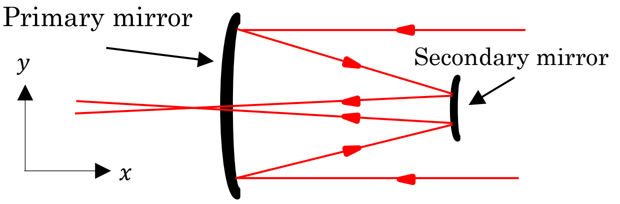

Fig. 1. Cassegrain telescope.

2.2. Calculation of mirror surface

To design surface of mirror, the function about surface x is expressed as

$$x = \frac{cy^2}{1 + \sqrt{1 - (1 + K)c^2y^2}} \quad (1)$$

where K is Conic constant and c is curvature(5). Even-order terms appear in the case of an aspheric surface, but this study does not consider them for simplicity. In case of the Cassegrain telescopes, the Conic constant, the curvature, the distance between primary and secondary mirror, and the diameter of the secondary mirror are determined by the diameter and the focus length of the primary mirror, the total focus length of the telescope, and the distance between the primary mirror and the focus of the telescope (Back distance). Table 1 shows the parameters determined with reference to the James Webb Space Telescope (JWST), which is scheduled to be launched in the future.

Table 1. Parameters of Cassegrain telescope.

Parameter	Value	Unit
Diameter of primary mirror	6.5	m
Focal length of primary mirror	8	m
Total focal length	131.4	m
Back distance	0.5	m
Conic constant of primary mirror	-1	–
Conic constant of secondary mirror	-1.28	–
Curvature of primary mirror	0.0625	1/m
Curvature of secondary mirror	0.963	1/m

2.3. Ray tracing

Ray tracing is used to simulate the optical system of a space telescope. Ray tracing is the simulation of the propagation of light rays according to physical laws. There are two types of ray tracing. Geometric optics treats lights as lines and wave optics treats light as waves. This study focuses on only geometric optics for simplicity. In ray tracing with geometric optics, the first step is to derive the intersection point of the ray vector and the mirror or lens. Next, the angle of incidence, which is the angle between the normal vector of the surface at the intersection point and the ray vector, is calculated. Finally, the angle of reflection equal to the angle of incidence is find and the angle of refraction is calculated using Snell's law.

3. Analysis of aberrations caused by changes in the positional relationship between the primary and secondary mirrors

Since the flexible structures to which the secondary mirror is connected are easy to vibrate, the secondary mirror is considered to be affected by the vibration. Therefore, the dynamic aberration seems to be dominated by the effect of the vibration of the secondary mirror. This study numerical analyzed the change of the image on the image sensor when the position of the secondary mirror moved. As a result of numerical analysis, Figure 2 shows the distribution of the light on the image sensor when the secondary mirror is moved 0.5 mm in the x direction, which is perpendicular to the mirror surface of the primary mirror. The graph legends show the position of the light source when the center of the primary mirror is the origin. The light that should be gathered at the origin as the focal point is spread out in the $\pm y$ direction. This is because the focal point moves in the x direction. Next, Figure 3 shows the distribution of the light on the image sensor when the secondary mirror is moved 0.5 mm in the y direction, which is parallel to the mirror surface of the primary mirror. The focal point moves in the $-y$ direction. Also, the focal point is blurred. Finally, Figure 4 shows the distribution of the light on the image sensor when the secondary mirror is rotated 0.5 mrad in the $-\theta$ direction, which is counterclockwise rotation. The focal point moves in the $-y$ direction. Also, the focal point is blurred. These result show that the changes in the image on the image sensor when the secondary mirror is moved in the y and rotational directions have a similar trend.

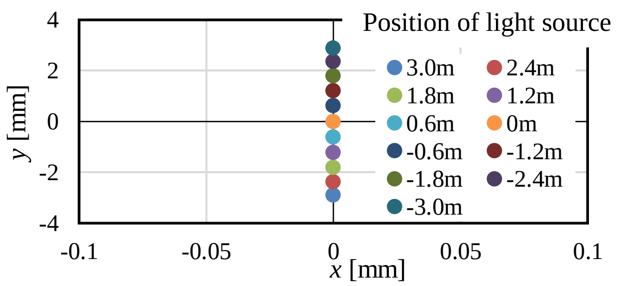

Fig. 2. Change of image by moving secondary mirror (x).

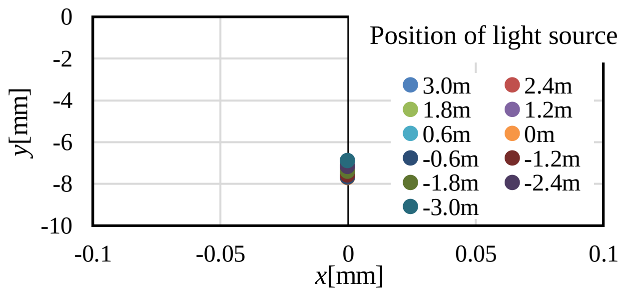

Fig. 3. Change of image by moving secondary mirror (y).

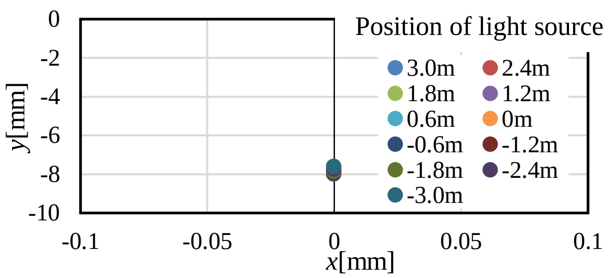

Fig. 4. Change of image by moving secondary mirror (θ).

4. Proposed method

This paper proposes a method to reduce the aberration using an image sensor instead of sensors to measure the deformation and vibration. However, since the image on the image sensor is analyzed numerically by ray tracing, the output equation is nonlinear. It is difficult to control a multi-output system in some cases of optical systems. Hence, separating the direction of the vibration in the telescope using the output is required. However, it is difficult to determine whether the vibration of the image is caused by the secondary mirror in the y direction or the rotation direction. Therefore, this study proposes to use mode shapes of the telescope and FFT analysis. First, modes included the vibration and those amplitude is obtained using FFT analysis to vibration data generated in the image on the image sensor. In addition, the ratio of the secondary mirror's mode shapes in the y direction and the rotation direction using mathematical model of the telescope. Actual vibration of the telescope can be estimated from vibration of image by using the information.

5. Modeling of the telescope and deriving the equation of the whole system

5.1. Model using the finite element method

This study makes a mathematical model of the space telescope to realize the proposed method. A telescope that is same scale as JWST is introduced as a modeling target. It consists of a primary mirror, a secondary mirror, and structures connecting the primary mirror and the secondary mirror like JWST. The structure is modeled using the finite element method (FEM). Elements of the model are assumed as linear beam elements that are deformable in bending and axial directions. The whole using FEM is shown in Figure 5. $\mathbf{q}_i$ that are coordinates of the i-th node and generalize coordinates $\mathbf{q}_e$ are shown in Equation (2) and (3). n is number of nodes.

$$\mathbf{q}_i = [u_i \quad v_i \quad \theta_i]^T \tag{2}$$

$$\mathbf{q}_e = [\mathbf{q}_1{}^T \quad \mathbf{q}_2{}^T \quad \cdots \quad \mathbf{q}_{n-1}{}^T \quad \mathbf{q}_n{}^T]^T \tag{3}$$

The mirrors that are assumed to be a rigid body are fixed at the central node of the primary and secondary mirror section in the FEM model. The mirrors fixed the central nodes are assumed to move with the movement of the nodes.

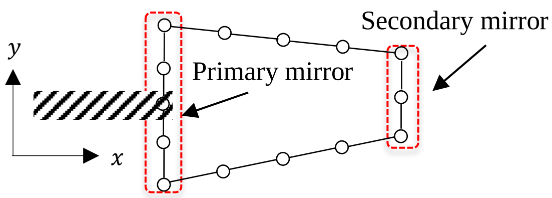

Fig. 5. Model of space telescope.

5.2. Combining a piezoelectric actuator with the FEM model

In this study, the position relationship between the primary and secondary mirrors is modified using reduce aberrations using actuators installed between the secondary mirror and the structure of the telescope. Piezoelectric actuators are selected for high responsiveness and accuracy. The piezoelectric actuator is assumed to be a single degree of freedom system to combine the actuator with the FEM model. The actuator is assumed that a mass M is attached to a spring of area A and length L that simulates an electrostriction element. Then, the response of the displacement u of the piezoelectric actuator to the electric field E is considered. Assuming the linearity of the generated force $AcdE$ with the application of the electric field E[6], the equation of motion is expressed as Equation (4). c is the electric stiffness, d_m is piezoelectric constant, and ζ is damping coefficient.

$$M\frac{d^2u}{dt^2} + \zeta\frac{du}{dt} + \frac{Ac}{L}u = Acd_mE \tag{4}$$

When the displacement of the piezoelectric actuators in the x, y, and rotational directions are denoted as x_a, y_a, and θ_a, the generalized coordinates are expressed as Equation (5).

$$\mathbf{q} = [\mathbf{q}_e \quad x_a \quad y_a \quad \theta_a]^T \tag{5}$$

The equation of the whole system including the FEM model and piezoelectric actuator can be expressed as Equation (6), where the mass matrix is $\mathbf{M}$, the damping matrix is $\mathbf{C}$, the stiffness matrix is $\mathbf{K}$, and the external force term is $\mathbf{F}$.

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F} \tag{6}$$

It is possible to perform numerical analysis that combines optical and mechanical systems by combining the equation and ray tracing.

6. Parameter estimation using Unscented Kalman Filter

This study proposes a method to estimate the vibration generated in the space telescope by using the mode shapes. However, since the mode shapes are determined by the model, the estimation accuracy may decrease if there is an error between the actual telescope and the model. This study proposes to use parameter estimation with the Unscented Kalman Filter (UKF) to correct the errors in the model[7]. Important parameters for estimating the vibration of the telescope from the image on the image sensor are the mode shapes of the model using FEM. So, this study estimates the y direction displacement in the mode shapes of the secondary mirror.

The physical coordinate $\mathbf{q}$ is expressed as Equation (7) when the eigenmode vector $\boldsymbol{\phi}$ of the Equation (6) and modal coordinate $\boldsymbol{\eta}$ are used[8].

$$\mathbf{q} = \boldsymbol{\phi}\boldsymbol{\eta} \tag{7}$$

The parameters to be estimated are contained in $\boldsymbol{\phi}$. The equation of the whole system using the modal coordinates is expressed as Equation (8).

$$\boldsymbol{\phi}^T\mathbf{M}\boldsymbol{\phi}\ddot{\boldsymbol{\eta}} + \boldsymbol{\phi}^T\mathbf{K}\boldsymbol{\phi}\boldsymbol{\eta} = \boldsymbol{\phi}^T\mathbf{F} \tag{8}$$

Mass and stiffness of each mode can be obtained using Equation (8). This study considers only the primary mode for simplicity. The expanded system with a new state variable containing the unknown parameter to be estimate is shown in Equation (9). The input is assumed to be zero.

$$\frac{d}{dt}\begin{bmatrix}\eta_1\\ \dot{\eta}_1\\ y_1\end{bmatrix} = \begin{bmatrix}\dot{\eta}_1\\ (\alpha {y_1}^2 + \beta y_1 + \gamma)\eta_1\\ 0\end{bmatrix} \qquad (9)$$

η_1 is the modal coordinate of the primary mode, y_1 is the unknown parameter. α, β, and γ are constants.

As a preliminary numerical analysis, a number in the eigenmode vectors that correspond to the displacement of the secondary mirror in the y direction in the primary mode is estimated. The eigenmode vector with errors is assumed to be the eigenmode vector when the stiffness of structures connecting the primary and the secondary mirror of the model are twice. The parameter estimation uses the data of the image change on the image sensor. The result of the parameter estimation is shown in Table 2. The ratio of the mode shapes of the secondary mirror in the y and rotation directions for the primary mode before parameter estimation is 11.5. This has an error of 26.8 % compared to 15.7 that is actual value. However, 11.5 is corrected to 15.5 and the error with the actual value is reduced to 1.27% by performing parameter estimation. Therefore, there is a possibility that the modeling error can be reduced by applying oarameter estimation with UKF.

Table 2. Results of parameter estimation.

Parameter	Value
Actual Value	15.7
Value with Error	11.5
Difference without UKF	-26.8 %
Estimated Value by UKF	15.5
Difference with UKF	-1.27%

7. Validation of the proposed method by numerically analysis

7.1. Validation of the proposed method using FFR analysis and mode shapes

In order to verify the proposed method, vibration is generated on the telescope in simulation. The vibration is estimated by the displacement of the image on the image sensor and the PD control is performed. The vibration is excited by applying a force of 2N in the y direction of the secondary mirror for 0.01s. The integration method is Newmark-beta method and numerical damping is added to reduce the integration error. In addition, Hamming window is applied to the data for FFT analysis to improve the accuracy of the FFT analysis. Table 3 shows the parameters of the PD control. Figure 6 shows the predicted and actual value of the vibration in y direction of secondary mirror, Figure 7 shows the predicted and actual value of the vibration in the rotation direction, and Figure 8 shows the vibration of the image on the image sensor when the PD control is applied. The PD control and displacement estimation are started after 1s in the numerical analysis for 3s because it is necessary to obtain some data for the FFT analysis. The analysis results show that the proposed method is able to separate the vibration in the y and rotation direction from the vibration of the image on the image sensor. In addition, 97.6% of the vibration on the image can be reduced by PD control. These results show the effectiveness of the proposed method.

Table 3. Parameters of PD control.

Parameter	Value
Proportional Gain	6.0×10^7
Derivative Gain	7.0×10^5

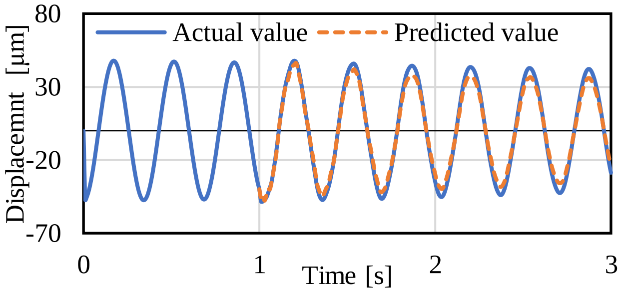

Fig. 6. Predicted and actual value of displacement of secondary mirror (y).

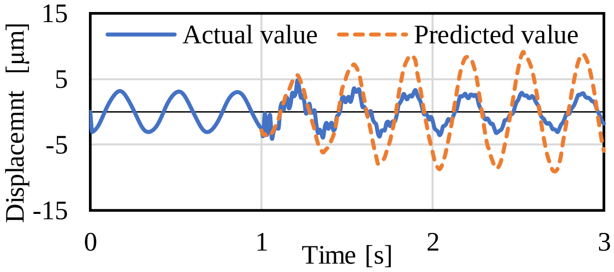

Fig. 7. Predicted and actual value of displacement of secondary mirror (θ).

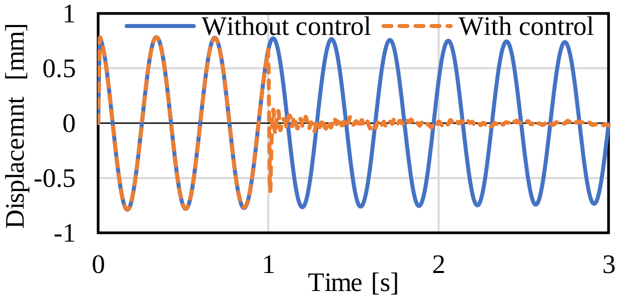

Fig. 8. Displacement of image.

7.2. Validation of parameter estimation by UKF

The effectiveness of parameter estimation by UKF on the proposed method is verified. the performance of the proposed method using FFT analysis and mode shapes and PD control with and without parameter analysis is compared. First, the displacement of the secondary mirror in y and rotation directions are separated using the data before and after the parameter estimation shown in Chapter 6. The separated displacement in the y and rotation directions are shown in Figure 9 and Figure 10. The results of PD control are shown in Figure 11. The separation errors are reduced from 2.99% to 0.893% in the y direction and from47.0% to 17.0% in the rotation direction by using the parameter estimation. Those results show the effectiveness of the parameter estimation for the proposed method. On the other hand, the vibration suppressions are 97.8% before parameter estimation, 97.6% after parameter estimation, and 97.6% in the case of no modeling error. In other words, there is no significant change with or without parameter estimation. The control performance needs to be verified with other control methods as well.

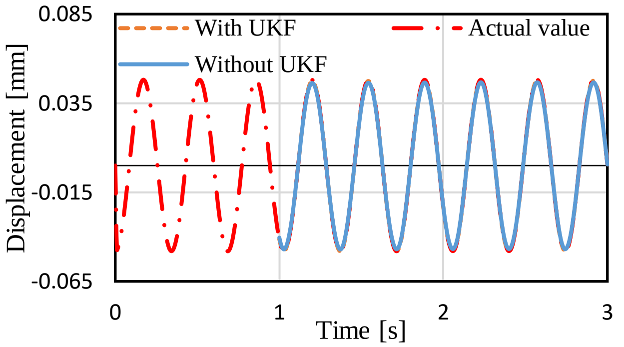

Fig. 9. Predicted and actual value of displacement of secondary mirror (y).

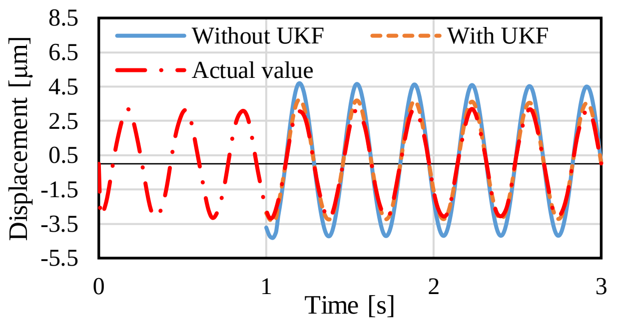

Fig. 10. Predicted and actual value of displacement of secondary mirror (θ).

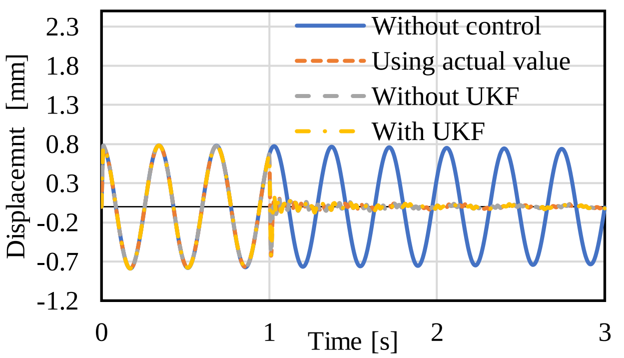

Fig. 11. Displacement of image.

8. Conclusions

In order to reduce the optical aberrations caused by mechanical vibrations in space telescopes without using additional sensors, this study is conducted to estimate and control the vibration of the telescope from the changes in the image on the image sensor. This study proposes a method to estimate the vibration from the image on the image sensor by combining mode shapes and FFT analysis. The results of numerical analysis show the effectiveness of the proposed method. Parameter estimation by UKF is performed to reduce the effect of modeling errors. Future tasks include conducting validation experiments and verifying the possibility of measuring the vibration using image analysis.

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