Corrugation Instabilities in Slow MHD Shocks

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Abstract

We study corrugation instabilities in slow shocks in ideal MHD. Stone & Edelman (1995) suggested that slow, oblique MHD shocks were unconditionally unstable based on a linear analysis, supported by a nonlinear simulation of an oblique three-dimensional (3-D) MHD shock. We explore this suggestion modeling 3-D oblique MHD shocks using Imogen, our recently developed MHD computer code. Using Imogen, we find, in agreement with Stone & Edelman, that the corrugation instability produces dense fingers which extend from the shock front into the postshock region, growing until they reach the end of the computational grid. We currently perform longterm simulations to determine the ultimate fate of the fingers.

Key words: MHD – shock waves — instabilities

1. INTRODUCTION

The high energy emission from a broad range of astrophysical sources is produced by shocks which form as plasma accretes onto a compact object. Such radiating shocks play important roles in the neutron star X-ray pulsars and in the Polar and Intermediate Polar white dwarf binaries. These systems all contain strongly magnetic compact objects and magnetohydrodynamics (MHD) effects must be taken into account. The effects of magnetic fields on the equilibrium flow properties and plasma emission mechanisms has garnered much interest over the years, much less effort has been directed toward the effects of the strong magnetic fields on the stability properties of the shocks. Further, although the stability of MHD shocks has been studied for over forty years, most of the attention has been directed toward fast MHD shocks, shocks corresponding to the fast wave mode. In the Polar and Intermediate Polar systems mentioned above, the shock corresponds to the slow wave mode. In either case, the physical problem is complex because perturbations to the shock front can generate MHD waves which are able to propagate upstream as well as downstream of the shock. The stability of such *slow* MHD shock waves to corrugation instabilities was initially investigated by Lessen & Deshpande (1967). Lessen & Deshpande found that oblique MHD shocks were unstable to perturbations in the plane defined by the directions of preshock and postshock magnetic fields. Oblique MHD shocks are unstable when the angle formed by the preshock magnetic field and the shock normal β is either small or close to 90°. They are stable for intermediate values of β. Lessen & Deshpande also found that the instability growth rates were proportional to the square of the Alfvénic Mach number, $\mathcal{M}_A$. They thus suggested that slow MHD shock waves were stable in the limit of strong magnetic field **B**. Stone & Edelman (1995) revisited the problem of the stability of slow MHD shocks to corrugation instabilities. They found that slow MHD shocks were unconditionally unstable if one considered more general perturbations, perturbations in the direction perpendicular to the plane defined by the preshock and postshock magnetic fields. Based on a linear stability analysis, supported by a representative nonlinear shock simulation, Stone & Edelman (1995) suggested that slow MHD shocks were unconditionally unstable. Either aperiodic instabilities would cause MHD shocks to degenerate into turbulence or overstable modes would lead to short timescale quasi-periodic variability. If confirmed, this conjecture would have far-reaching consequences because of the ubiquity of MHD shocks. Here, we investigate the corrugation instability in MHD shocks for a broad range of conditions using a newly developed nonlinear numerical MHD code.

2. PHYSICAL PROBLEM

We envision a shock embedded in a flow of magnetic plasma, homogeneous in front of and behind the shock where the magnetic field and unperturbed flow are antiparallel. The properties of the preshock flow are defined by β, its sonic Mach number, $\mathcal{M}_s = v_{in}/c_s$, and its Alfvénic Mach number, $\mathcal{M}_A = v_{in}/v_A$, where v_{in}, c_s, and v_A are the flow speed, sound speed, and Alfvén speed, re-

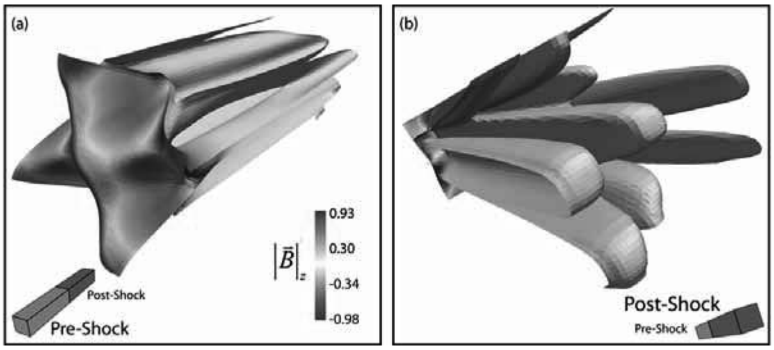

Fig. 1. Mass contours of the corrugation instability for a parallel shock with sonic Mach number of 10 and Alfvénic Mach number of 0.5, (a) as viewed from the preshock region, and (b) as viewed from the post-shock region. The contour coloring signifies magnetic wave generation as the magnitude of the transverse component, in this case z, of the magnetic field, which is initially zero throughout the simulated domain.

spectively. The flow and the field remain aligned behind the shock but they may refract. The postshock flow conditions are defined by the MHD shock jump conditions (*e.g.*, see Stone & Edelman 1995). Given these initial conditions we model the evolution of MHD shocks using the nonlinear 3-D MHD computer code, Imogen. Imogen is a high-performance MHD computer code developed by the Imamura group (Ernst *et al.* 2009). Imogen was created in 2007 after we conducted examinations of existing and publicly available computer codes and found them lacking the functionality and extensibility necessary to achieve our research goals. Imogen has been rigorously tested for both hydrodynamic and MHD cases and agrees well with published test problems (Ernst *et al.* 2009). With a capable MHD code, the problem in modeling corrugation instabilities arises from the artificial boundary conditions for the edges of the computational domain, which must be transparent to outflow while maintaining an inflow that is modified by wave propagation from the instability into the preshock region. In the artificial boundary condition community, this is considered a case of the turbulent outflow problem, a difficult and largely unsolved problem (Colonius 2004). Our solution to the outflow condition is to use a technique of dissipative interpolation based on the assumption that the waves die out over some finite distance along the external domain solution. The interpolation profile is anisotropic to ensure that advected fields are treated according to localized propagation directions, preventing any numerical noise generated by the artificial boundaries from entering the domain of interest.

3. PRELIMINARY RESULTS

We have performed 3D nonlinear simulations of corrugation instabilities in strong, sonic Mach numbers of 10, slow MHD shock waves with Alfvénic Mach numbers of 0.125 and 0.5, and incoming flow attack angles of $\beta = 0°$, $10°$, $22.5°$, $45°$, and $60°$. The simulations were conducted on a grid with cell dimensions of 300x48x48 and evolved for approximately 100 Alfvén crossing times. The crossing length was defined to be the smallest spatial grid length.

Confirming the results of previous inquiries, we find the shock waves to be generally unstable for this parameter regime. Only the parallel shock, $\beta = 0°$ and Alfvénic Mach of 0.125 case appeared stable to long-term evolution. Of particular interest in the unstable case is a growth behavior in the post-shock region, where the corrugation of the front is accompanied by distinct finger-like formations that protrude into the post shock region as shown in Figure 1 (a) & (b). In all cases these fingers grow steadily as the instability evolves, eventually extending beyond the edge of the computational domain and imposing a limit on the maximum simulation time for a particular grid size.

4. ONGOING WORK

Recently, we improved our computing power through increases in memory, processors, and computational speed. We are extending the computational domain to enable longer shock evolutions to determine the ultimate outcome of corrugation instabilities, expanding the range of sonic Mach numbers we consider, and adding optically thin cooling (*e.g.*, see Bertschinger 1986, Edelman 1991a, Strickland & Blondin 1995).

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