Integrated Optimization of Guidance Navigation and Control Strategy via Stochastic Trajectory Optimization Approach

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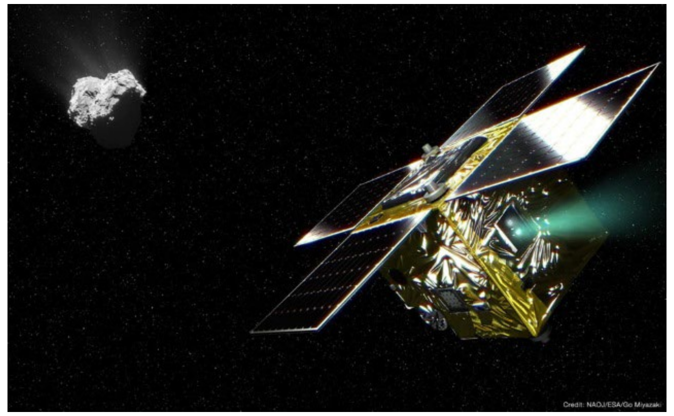
Research Background

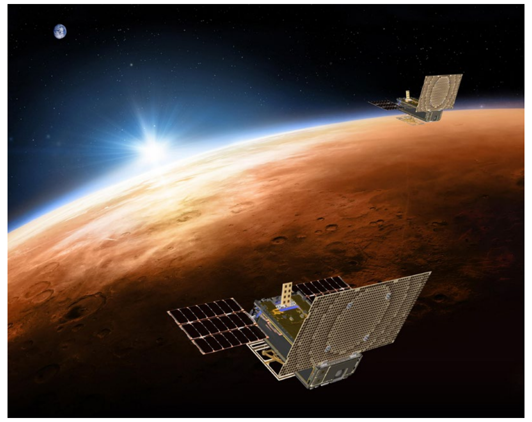
Deep Space Exploration by Small Satellites

- Advances in small satellite technology have led to deep space exploration by small satellites.
  - Propulsion (trajectory control)
  - Communication & Orbit determination

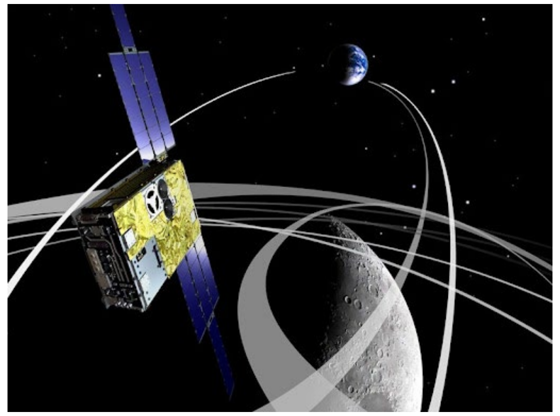
- Small satellites will realize deep space exploration at higher frequency and lower cost.
  - Mass < 100kg (Conventional:~1000kg)
  - Cost ~$10M (Conventional:~$100M)
  - 10 CubeSats will be launched by NASA Artemis-1 (2022)



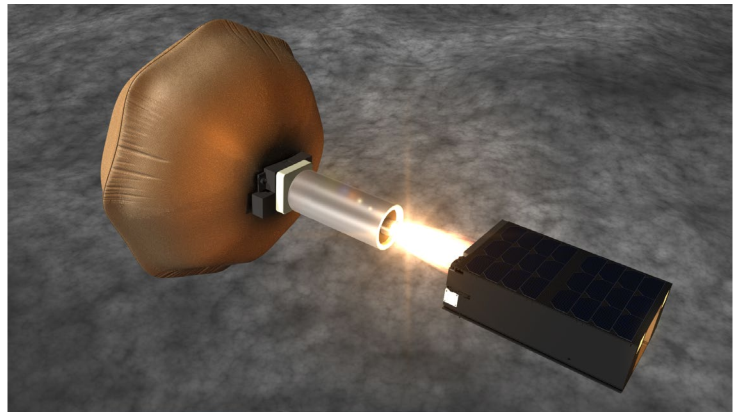
PROCYON (2014, UTokyo/JAXA)
65kg



MarCO (2018, NASA)
14kg



EQUULEUS (2022, UTokyo/JAXA)
10kg



OMOTENASHI (2022, JAXA)
13kg

Research Background

Key Problems of Trajectory Design in Deep Space Exploration by Small Satellites

- Low trajectory control capability
  - Nominal trajectory is designed as ΔV for nominal control is small and time of flight is long.
  - **ΔV for trajectory correction is** relatively more **important** than conventional missions.

ΔV Budget for EQUULEUS

	LGA 2 (full success)		EML2 arrival (8-month op.)		1 month @ EML2 (extra success)		1 year @ EML2	
Maneuver	DVμ[m/s]	DVσ [m/s]	DVμ[m/s]	DVσ [m/s]	DVμ[m/s]	DVσ [m/s]	DVμ[m/s]	DVσ [m/s]
DV1	10.000	0.333	10.000	0.333	10.000	0.333	10.000	0.333
Gravity Loss during DV1	1.000	0.000	1.000	0.000	1.000	0.000	1.000	0.000
TCM1+CUM1	10.000	3.333	10.000	3.333	10.000	3.333	10.000	3.333
DV2	0.200	0.000	0.200	0.000	0.200	0.000	0.200	0.000
TCM2+CUM2	5.000	1.667	5.000	1.667	5.000	1.667	5.000	1.667
DV3	0.000	0.000	8.400	0.000	8.400	0.000	8.400	0.000
TCM3+CUM3	0.000	0.000	5.000	1.667	5.000	1.667	5.000	1.667
Unloading (per year)	1.101	0.000	3.699	0.000	4.192	0.000	9.699	0.000
Station Keeping (per year)	0.000	0.000	0.000	0.000	1.644	0.000	20.000	0.000
Total DVμ [m/s]		**27.3**		**43.3**		**45.4**		**69.3**
DV μ+σ [m/s]		**31.0**		**47.4**		**49.5**		**73.4**
DV μ+ 3σ [m/s]		**38.5**		**55.6**		**57.7**		**81.6**

- Cost of orbit determination
  - Orbit determination is performed by communication to ground stations using large deep space antennas.
  - **Operation cost is a major burden** for low-cost deep space exploration by small satellites.

Research Questions

1. How can ΔV for trajectory correction maneuvers (TCM) be reduced?

2. When is the optimal operation schedule for efficient orbit determination (OD)?

TCM Formulation

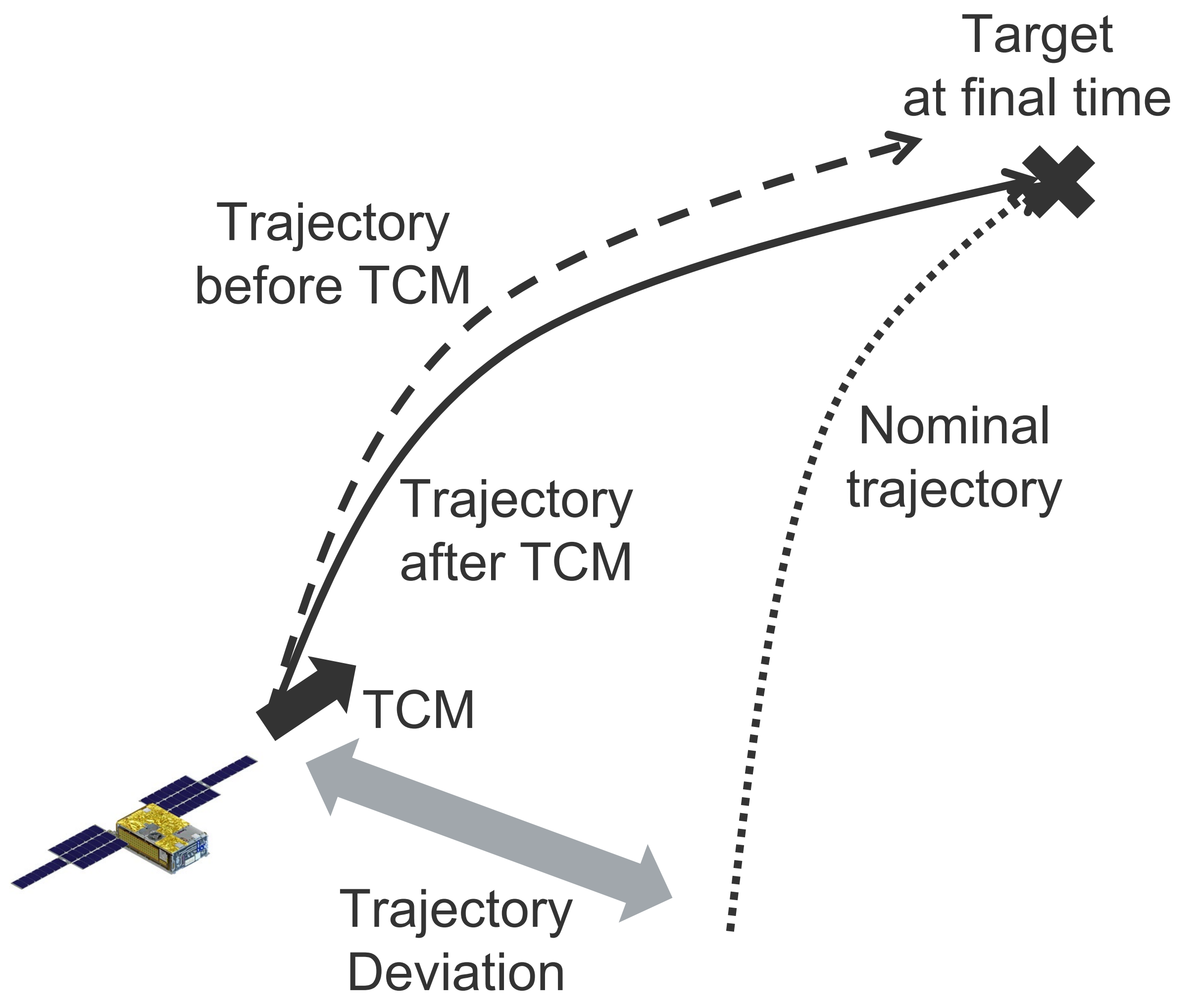
Fixed Time-of-Arrival (FTA) Guidance Formulation

$$\boldsymbol{u}_{k+1} = -\left[\Phi^{-1}_{rv,f,k+1}\Phi_{rr,f,k+1} \quad I \right]\left(-\left(\boldsymbol{x}^{-}_{k+1} - \widehat{\boldsymbol{x}}^{-}_{k+1}\right) + \Phi_{k+1,k}\left(\left(\boldsymbol{x}^{-}_{k} - \widehat{\boldsymbol{x}}^{-}_{k}\right) + B\delta u_k\right) + \delta \boldsymbol{w}_{k+1,k}\right)$$

Sensitivity of Dynamics | Orbit Determination Error at TCM | OD Error at Previous TCM | Control Error of Previous TCM | Dynamics Error

Factors that determine the magnitude of TCM

- Sensitivity of dynamics (depending on TCM timing)
- Orbit determination error at TCM (depending on OD timing)
- Remaining error of previous TCM
  - Propagation of error (depending on TCM interval time)
  - OD error at previous TCM (depending on OD timing)
  - Control error of previous TCM
  - Dynamics Error from previous TCM time (depending on TCM interval time)



→TCM ΔV is determined based on these combined effects.
TCM and OD timing have large effects on TCM ΔV.

Research Objective

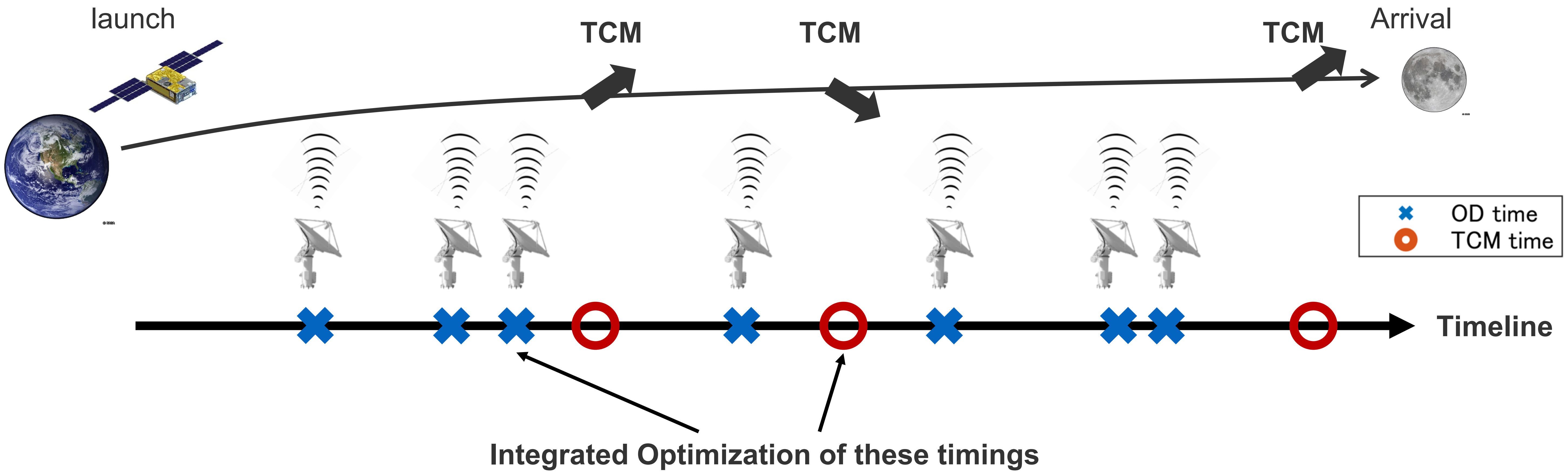
Integrated Optimization of Trajectory Correction and Orbit Determination Scheduling

①Optimization of TCM Timing

- Earlier TCM can reduce ΔV because of sensitivity, but it results worse guidance accuracy.
- ΔV is minimized keeping necessary guidance accuracy.

② Optimization of OD Timing

- OD accuracy is dependent on OD timing, and improvement of OD accuracy can reduce TCM ΔV.
- OD is performed when it can reduce TCM ΔV with the number of OD limited.



Related Works

① Optimization of TCM Timing

- Spacing Rule
  - Breakwell(1962)
  - Kawaguchi and Matsuo(1996)
- Approximation in Spacing Rule (Limitation of the previous method)
  - Dynamics ： Linear
  - OD error ： Fixed
  - Probabilistic Distribution ： Not considered
- Modern stochastic trajectory optimization formulation is adopted in this research to overcome these limitations

② Optimization of OD Timing

- Gentile et al. (2019)
  - Trace of covariance matrix at final time is minimized by optimizing observation numbers, timings, and methods
- No research for integrated optimization of TCM timing and OD timing

Methods

Stochastic Trajectory Optimization

- Trajectory Optimization with considering various uncertainties
  - Initial state errors
  - Dynamics errors
  - Orbit determination errors
  - Control errors
- Various researches were conducted in recent years
  - Greco et al. (2020), Ozaki et al. (2020), Oguri and McMahon (2021)
- Tube Stochastic Optimal Control (TSOC) (Ozaki et al. (2020))
  - The state of trajectory is considered as probabilistic variables, and its distribution is parameterized.
  - The stochastic trajectory optimization problem is converted to the deterministic trajectory optimization problem of the parameter of the distribution, and it is solved by numerical optimization algorithm.
  - **Orbit determination errors are not considered** in the previous research.

Proposed Method Summary

- The problem of minimizing TCM ΔV with fixed numbers of OD is formulated via stochastic trajectory optimization.
- Augmented state with true state and estimated state is considered to handle orbit determination error.
- The problem is converted to deterministic optimization problem by parameterization of probabilistic distribution and solved numerically.

$$\begin{aligned}
\underset{t_k \in \mathcal{T}_k,\ t_l \in \mathcal{T}_l}{\text{minimize}} \quad & E\left[\sum_{k}^{N_k} \|\boldsymbol{U}_k + \delta \boldsymbol{U}_k\|_2\right] \\
\text{subject to} \quad & p(\boldsymbol{Z}_i) = \mathcal{F}_{i,i-1}\left(p\left(\boldsymbol{Z}_{i-1}\right)\right) \\
& p\left(\boldsymbol{Z}_0\right) = p_0\left(\boldsymbol{Z}_0\right) \\
& p\left(\boldsymbol{U}_k\right) = \mathcal{M}_k\left(p\left(\boldsymbol{Z}_k^-\right)\right) \\
& P\left(\boldsymbol{C}_i\left(\boldsymbol{Z}_i\right) \leq \boldsymbol{0}\right) \geq 1 - \Delta_i \qquad t_i \in \mathcal{T} \\
& \boldsymbol{C}\left(t_k,\ t_l\right) \leq \boldsymbol{0},
\end{aligned}$$

Stochastic Trajectory Optimization

$$\begin{aligned}
\underset{t_k \in \mathcal{T}_k,\ t_l \in \mathcal{T}_l}{\text{minimize}} \quad & \frac{1}{N} \sum_{k}^{N_k} \sum_{n=1}^{N} \|\boldsymbol{U}_k^{(n)} + \delta \boldsymbol{U}_k^{(n)}\|_2 \\
\text{subject to} \quad & \boldsymbol{\theta}_{Z,i} = \mathscr{F}_{i,i-1}\left(\boldsymbol{\theta}_{Z,i-1}\right) \\
& \boldsymbol{\theta}_{Z,0} = \bar{\boldsymbol{\theta}}_{Z,0} \\
& \boldsymbol{\theta}_{U,k} = \mathscr{M}_k\left(\boldsymbol{\theta}_{Z,k}^-\right) \\
& \boldsymbol{C}_Z\left(\boldsymbol{\theta}_{Z,i}\right) \leq \boldsymbol{0} \qquad t_i \in \mathcal{T} \\
& \boldsymbol{C}\left(t_k,\ t_l\right) \leq \boldsymbol{0}.
\end{aligned}$$

Deterministic Trajectory Optimization

Formulation of Stochastic Trajectory Optimization Problem

Formulations of Optimization

- Objective function: Mean of TCM ΔV
- Constraint: Final guidance accuracy
- Optimization variables: TCM and OD timings

$$
\begin{aligned}
\underset{t_k \in \mathcal{T}_k,\ t_l \in \mathcal{T}_l}{\text{minimize}} \quad & \frac{1}{N} \sum_{k}^{N_k} \sum_{n=1}^{N} \|\boldsymbol{U}_k^{(n)} + \delta \boldsymbol{U}_k^{(n)}\|_2 \\
\text{subject to} \quad & \boldsymbol{\theta}_{Z,i} = \mathscr{F}_{i,i-1}\left(\boldsymbol{\theta}_{Z,i-1}\right) \\
& \boldsymbol{\theta}_{Z,0} = \bar{\boldsymbol{\theta}}_{Z,0} \\
& \boldsymbol{\theta}_{U,k} = \mathscr{M}_k\left(\boldsymbol{\theta}_{Z,k}^{-}\right) \\
& \boldsymbol{C}_Z\left(\boldsymbol{\theta}_{Z,i}\right) \leq \boldsymbol{0} \qquad t_i \in \mathcal{T} \\
& \boldsymbol{C}\left(t_k,\ t_l\right) \leq \boldsymbol{0}.
\end{aligned}
$$

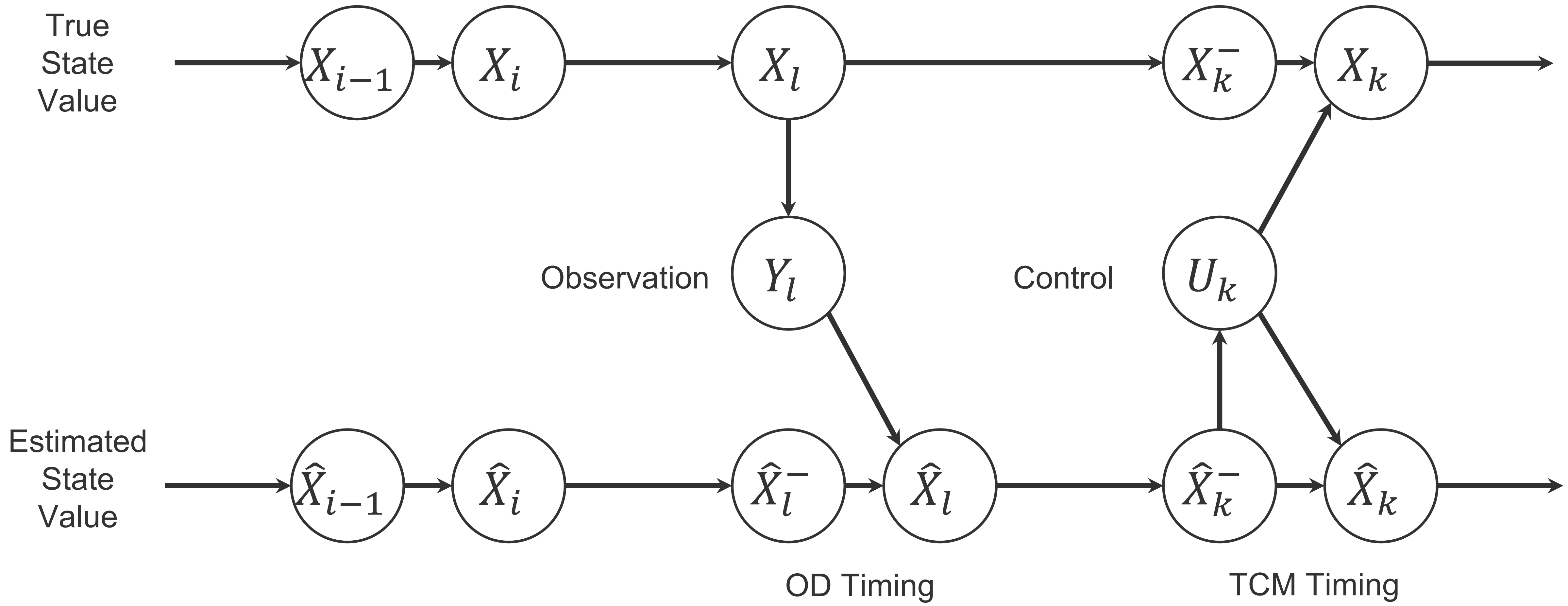
Issues

1. Propagation of parameters of probability distribution of state variables
2. Calculation of objective function from parameters of probability distribution of control variables
3. Calculation of constraint on state variables
4. Numerical optimization algorithm

Propagation of state

Propagation of probability distribution of state variables

- Coupling of true state and estimated state
  - True state is propagated by dynamics and controls (function of **estimated state**)
  - Estimated state is propagated by Kalman Filter and updated by observations (function of **true state**)
  - Augmented state $\boldsymbol{Z} = [\boldsymbol{X}^T \quad \hat{\boldsymbol{X}}^T]^T$ consisting of true state and estimated state is adopted to considering orbit determination error.



Parameterization of probability distribution

- Gaussian Approximation
  - **Linearization** (like EKF)
  - Statistical Linearization
  - Unscented Transform (like UKF)
  - Cubature Rules (like CKF)
  - Polynomial Chaos Expansion(PCE)

$$\mathbf{Z} \sim \mathcal{N}(\boldsymbol{\mu}', \Sigma')$$

$$p(\mathbf{Z}) \approx \frac{\exp\left(-\frac{1}{2}(\mathbf{Z}-\boldsymbol{\mu}')^{\mathrm{T}}\Sigma'^{-1}(\mathbf{Z}-\boldsymbol{\mu}')\right)}{\sqrt{(2\pi)^d|\Sigma'|}}$$

$$\boldsymbol{\theta}_{Z,i} = [\boldsymbol{\mu}'_i \ \Sigma'_i] .$$

- Monte Carlo Approximation
  - PCE Monte Carlo (PCE-MC)
  - Normal Monte Carlo (like PF)

$$p(\mathbf{Z}) \approx \frac{1}{N}\sum_{n=1}^{N}\delta\left(\mathbf{Z}-\mathbf{Z}^{(n)}\right)$$

$$\boldsymbol{\theta}_{Z,i} = \left[\mathbf{Z}_i^{(1)} \ \mathbf{Z}_i^{(2)} \ \cdots \ \mathbf{Z}_i^{(N)}\right]$$

1. Propagation of parameters of probability distribution of state variables
Parameters Propagation (Gaussian Approximation, Linearization)

At TCM Timings

$$\boldsymbol{\mu}'_k = \begin{bmatrix} I & C_k \\ O & I + C_k \end{bmatrix} \begin{bmatrix} \Phi_{k,l} & O \\ O & \Phi_{k,l} \end{bmatrix} \boldsymbol{\mu}'_l$$

$$\Sigma'_k = \begin{bmatrix} I & C_k \\ O & I + C_k \end{bmatrix} \begin{bmatrix} \Phi_{k,l} & O \\ O & \Phi_{k,l} \end{bmatrix} \Sigma'_l \begin{bmatrix} \Phi_{k,l} & O \\ O & \Phi_{k,l} \end{bmatrix}^T \begin{bmatrix} I & C_k \\ O & I + C_k \end{bmatrix}^T + \begin{bmatrix} \Gamma \\ O \end{bmatrix} Q_{k,l} \begin{bmatrix} \Gamma \\ O \end{bmatrix}^T + \begin{bmatrix} B \\ O \end{bmatrix} S_k \begin{bmatrix} B \\ O \end{bmatrix}^T$$

At Observation Timings

$$\boldsymbol{\mu}'_l = \begin{bmatrix} I & O \\ K_l H_l & I - K_l H_l \end{bmatrix} \begin{bmatrix} \Phi_{l,l-1} & O \\ O & \Phi_{l,l-1} \end{bmatrix} \boldsymbol{\mu}'_{l-1}$$

$$\Sigma'_l = \begin{bmatrix} I & O \\ K_l H_l & I - K_l H_l \end{bmatrix} \begin{bmatrix} \Phi_{l,l-1} & O \\ O & \Phi_{l,l-1} \end{bmatrix} \Sigma'_l \begin{bmatrix} \Phi_{l,l-1} & O \\ O & \Phi_{l,l-1} \end{bmatrix}^T \begin{bmatrix} I & O \\ K_l H_l & I - K_l H_l \end{bmatrix}^T$$
$$+ \begin{bmatrix} \Gamma \\ K_l H_l \Gamma \end{bmatrix} Q_{k,l} \begin{bmatrix} \Gamma \\ K_l H_l \Gamma \end{bmatrix}^T + \begin{bmatrix} O \\ K_l \end{bmatrix} R_l \begin{bmatrix} O \\ K_l \end{bmatrix}^T$$

→ Parameters of probability distribution can be calculated if TCM and OD timings are determined.

2. 3. Calculation of Objective Function and Constraints

Objective Function

- Approximation by Monte Carlo
  - No analytical (closed-form) function of parameters
  - Calculated by sampling from distribution of control

$$E\left[||\boldsymbol{U}_k + \delta\boldsymbol{U}_k||_2\right] \approx \frac{1}{N}\sum_{n=1}^{N} ||\boldsymbol{U}_k^{(n)} + \delta\boldsymbol{U}_k^{(n)}||_2$$

Constraints

- Constraint on final guidance accuracy

$$E\left[\boldsymbol{X}_f\right] = X_f^*$$

$$V\left[\boldsymbol{X}_f\right] \leq \overline{\Sigma_{\boldsymbol{X}_f}}$$

- Constraint on OD and TCM timings

$$t_l - t_{l-1} \geq \delta t_{od}$$

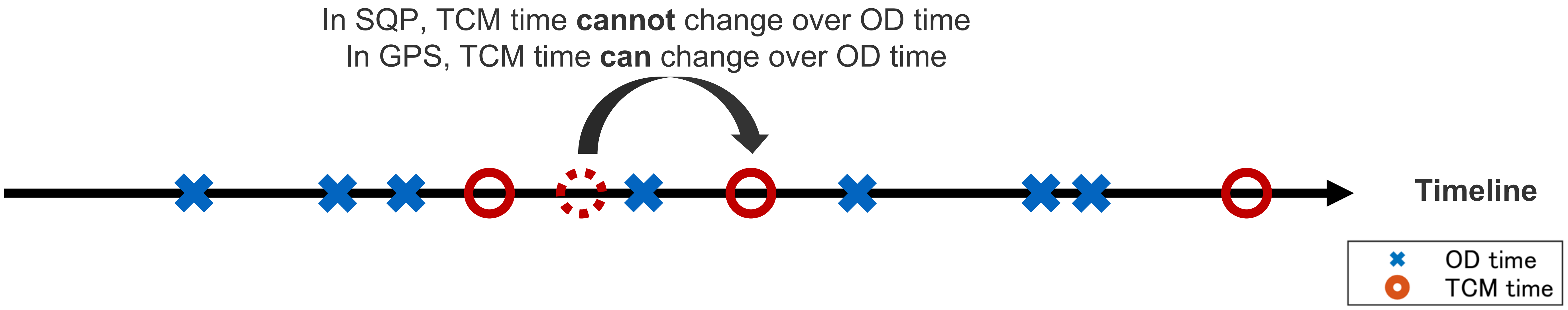
$$t_k - t_{l(k)} = \delta t_{cf}$$

$$t_l \leq t_{l(k)} - \delta t_{\mathrm{od}} \text{ or } t_l \geq t_k$$

→Objective function and constraints can be calculated from parameters of probability distribution.

Numerical Optimization Algorithm

- Sequential Quadratic Programming (SQP)
  - Can search for solutions continuously, but the objective function must be twice differentiable.
  - If the TCM time exceeds the orbit determination time, the objective function changes discontinuously (because the orbit determination error changes discontinuously at the orbit determination time), so such a search is not possible.
- Generalized Pattern Search (GPS)
  - Applicable even when the objective function is discontinuous.
  - Continuously searching for solutions is not as good as SQP, but it is possible to reach solutions that SQP cannot reach.



Formulation of Proposed Method (Summary)

Formulations of Optimization

- Objective function: Mean of TCM ΔV
- Constraint: Final guidance accuracy
- Optimization variables: TCM and OD timings

$$
\begin{aligned}
\underset{t_k \in \mathcal{T}_k,\ t_l \in \mathcal{T}_l}{\text{minimize}} \quad & \frac{1}{N} \sum_{k}^{N_k} \sum_{n=1}^{N} \| \boldsymbol{U}_k^{(n)} + \delta \boldsymbol{U}_k^{(n)} \|_2 \\
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& \boldsymbol{C}_Z \left(\boldsymbol{\theta}_{Z,i} \right) \leq \boldsymbol{0} \qquad t_i \in \mathcal{T} \\
& \boldsymbol{C} \left(t_k,\ t_l \right) \leq \boldsymbol{0}.
\end{aligned}
$$

Calculation method

1. Propagation of parameters of probability distribution of state variables is calculated if TCM and OD timings are determined.

2. Objective function is calculated by Monte Carlo method.

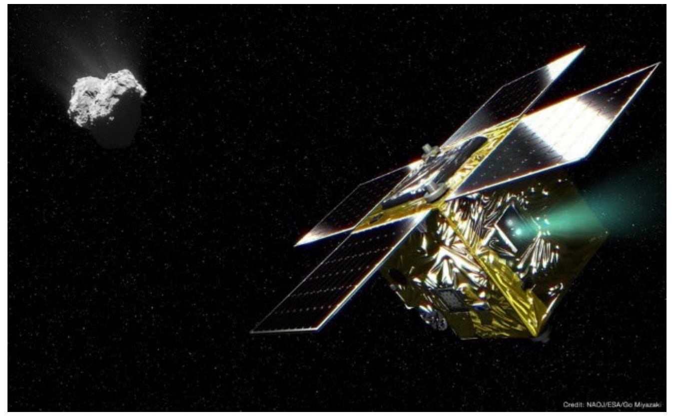
3. Constraint on state variables is calculated by mean and covariance of state variables.

4. SQP or GPS is used for numerical optimization algorithm.

Numerical Simulation Example: Optimization at PROCYON Trajectory

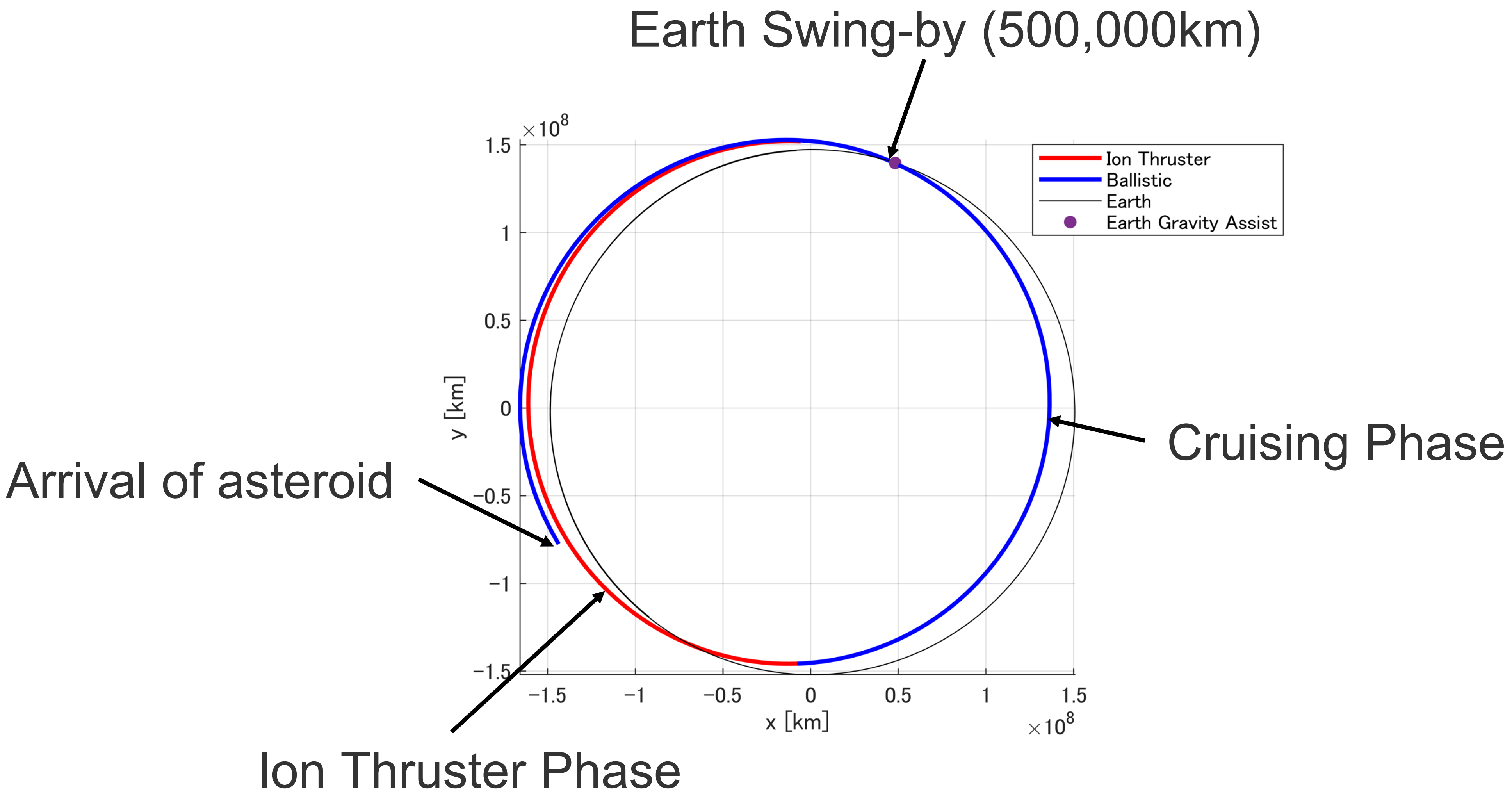
PROCYON Trajectory

- Ion Thruster→Cruising→Earth Swing-by→Cruising→Arrival of asteroid
- Dynamics：Sun, Earth, Moon, Mars, Solar Radiation Pressure
- Observations：Direct observation
- Propagator： jTOP(jPRO)

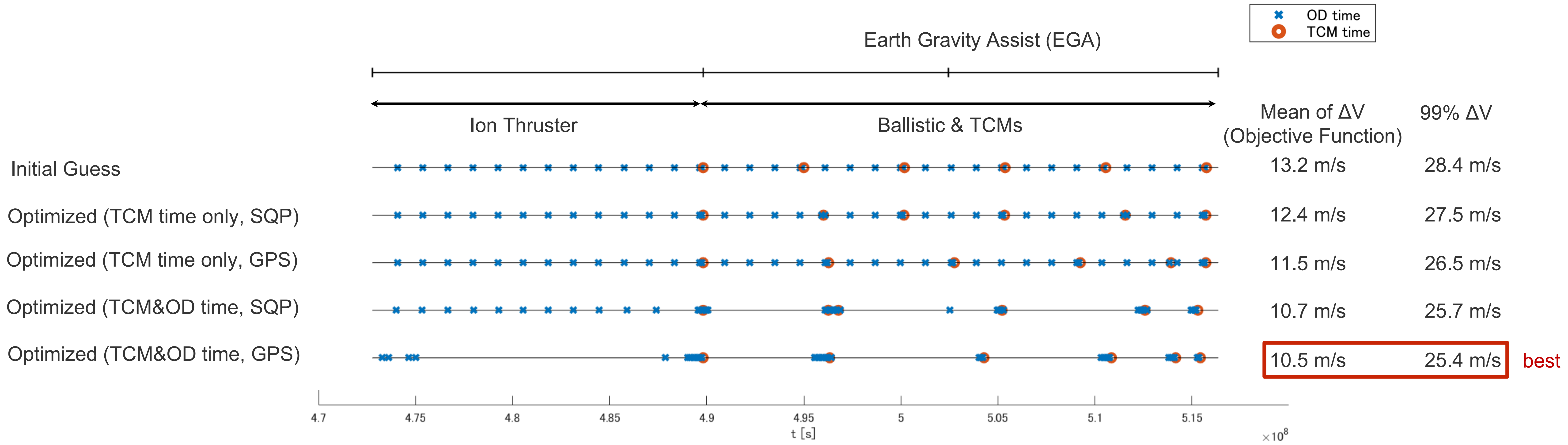


Conditions

- ODs in all phases (39 times)
- TCM in cruising phases (6times)
- Initial errors：position 1000km, velocity 0.1m/s
- Final position constraint：120km
- Control errors：0.02m/s
- Dynamics errors：1.12e-7km/s^2
- Observation errors：position 100km, velocity 0.1m/s
- Parameterization：Gaussian (Linearization)



Optimized Solution: Optimization at PROCYON Trajectory

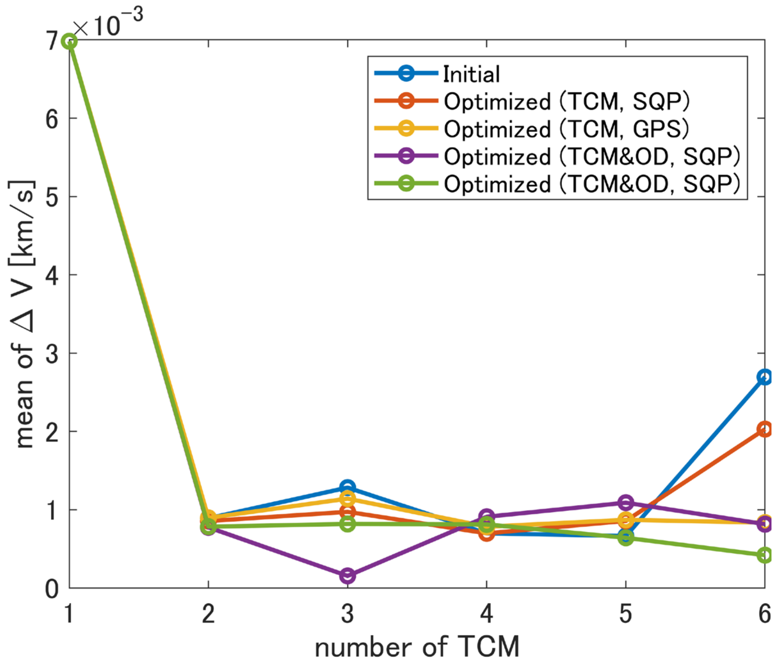


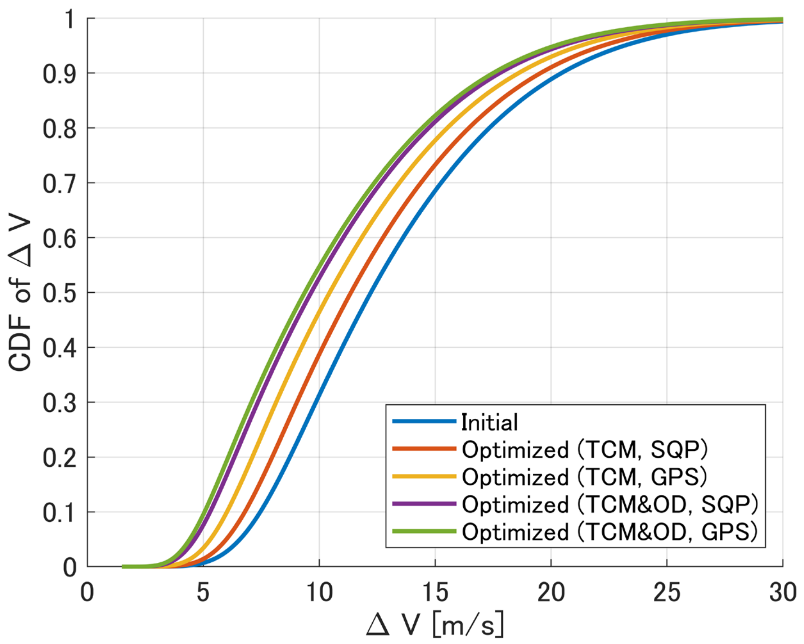
Optimize TCM time only

- SQP has not been able to overcome the orbit determination time from the initial solution.
- GPS overcomes the orbit determination time to reach a more optimal solution.

Optimize TCM & OD time

- Better solutions are obtained compared to the case of TCM-only optimization
- The TCM time has not been able to overcome the EGA time in SQP.
- In GPS, third TCM overcomes the EGA time to get a better solution than SQP.





Conclusions

- Integrated optimization of trajectory correction and orbit determination is important for future low-cost and frequent deep space missions.
- Optimization of TCM and OD timings formulated via stochastic trajectory optimization method is proposed.
- Orbit determination error can be handled by using augmented state.
- The proposed method is applied to PROCYON trajectory and the results show that optimization of both of TCM and OD timings can generate better solution than that of TCM timings only.