Testing the No-Hair Theorem with Observations of Astrophysical Black Holes in the Electromagnetic Spectrum

Dimitrios Psaltis[1,2,3] and Tim Johannsen[2]

[1] Astronomy Department, University of Arizona, 933 N. Cherry Ave, Tucson, AZ 85721, USA
[2] Physics Department, University of Arizona, 1118 E. 4th St., Tucson, AZ 85721, USA
[3] Institute for Theory and Computation, Harvard-Smithsonian Center for Astrophysics,
60 Garden St., Cambridge, MA 02138, USA
E-mail(DP): dpsaltis@email.arizona.edu, (TJ): timj@physics.arizona.edu

Abstract

The Kerr spacetime of spinning black holes is one of the most intriguing predictions of Einstein's theory of general relativity. The special role this spacetime plays in the theory of gravity is encapsulated in the no-hair theorem, which states that the Kerr metric is the only realistic black-hole solution to the vacuum field equations. Recent and anticipated advances in the observations of black holes throughout the electromagnetic spectrum have secured our understanding of their basic properties while opening up new opportunities for devising tests of the Kerr metric. In this paper, we argue that imaging and spectroscopic observations of accreting black-holes with current and future instruments can lead to the first direct test of the no-hair theorem.

Key words: relativity — black hole physics

1. Introduction

According to the general relativistic no-hair theorem, the Kerr metric is the only axisymmetric, vacuum solution to the Einstein field equations that possesses a horizon and no time-like loops (see Heusler 1996 and references therein). Together with the cosmic censorship conjecture, which states that a naked singularity cannot be formed by an astrophysical process (see, however, Shapiro et al. 1995), this theorem naturally leads to the expectation that all astrophysical objects that have been identified as black-hole candidates are indeed described by the Kerr metric.

The Kerr nature of astrophysical black holes is a prediction that needs to be tested observationally. It is still mathematically possible within general relativity that these astrophysical objects are described by a metric with a naked singularity and no horizon, such as the Manko & Novikov (1992) metric, violating the cosmic censorship hypothesis. Alternatively, the massive objects we have identified as black-hole candidates may be ultra massive "stars" (such as boson stars, Q stars, gravastars, etc.) supported by fields and matter in conditions that we have not encountered in terrestrial experiments (see the discussion in Psaltis 2006; Barceló et al. 2008; Narayan & McClintock 2008). Finally, the theory of general relativity itself may break down in the strong-field regime, with the more complete theory leading to a black hole solution that is not described by the Kerr metric (e.g., Yunes & Pretorious 2009; also Psaltis et al. 2008 and Barausse & Sotiriou 2008). Testing the no-hair theorem with astrophysical black holes offers the unique opportunity for both rejecting alternative interpretations of their nature and for verifying general relativity in the strong-field regime.

The substantial improvement in observational techniques of the last decade has led to the identification of at least four observables from accreting black holes that depend on the spacetimes very close to their event horizons and allow, in principle, for a test of the no-hair theorem (see also Psaltis 2008): *(i)* the high-resolution images of the inner accretion flows (Doeleman et al. 2008; Broderick et al. 2009), *(ii)* the relativistically broadened iron lines in their X-ray spectra (e.g., Reynolds & Nowak 2003; Fabian 2007; Nandra et al. 2007; Miller 2007), *(iii)* the maxima of the thermal spectra from their accretion disks (e.g., Shafee et al. 2006, Narayan et al. 2007), and *(iv)* the quasi-periodic oscillations in their X-ray lightcurves (Psaltis 2004; McClintock & Remillard 2006).

On the theoretical front, there have also been significant recent advances in the development of frameworks with which observations of black holes can be used to test quantitatively the Kerr metric and search for violations of the no-hair theorem. These involve, for example, the expansion of the black-hole spacetime into multipoles with coefficients that can be measured observationally (Ryan 1995), or parametric deviations of the Schwarzschild (Collins & Hughes 2004) and of the Kerr spacetimes (Glampedakis & Babak 2006; Vigeland & Hughes 2009) for black holes with zero or finite spins, respectively. Although these studies focused on the emission of gravitational waves from inspirals of compact objects onto supermassive black holes, the basic methods they advocated can be extended to analyze and understand observations of black holes in the electromagnetic spectrum.

In this paper, we describe a parametric framework with which tests of the no-hair theorem can be formulated and performed with imaging and spectroscopic observations. We then explore the observable implications of a violation of the no-hair theorem and discuss the strategies with which a test of the theorem can be performed in the near future.

2. Parametrizing Violations of the No-Hair Theorem

The Kerr metric is uniquely determined by only two parameters: the mass and the spin of the black hole (we do not consider here the unlikely possibility that an astrophysical black hole will have a net charge). This allows us to define a formal test of the no-hair theorem, based on the work of Ryan (1995), in the following way (see also Collins & Hughes 2004; Glampedakis & Babak 2006; Gair et al. 2008; Vigeland & Hughes 2009).

We can, in principle, expand the exterior metric of any compact object in multipoles (Geroch 1970; Hansen 1974) and use observations to measure the coefficients of the expansion. Because of the no-hair theorem, only two of the multipole coefficients for the spacetime of a black hole are independent. The coefficient of the monopole is the mass M of the black hole and of the dipole is its spin a. All higher-order coefficients will depend on the first two, in the particular way dictated by the Kerr metric. Testing the no-hair theorem requires measuring at least the coefficient of the quadrupole q and verifying whether it satisfies the Kerr relation $q = -a^2$.

Four different approaches have been explored so far for introducing additional non-Kerr hair to the spacetimes of compact objects. Ryan (1995) studied a general expansion of stationary, axisymmetric, and asymptotically flat spacetimes in Geroch-Hansen multipoles. Collins & Hughes (2004) as well as Vigeland & Hughes (2009) added Weyl-sector bumps to the Schwarzschild and Kerr spacetimes. Glampedakis & Babak (2006) used the Hartle-Thorne metric, which is valued for slowly spinning compact objects in general relativity, and allowed its quadrupole moment to attain non-Kerr values. Finally, Gair et al. (2008) considered a coupled set of multipole moments in the Manko & Novikov (1992) spacetime that depends on three parameters.

All the above approaches were developed originally in order to test general relativity with future observations of the gravitational waves generated during inspirals into supermassive black holes (see Hughes 2006). The calculation of the waveforms of the gravitational waves themselves requires the solution of the time-dependent Einstein field equations on the parametric post-Kerr background. As a result, the validity of general relativity is assumed implicitly in the computation of these waveforms. This is not the case, however, when predicting observables in the electromagnetic spectrum, which can be calculated by requiring only the validity of the equivalence principle.

As a first approach to testing the no-hair theorem, we use the parametric post-Kerr spacetime obtained by Glampedakis & Babak (2006). This approach uses a single parameter associated to the quadrupole moment of the spacetime to quantify potential deviations from the Kerr metric, making it the simplest and most concise possible avenue for testing the no-hair theorem. Moreover, the complete metric of Glampedakis & Babak (2006) remains a valid solution to the vacuum Einstein field equations, allowing us to perform a self-consistent test of the theorem and of the black-hole identification of the compact object, within general relativity. The key drawback of this metric is that it cannot be used in describing the exterior spacetimes of rapidly spinning black holes. It will, of course, be optimal to perform the tests of the no-hair theorem described below with all four of the above formalisms in order to explore the robustness of the results.

Following Glampedakis & Babak (2006), we start with the most general axisymmetric spacetime of a slowly spinning compact object in general relativity, allowing for its quadrupole moment q to take arbitrary values. We express the coefficient of the quadrupole multipole as

$$q = -a^2 + \epsilon \qquad (1)$$

with the parameter ϵ measuring the degree of violation of the no-hair theorem and all the multipole coefficients expressed in geometric units. We then study the trajectories of photons and particles in this spacetime and identify the implications for various observables of the presence of a non-Kerr quadrupole in the spacetime of a black hole.

3. The observational appearance of black holes that violate the no-hair theorem

We explored in detail the Glampedakis & Babak (2006) metric in Johannsen & Psaltis (2009), addressing its potential for testing the no-hair theorem with observations in the electromagnetic spectrum. We identified a number of properties of the spacetimes that are significantly

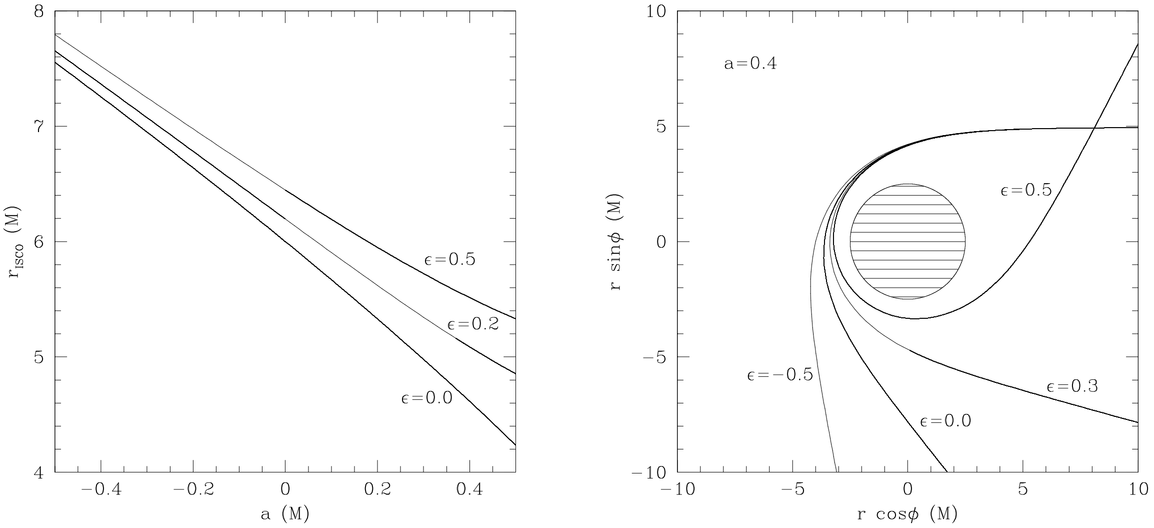

Fig. 1. *(Left)* The dependence of the location of the innermost stable circular orbit around a black hole on the black-hole spin a and on the parameter ϵ that measures the degree of violation of the no-hair theorem. *(Right)* The trajectories of photons on the equatorial plane in the vicinity of the black-hole horizon for spacetimes with different values of the parameter ϵ (Johannsen & Psaltis 2009).

affected by the presence of a non-Kerr quadrupole, with observable consequences.

(i) The location of the innermost stable circular orbit (ISCO). This is shown in Figure 1, as a function of the spin of the black hole, for different values of the parameter ϵ. The location of the maximum emission from the accretion flow around a black hole is expected to be very close to that of the ISCO (see, e.g., Krolik & Hawley 2002 and references therein). As a result, the maximum temperatures of geometrically thin accretion disks (Shafee et al. 2006) as well as the brightness profiles of the VLBI images from Sgr A* (Broderick & Loeb 2006; Noble et al. 2007; Dexter et al. 2009) will depend on the value of the quadrupole. Moreover, the location of the ISCO determines the maximum redshift of relativistically broadened iron lines. Observations of such lines have been used in the past to infer the spins of black holes using the Kerr metric (e.g., Brenneman & Reynolds 2006; Miller 2007), but are also very sensitive to the quadrupole moment of the spacetime.

(ii) The radius of the photon orbit. The radius of the photon orbit is affected at the same level as the radius of the ISCO and determines the size of the shadow of the black hole on images of the accretion flow around it (see Bardeen 1973; Falcke et al. 2000). Interferometric observations of Sgr A* in the near future will allow for a measurement of the detailed structure of the black-hole shadow in this system (Fish & Doeleman 2009) and will place a strong constraint on its quadrupole moment.

(iii) The velocity of matter at the ISCO. The accreting material is expected to follow quasi-Keplerian orbits, while slowly drifting towards the black hole. The high velocity of matter in these orbits introduces significant Lorentz boosts to the emitted radiation and causes the approaching region of the flow to appear bluer and brighter than the receding one. The amount of Lorentz boost depends on both the dipole and quadrupole moments of the spacetime. Therefore, measurements of the relative brightness of the blue and red wings of relativistic broadened lines as well as of the different regions in the images of accretion flows can be used in constraining the relative magnitudes of the dipole and quadrupole moments.

(iv) The detailed trajectories of photons that propagate close to the black hole. The amount of gravitational lensing in the vicinity of the black hole is affected significantly by the quadrupole moment of the spacetime, as shown in Figure 1. This leads to non-trivial deformations of black hole images and of the profiles of relativistically broadened iron lines, which will be detectable in future high signal-to-noise observations.

One of the main difficulties in constraining the violation of the no-hair theorem using observations in the electromagnetic spectrum arises from the existence of degeneracies between changing the dipole (i.e., the spin of the black hole) and the quadrupole moments of the spacetime. This would indeed be a problem if we were to use observables that depend only on the location of the ISCO (c.f. Figure 1), such as the maximum temperature of geometrically thin accretion disks and the maximum redshift of broadened iron lines. However, the detailed profiles of the continuum and line spectra as well as of the images from accretion flows depend also very strongly on the velocity of matter at the ISCO and on the self-lensing of the radiation emitted near the black hole. As a result, high signal-to-noise observations encode independent signatures of the dipole and quadrupole moments of the black-hole metrics that break the degeneracy between them.

4. Conclusions

Astrophysical observations of black holes offer the unique opportunity of testing the Kerr metric and thus the no-hair theorem, which is one of the most extreme general relativistic predictions. In this paper, we presented a framework for such tests using observations of accreting black holes in the electromagnetic spectrum.

There are at least two types of observations of accreting black holes that will become possible in the near future and carry the potential of performing such tests. First, radio and sub-mm observations of the black-hole in the center of the Milky Way will be able to produce snapshots of the innermost accretion flow, resolving the shadow of the black hole (Doeleman et al. 2008; Fish & Doeleman 2009). Second, the broad iron lines and detailed continuum spectra that will be observed from many accreting black holes with the International X-ray Observatory will offer an alternative approach to testing the no-hair theorem. These observations, in conjunction with the anticipated detection of gravitational waves with LISA (see Hughes 2006) and the high-resolution images of the stars in the vicinity of Sgr A* (Will 2008), will allow us in the near future to map in detail the spacetimes of black holes.

References

Barausse, E., & Sotiriou, T. 2008, PRL, 101, 9001

Barceló, C., et al. 2008, PRD, 77, 4032

Bardeen, J. 1973, in Black Holes, eds. DeWitt & DeWitt, p. 215

Brenneman, L.W., & Reynolds, C.S., 2006, ApJ, 652, 1028

Broderick, A. E., Fish, V. L., Doeleman, S. S., & Loeb, A. 2009, ApJ, 697, 45

Broderic, A. E., & Loeb, A. 2006, ApJ, 636, L109

Collins, N. A., & Hughes, S. A. 2004, PRD 69, 124022

Dexter, J., Agol, E., & Fragile, P. C. 2009, ApJ, 703, L142

Doeleman, S., et al. 2008, Nature, 455, 78

Fabian, A., 2007, in IAU Symposium 238, arXiv:0612435

Falcke, H., Melia, F., & Agol, E. 2000, 528, L13

Fish, V. L., & Doeleman, S. S. 2009, in IAU Symposium 261, in press, arXiv:0906.4040

Gair, J. R., Li, C., & Mandel, I. 2008, PRD, 77, 024035

Glampedakis, K., & Babak, S. 2006, CQG 23, 4167

Geroch, R, 1970, J. Math. Phys., 11, 2580

Hansen, R. O. 1974, J. Math. Phys., 15, 46

Heusler 1996, Black Hole Uniqueness Theorems (Cambridge: University Press)

Hughes, S. 2006, in Sixth International Lisa Symposium, arXiv:gr-qc/0609028

Johannsen, T., & Psaltis, D., 2009, in preparation

Krolik, J. H., & Hawley, J. F. 2002, 573, 754

Manko, V. S., & Novikov, I. D. 1992, CQG 9, 2477

McClintock, J. E., & Remillard, R. A. 2006, ARA&A, 44, 49

Miller, J. M. 2007, ARA&A, 45, 441

Nandra, K. et al. 2006, Astron. Nachr., 327, 1039, arXiv:astro-ph/0610585

Narayan, R., & McClintock, J. 2008, New Astron. Rev. 51, 733

Narayan, R., McClintock, J., & Shafee, R. 2007, in Astrophysics of Compact Objects, arXiv:0710.4073

Noble, S. C., et al. 2007, CQG 24, S259, arXiv:astro-ph/0701778

Psaltis, D. 2004, in X-ray Timing 2003: Rossi and Beyond, arXiv:0402213

Psaltis, D. 2006, in Compact Stellar X-ray sources, eds. W. H. G. Lewin and M. van der Klis (Cambridge:University Press)

Psaltis, D. 2008, Liv. Rev. in Relativity, 11, 9, arXiv:0806.1531

Psaltis, D., Perrodin, D., Dienes, K, & Mocioiu, I. 2008, PRL, 100, 1101

Reynolds, C. S., & Nowak, M. A. 2003, Phys. Rep. 377, 389

Ryan, F. D. 1995, PRD 52, 5707

Scafee, R. et al. 2006, ApJ, 636, L113

Shapiro, S. L., Teukolsky, S. A., & Winicour, J. 1995, PRD, 52, 6982

Vigeland, S. & Hughes, S. 2009, PRD, submitted (arXiv:0911.1756)

Will, C. M. 2008, ApJ, 674, L25

Yunes, N., & Pretorius, F. 2009, PRD, 79, 4043