Reducing Uncertainty in EFD and CFD Through Data/Model Fusion

JAXA Workshop

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Acknowledgements

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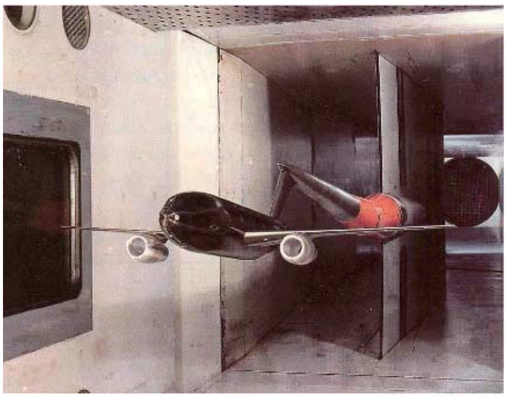
NASA Ames Rotorcraft Branch

Background

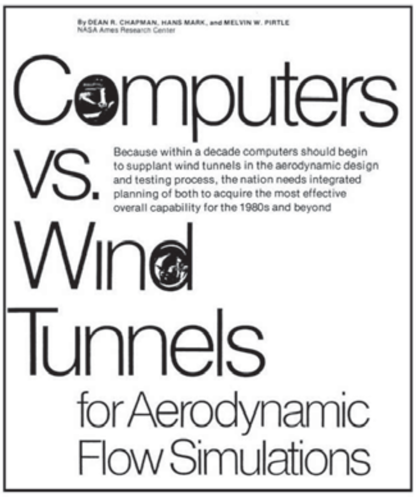
As a graduate student working at NASA Ames Research Center I was well exposed to both the EFD and CFD communities.

At the time, they didn't always get along but we were on the same team.



Background

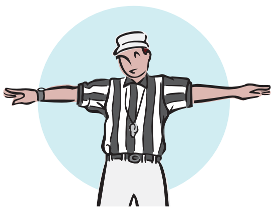
Despite 30 years of advances in CFD, we now understand that it is nothing more than the third approach. CFD synergistically complements pure theory and experiments but it can never replace either of these other approaches.


By DEAN R. CHAPMAN, HANS MARK, and MELVIN W. PIRTLE
NASA Ames Research Center

Computers vs. Wind Tunnels for Aerodynamic Flow Simulations

Because within a decade computers should begin to supplant wind tunnels in the aerodynamic design and testing process, the nation needs integrated planning of both to acquire the most effective overall capability for the 1980s and beyond

Hans Mark, et. al. 1975



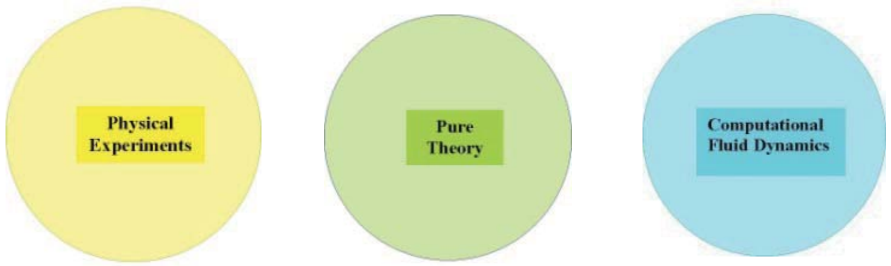

41. THE COMPUTER: RUIN OF SCIENCE AND THREAT TO MANKIND (1980/1982)

Contents

Clifford Truesdell, 1984

Background

The future advancement of fluid dynamics will rest upon a proper balance of all three methods.



Design of future aircraft and spacecraft will require even greater coupling between physical disciplines and better fidelity of their respective models (e.g., hypersonic aircraft)

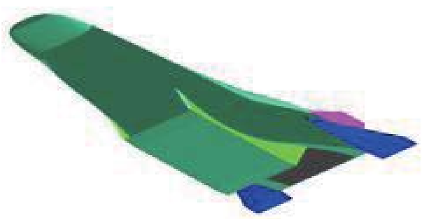


Figure: Notional NASA air breathing hypersonic aircraft design

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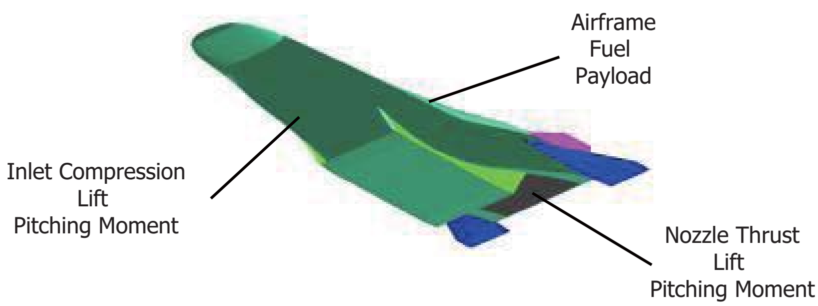
Background

Hypersonic Aircraft Example :

- Strong interactions between vehicle components
- Aerodynamics, propulsion, control, structure, tank, thermal protection, etc.
- Highly integrated engine and airframe
- Much of vehicle is engine inlet / nozzle
- Large propulsive lift and pitching moments – strong contributor to trim, stability & control
- Large Mach number and dynamic pressure variations in flight
- Severe aerodynamic heating
- Thermal protection must be integrated with structure
- High fuel mass fraction required - majority of volume accommodates fuel



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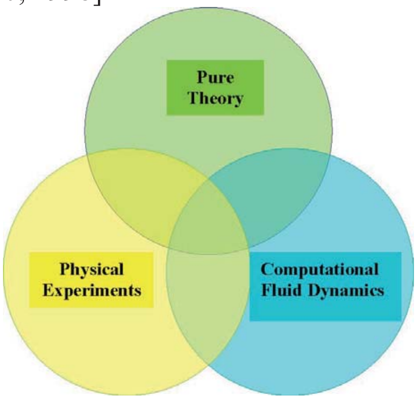
Background

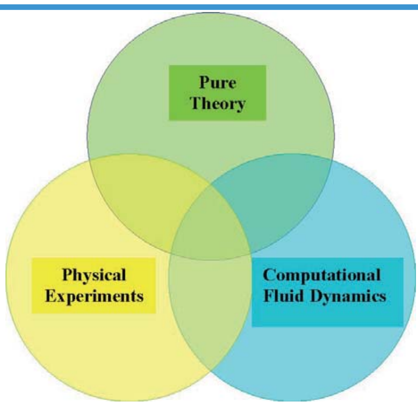
We can develop a framework by which this synergy is accessible through the use of tools from scattered data approximation (machine learning tools) and Generalized Regularization (GR)

[A.N. Tikhonov and V.Y. Arsenin, *Solution of Ill - Posed Problems*, 1977]

[M. Ulbrich, TUM-M9810, 1998]



Background



Specifically, mathematical analysis of experimental data is treated as an ill-posed problem.
Its regularization involves the introduction of additional information regarding the physical system.

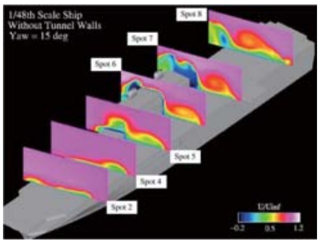
We can then utilize a-priori mathematical models of physical systems at appropriate orders of fidelity for regularization. We can then investigate its potential in reducing uncertainty in EFD and CFD.

Motivation

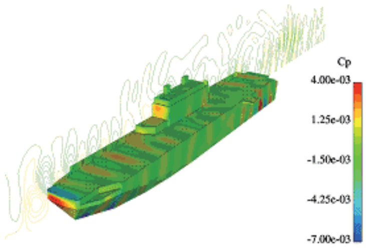
What are some of the potential applications of a fusion of mathematical models, computational methods, and experimental data?

- Filling in the blanks in experiments (including reducing uncertainty in EFD)
- Accelerating through test matrices (steering and predicting the amount of data needed)
- Accelerating CFD solutions using data (including reducing uncertainty in CFD)
- Noise filtering of data
- Knowledge discovery through parameter estimation

[Kurt Long, NASA Ames]



[Gloria Yamauchi, NASA Ames]



[Gary Sivak, Air Force Research Lab]

Approach

To begin, define:

$$d \equiv \text{ denotes the dimensionality of the approximation problem}$$

$$x_i \equiv \left(x_{1,i},\ldots,x_{d,i}\right) = d \text{ dimensional input}$$

$$\phi_k\left(x\right) \equiv k^{\text{th}} \text{ basis function}$$

$$c_k \equiv \text{ coefficients corresponding to the basis functions}$$

$$u(x) \equiv \text{ exact response of an unknown or underlying physical process}$$

$$f(x) \equiv \text{ observable output } \Rightarrow F\left(u\right) = f$$

$$\mu \equiv \text{ random noise of measurements at coordinate } x_i$$

$$f_e(x_i) \equiv \text{ experimental data points } = f(x_i) + \mu_i$$

$$u_a(x) \equiv \text{ approximation of } u(x) \;\Rightarrow\; u_a(x) = \sum_k \Phi_k\left(x\right) c_k$$

The function $F\left(u_a\right) = f_a$ is designed to pass approximately through the experimental data points $f_e(x_i)$

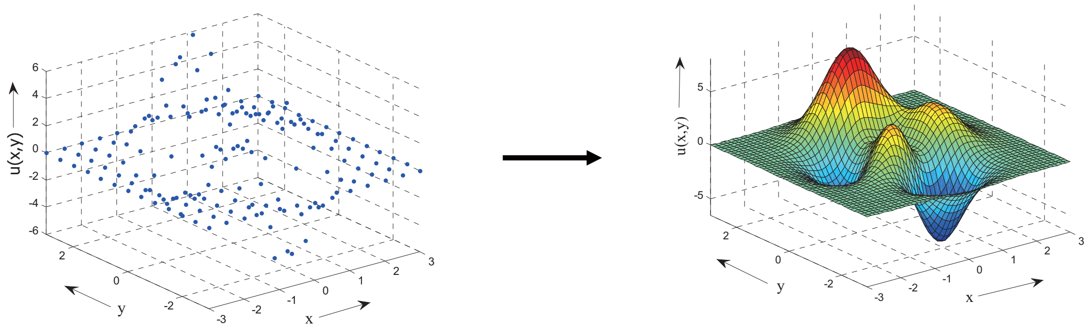
Approach

Basic Problem:

The construction of a response surface from the minimization of the standard square error,

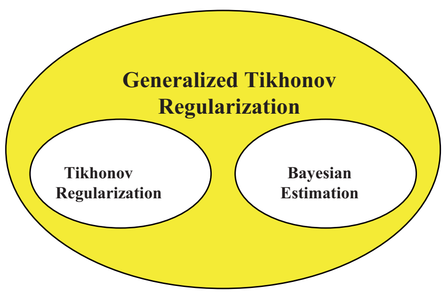
$$\varepsilon = \sum_{i=1}^{s} \left[f_e(x_i) - F(u_a(x_i)) \right]^2$$

constitutes an ill-posed problem since u_a is nonunique and sensitive to the noise in data f_e.



Approach

Solution:



The general idea in GTR is the minimization of

$$\varepsilon = A + \Lambda B$$

A is a scalar measure of the agreement between f_e and $F(u_a)$. In Bayesian terms A is related to *a posteriori* knowledge.

B is a stabilizing functional (i.e. regularizing operator) and is related to *a priori* information.

Λ is a positive scalar known as the regularization that controls the relevance of the *a posteriori* and *a priori* information to the approximation u_a.

Approach

$$\varepsilon = A + \Lambda B$$

The regularizing operator, B, is classified as either quantitative or qualitative.

Can range from systems of time-dependent nonlinear partial differential equations, to statistical correlations, to heuristics.

Finding an approximate solution reduces to
(a) finding regularization operators B and
(b) determining the regularization parameter Λ from supplementary information pertaining to the problem, e.g., pertaining to the noise level in f_e.

B is not unique and depends on the type of physical process and the data type.

Approach

In this context, the GTR formulation acts as a Swiss Army knife and gives us access to a number of popular methods in inverse problems.

Popular inverse problem methods include:

- Regularization by noise filtering
- Regularization by projection
- Conventional Tikhonov regularization $\Rightarrow A = \sum_{i=1}^{s} \left[f_e(x_i) - F(u_a(x_i)) \right]^2$ and B is a smoothing functional in variational form.
- Scattered data approximation for Response Surface Modeling (RSM) – includes support vector machines, radial basis functions, artificial neural networks
- Kalman Filter
- Bayesian Estimation

Approach

Bayesian Estimation :

With $A = -2\log p(f_a \mid f_e)$, $B = -2\log p(f_a)$, and $\Lambda = 1$, then ε becomes the figure-of-merit function from Bayesian estimation theory.

Therefore, one can think of the GTR framework as a *deterministic* form of Bayesian estimation.

Kalman Filter :

Assume the measurement errors are Gaussian. The probability density functions act as if a differential operator is used to form B and operates on $u_0 - u_a$, where u_0 is the solution to a mathematical model of the time-dependent physical system of interest. The solution to the resulting Euler-Lagrange equation is $u_a = u_0 + u_{cor}$, where $u_{cor}(t_k) = F\left(\left(u_e(t_k) - u_0(t_k)\right), \tau\right)$.

The basis functions for u_{cor} are the Green's function to a dynamic equation and τ is computed to minimize $\sum (u_0 - u_a)^2$ during the time marching.

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Approach

Back to Generalized Tikhonov Regularization

$$\Lambda \to \infty,\quad f_a(x) \to f_0(x) \qquad \text{and} \qquad \Lambda \to 0,\quad f_a(x) \to f_e(x)$$

One approach to determining the correct value for Λ is assuming the error is random, statistically independent and uniformly distributed in the interval $[-\tau,\ \tau]$. This will give,

$$\sum_{i=1}^{s}\left[f_e(x_i) - F(u_a(x_i))\right]^2 = E\left[\sum_{i=1}^{s}\left[f_e(x_i) - F(u_a(x_i))\right]^2\right] = E\left[\sum_{i=1}^{s}\mu_i^2\right] = s\cdot \mathrm{Var}(\mu) = s\frac{\tau^2}{3}$$

where $E[\cdot]$ denotes the operator of mathematical expectation.

Otherwise you can determine Λ such that $\mathrm{Max}\left|f_e(x_i) - F(u_a(x_i))\right| \le \tau$

Note in both formulations, Λ is solved for iteratively and requires the solution of $f_a(x)$ within each iteration.

The direct solution of Λ is still an area of active research.

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Approach

Back to Generalized Tikhonov Regularization

Interesting Property

$$\Lambda \to \infty,\quad f_a(x) \to f_0(x) \qquad \text{and} \qquad \Lambda \to 0,\quad f_a(x) \to f_e(x)$$

One approach to determining the correct value for Λ is assuming the error is random, statistically independent and uniformly distributed in the interval $[-\tau,\ \tau]$. This will give,

$$\sum_{i=1}^{s}\left[f_e(x_i)-F(u_a(x_i))\right]^2 = E\left[\sum_{i=1}^{s}\left[f_e(x_i)-F(u_a(x_i))\right]^2\right] = E\left[\sum_{i=1}^{s}\mu_i^2\right] = s\cdot \mathrm{Var}(\mu) = s\frac{\tau^2}{3}$$

where $E[\square]$ denotes the operator of mathematical expectation.

Otherwise you can determine Λ such that $\operatorname{Max}\left|f_e(x_i)-F(u_a(x_i))\right| \le \tau$
Note in both formulations, Λ is solved for iteratively and requires the solution of $f_a(x)$ within each iteration.

The direct solution of Λ is still an area of active research.

Approach

GTR can use physical model in nonvariational form in place of B. A large class of mechanics problems can be described by the equations:

$$L\left[u_0(x)\right] = g(x), \quad x \in \Omega$$
$$\beta\left[u_0(x)\right] = 0, \quad x \in \partial\Omega$$

Without a proxy, the minimum of $\varepsilon(u_a)$ satisfies

$$L\left[u_a(x)\right] = g(x) + 2\sum_{i=1}^{s}\left[\frac{u_e(x_i)-u_a(x_i)}{\Lambda}\right]\delta(x-x_i), \qquad x \in \Omega$$
$$B\left[u_a(x)\right] = 0, \qquad x \in \partial\Omega$$

Where $u_0(x)$ is the solution to the differential equation.

Approach

The analytical solution to this modified mathematical model is

$$u_a(x) = u_0(x) + \sum_{i=1}^{s} G(x, x_i) c_i \quad \text{with} \quad c_i = \left\{ \left(\mathbf{G} + \frac{1}{2} \Lambda \mathbf{I} \right)^{-1} (\mathbf{u}_e - \mathbf{u}_0) \right\}_i$$

where $G(x, x_i)$ is the Green's function to $L[\square]$, $G(x_j, x_i) = \{\mathbf{G}\}_{j,i}$, and $\Lambda \mathbf{I}$ is a diagonal matrix.

Note that with $\Lambda \to 0$ we have $u_a(x_i) \to u_e(x_i)$ but the slope and curvature of $u_0(x)$ is used for interpolation. This means we can use the qualitative information from $u_0(x)$ and are not constrained by the quantitative properties.

In other words, additional information of *any* fidelity can be used.

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Approach

GTR possesses a number of useful properties. However:

- The Green's functions for practical models are usually unavailable. Is it possible to use existing numerical solutions in the literature for fusion?
- Is it possible to find a value for Λ that satisfies $s\frac{\tau^2}{3}$ without the trouble of solving for $u_a(x)$ explicitly?
- Rather than using a single value for Λ is it possible to use a distributed one?
- Is the full data set required to accomplish data-model fusion?
- Is it possible to do all of this with minimal user interaction?
- Can we accelerate through experiments with incomplete mathematical models or no model at all (Black-Box modeling)?
- Can we reduce uncertainty in EFD and CFD?

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Approach #1

$$\varepsilon = A + \Lambda B$$

We can also determine the minimum of $\varepsilon(u_a)$ through a conventional computational mechanics form using the Method of Weighted Residuals (MWR).

$$2\sum_{j=1}^{N}\left(\sum_{i=1}^{s}\phi_j(x_i)\psi_k(x_i)\right)c_j + \Lambda\left\langle \psi_k(x), L\left[u_a(x)\right]\right\rangle + \Lambda\ b\left\langle \psi_k(x),\ q\right\rangle =$$

$$2\sum_{i=1}^{s} f_e(x_i)\psi_k(x_i) + \Lambda\left\langle g, \psi_k(x)\right\rangle \quad \text{for } k = 1,\ldots,N$$

with the constraint $\sum_{i=1}^{s}\left[f_e(x_i) - F\left(u_a(x_i)\right)\right]^2 \le s\dfrac{\tau^2}{3}$

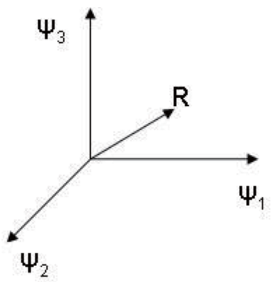
Approach #1

The MWR can be described as a numerical approximation method that solves for the coefficients c_i by setting the inner product of the weighted residual equation to zero:

$$\left\langle R, \psi_k \right\rangle = 0, \quad k = 1,\ \ldots,\ s$$

where $\mathrm{L}[u_a] - g = R$ and

$\psi_k \equiv$ weighting function



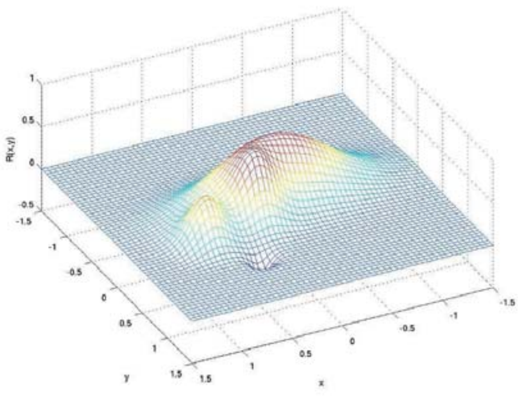


Figure: Distribution of the equation residual R.

Approach #1

In this context, the MWR gives us access to many of the popular computational methods in a single framework.

Finite volume method: $\psi_k = \begin{cases} 1 \text{ in } \Omega_k \\ 0 \text{ outside } \Omega_k \end{cases}$

Collocation method: $\psi_k = \delta\left(x - x_k\right)$ where δ is the Dirac delta function

Least-squares method: $\psi_k = \dfrac{\partial R}{\partial c_k}$

Method of moments: $\psi_k = x^k$

Generalized Galerkin method: $\psi_k = F\left(\Phi_k\right) \quad \Rightarrow$ finite elements

The form of the bases $\Phi_i(x)$ and the integration quadrature can also give us access to the finite difference and spectral methods.

Approach #1

So our numerical formulation can be written as the well-conditioned system

$$\left(\mathbf{H} + \Lambda\mathbf{M}\right)\mathbf{c} = \mathbf{e} + \Lambda\mathbf{p} \quad \text{with} \quad \sum_{i=1}^{s}\left(f_e\left(x_i\right) - F\left(u_a(x_i)\right)\right)^2 \leq s\frac{\tau^2}{3}$$

$\mathbf{H} \equiv$ matrix containing the independent variable information of the measurement points
$\mathbf{M} \equiv$ conventional discretization matrix from computational mechanics
$\mathbf{e} \equiv$ vector representing the observational data
$\mathbf{p} \equiv$ forcing term vector of the *a priori* mathematical model with boundary conditions

- You can use the numerical method of your choice
- Λ acts as a penalty coefficient
- The data is embedded in the equation solution and acts an initial and/or boundary condition. **We have the potential to steer the CFD solution using EFD.**

Approach #1

So our numerical formulation can be written as the well-conditioned system

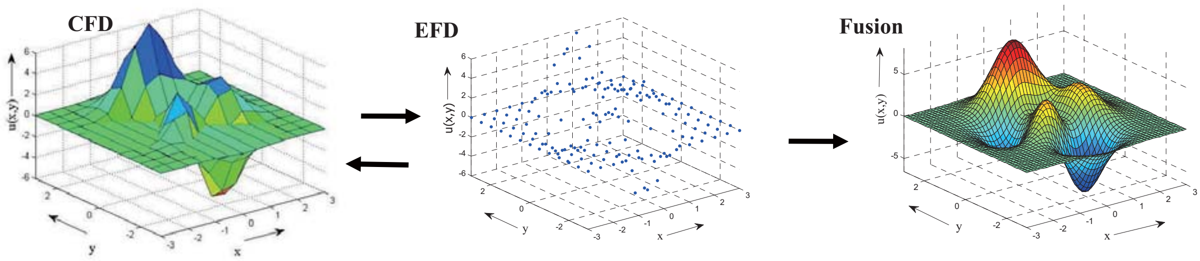
$$(\mathbf{H}+\Lambda\mathbf{M})\mathbf{c}=\mathbf{e}+\Lambda\mathbf{p} \quad \text{with} \quad \sum_{i=1}^{s}\left(f_e(x_i)-F\left(u_a(x_i)\right)\right)^2 \le s\frac{\tau^2}{3}$$

CFD:

- We control the approximation method and its discretization.
- We use Λ to balance cost and fidelity of the *a priori* model against the cost of the experimental data in the approximation.

EFD:

- We control the quality and distribution of the experimental data - DoE.
- We balance the cost of the experimental data against the *a priori* model in the approximation.



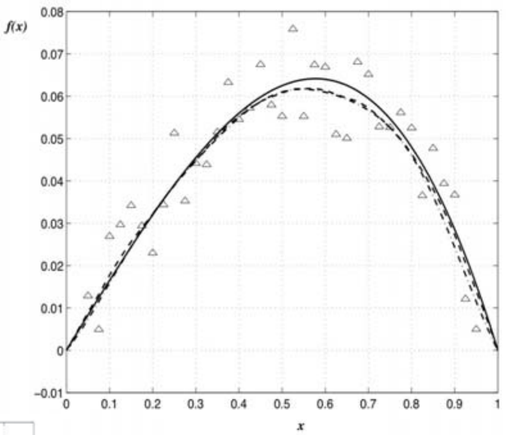
Approach #1 Results

Differential Equation Fused with Empirical Data
[B. Zeldin and A. J. Meade, *AIAA J.*, **35** (11) (1997)]

$$a\frac{d^2u}{dx^2}+x=0, \quad x\in(0,1), \quad u(0)=u(1)=0$$

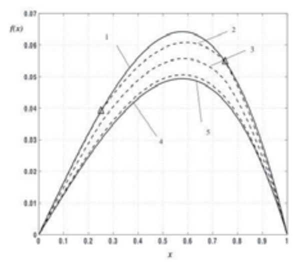
$$(\mathbf{H}+\Lambda\mathbf{M})\mathbf{c}=\mathbf{e}+\Lambda\mathbf{p} \quad \text{with} \quad \sum_{i=1}^{s}\left(u_e(x_i)-u_a(x_i)\right)^2 \le s\frac{\tau^2}{3}$$

$a=1.0$ for $u(x)$ and $a=1.3$ for $u_0(x)$ with $|\mu_i|\le +0.014$.

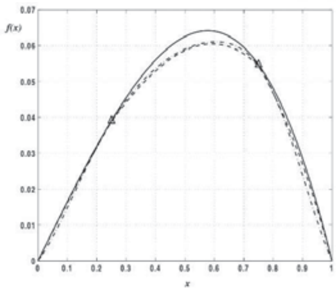


(a)

Interpolation of data with noise.
Dashed curve – regularized analytical solution.
Dashed-dot curve – regularized ANN model with 7 RBFs



Response of regularized ANN system with insufficient data. 1 – response of the mechanical system,
2 – regularized response Λ = 0,
3 – regularized response Λ = 0.3,
4 – regularized response Λ = 3.0,
5 –response of a-priori mathematical model



(b)
Interpolation with insufficient data

Approach #1 Results

The earliest and most common application of data/physical model fusion is in the dynamic modeling of weather.

Recognition by V. Bjerknes in 1904 that weather forecasting is fundamentally an initial-value problem and basic system of equations already known

- L. F. Richardson's (1922) attempt at practical Numerical Weather Prediction
- Late 1940s: First successful dynamical-numerical forecast made by Charney et al.
- 1960s: Edward Lorenz shows the atmosphere is chaotic and its predictability limit is about two weeks
- 1980s: Kalman Filtering applied to meteorology

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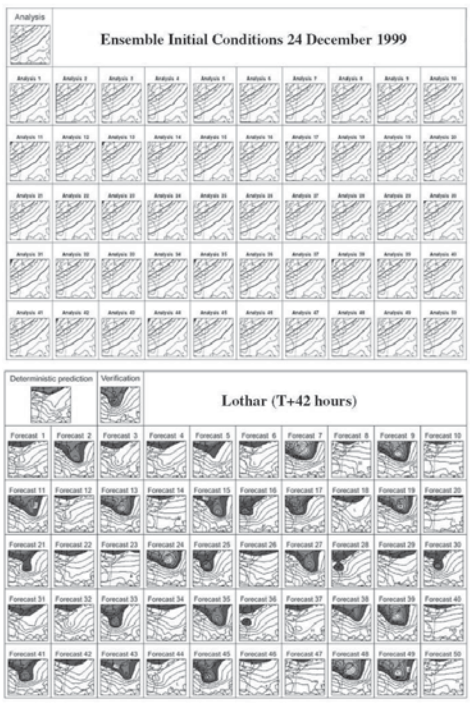
Approach #1 Results

Kalman Filtering for Numerical Weather Modeling
[*SIAM News*, **36** (8) (1997)]

Kalman filter are provably optimal if the differential equations for the system are linear. Excellent for real-time linear computations.

SIAM 1997 article stated that the Kalman filter doesn't handle any serious nonlinearity and proposed remedies were just too complicated. This resulted in the modern Ensemble Kalman Filter.



Slight changes in initial data result in dramatically different forecasts. Top, weather forecasters' best estimate of atmospheric conditions (contours represent isobars of pressure) on December 24 ("Analysis"), along with 50 slight perturbations. Bottom, the ECMWF model's best-guess forecast ("Deterministic prediction"), along with the 50 ensemble members and a map of the conditions actually observed ("Verification"), 42 hours later; shading indicates low pressure. Ensemble methods, as results like those show, can detect in advance the possibility of weather that differs markedly from the best-guess forecast. Courtesy of the European Center for Medium-Range Weather Forecasts.

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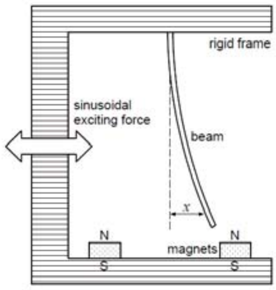
Approach #1 Results

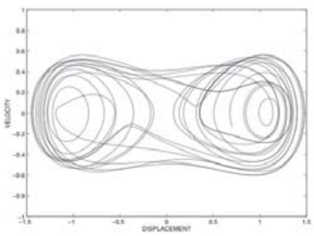
The Inhomogenous Duffing's Equation Merged with Empirical Data

[A. J. Meade and R. Moreno, *Intl. Journal of Smart Engineering System Design,* 1 (1998)]

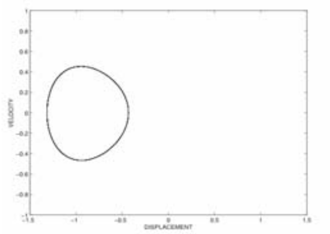
$$\frac{d^2x}{dt^2} + 2\mu\frac{dx}{dt} - \frac{1}{2}\left(x - x^3\right) = F_0\cos\left(\omega t\right)$$

$$\left(\mathbf{H} + \Lambda\mathbf{M}\right)\mathbf{c} = \mathbf{e} + \Lambda\mathbf{p} \quad \text{with} \quad \sum_{i=1}^{s}\left(\frac{dx_e}{dt}\left(t_i\right) - \frac{dx_a}{dt}\left(t_i\right)\right)^2 \le s\frac{\tau^2}{3}$$

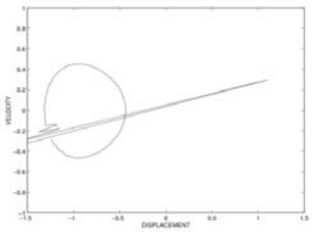




Chaotic phase space trajectory of the a-priori model for omega = 0.86



Desired nonchaotic trajectory.



Response of the model empirical merging.

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Approach #1 Results

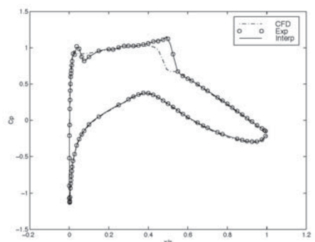
CFD Code Merged with Empirical Data

[W. Wang, M.S. Thesis, August 1998)]

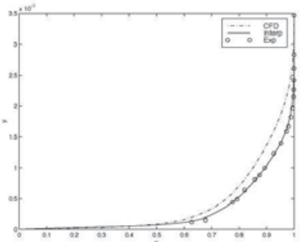
We have also had success with this formulation using a Thin-Layer Navier-Stokes solver in finite difference form fused with experimental surface pressure and boundary layer profile.

$$\left(\mathbf{H} + \Lambda\mathbf{M}\right)\mathbf{c} = \mathbf{e} + \Lambda\mathbf{p} \quad \text{with data as the BCs}$$

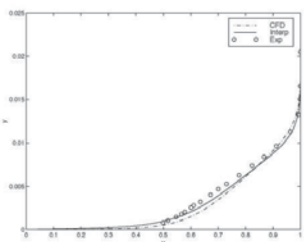
Case A	C_L	*error*
Experiment	-.1210	
CFD	-.0991	18.1%
Fused	-.1215	0.39%



Comparison of Pressure Coefficient Distribution for the RAE 2822 Airfoil at $M_\infty = 0.725$, $\alpha_{num} = 2.10$, $Re = 6.5 \times 10^6$



Comparison of Velocity Profile for the RAE 2822 Airfoil at $M_\infty = 0.676$, $\alpha_{num} = -2.30$, $Re = 5.7 \times 10^6$, $x = 0.179$



Comparison of Velocity Profile for the RAE 2822 Airfoil at $M_\infty = 0.676$, $\alpha_{num} = -2.30$, $Re = 5.7 \times 10^6$, $x = 0.90$ where no experimental data are used for fusing, but exist

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Approach #2

GTR questions:

- The Green's functions for practical models are usually unavailable. Is it possible to use existing numerical solutions in the literature for fusion?
- Is it possible to find a value for Λ that satisfies $s\frac{\tau^2}{3}$ without the trouble of solving for $u_a(x)$ explicitly?
- Rather than using a single value for Λ is it possible to use a distributed one?
- Is the full data set required to accomplish data-model fusion?
- Is it possible to do all of this with minimal user interaction?
- Can we accelerate through experiments with incomplete mathematical models or no model at all (Black-Box modeling)?
- Can we reduce uncertainty in EFD and CFD?

Approach #2

An alternative method has been developed in hope of addressing the remaining concerns.

Starting with a straightforward reformulation using the solution to a numerical model, $u_{CFD}(x)$, from the literature

$$L\left[u_a(x)-u_{CFD}(x)\right]=0+2\sum_{i=1}^{s}\left[\frac{u_e(x_i)-u_a(x_i)}{\Lambda_i}\right]\delta\left(x-x_i\right), \qquad x\in\Omega$$
$$B\left[u_a(x)-u_{CFD}(x)\right]=0, \qquad x\in\partial\Omega$$

where $L[\cdot]$ and $B[\cdot]$ are now *linear* operators.

Approach #2

If we define $e(x) \equiv u_e(x) - u_{CFD}(x)$

and use $G(x, x_i) = \phi_i(x)$,

then our solution to this modified form is

$$\sum_{i=1}^{N} \phi_i(x) c_i = u_a(x) - u_{CFD}(x) \square u(x) - u_{CFD}(x) = e(x) - \mu(x)$$

This indicates that we need an approximation technique that can also address $\mu(x)$.

We will construct a *scattered data approximation* (i.e. response surface method) from the data $\{e(x_1),\ e(x_2),\ \ldots,\ e(x_s)\}$ and build to a tolerance τ equal to the maximum estimated measurement error $(\max |\mu_i| \leq \tau)$.

Approach #2

These scattered data approximation techniques include:

- Kriging method (trouble with over 20 inputs and over 1000 data sets)
- Artificial neural networks, specifically radial basis function networks (RBFN), generalized regression neural networks (GRNN), and support vector machines (SVM).
- Greedy algorithms. This includes algorithms like Proper Orthogonal Decomposition (POD). We have developed our own greedy algorithm we call Sequential Function Approximation (SFA).

Approach #2

Our SFA algorithm is a variation of Orr's Forward Selection training method and Platt's Resource Allocating network from machine learning that seeks to improve the computational efficiency through the MWR.

For convenience, we use the Gaussian radial basis function since it is unaffected by the dimensions d.

We set

$$\phi(x,\sigma_n,x_n)=\exp\left(\frac{-(x-x_n)\cdot(x-x_n)}{\sigma_n{}^2}\right)\quad \text{so then}$$

$$\begin{aligned}R_n(x,\sigma_n,x_n)&=u(x)-u_a^{(n)}(x)=u(x)-u_a^{(n-1)}(x)-c_n\phi(x,\sigma_n,x_n)\\&=R_{n-1}-c_n\phi_n\end{aligned}$$

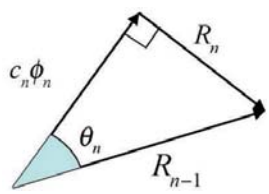
Approach #2

The objective is to determine c_n, σ_n, and x_n that minimize the residual R_n. Through the MWR we can reformulate the residual equation as a minimization problem,

$$\langle R_n,R_n\rangle=\langle R_{n-1},R_{n-1}\rangle-2c_n\langle R_{n-1},\phi_n\rangle+c_n^2\langle\phi_n,\phi_n\rangle \quad \text{which for an optimum } c_n;\ c_n=\frac{\langle R_{n-1},\phi_n\rangle}{\langle\phi_n,\phi_n\rangle}$$

$$\text{becomes}\Rightarrow\quad \frac{\langle R_n,R_n\rangle}{\langle R_{n-1},R_{n-1}\rangle}=1-\frac{\langle R_{n-1},\phi_n\rangle^2}{\langle\phi_n,\phi_n\rangle\langle R_{n-1},R_{n-1}\rangle}\quad \text{or}\quad \|R_n\|_2=\|R_{n-1}\|_2\sin\theta_n$$

where θ_n is the angle between R_{n-1} and ϕ_n. So this formulation should give $\|R_n\|_2<\|R_{n-1}\|_2$ as long as $\theta_n\neq\frac{\pi}{2}$.



Approach #2

This formulation has characteristics similar to Proper Orthogonal Decompositon（POD）, empirical basis function method, and the Krylov method.

We've used the SFA algorithm successfully with differential equations and conventional finite element basis functions in the mesh-free solution of differential equations.

We plan on using this mesh-free approach to determine whether or not CFD solutions can be steered by EFD results during calculations.

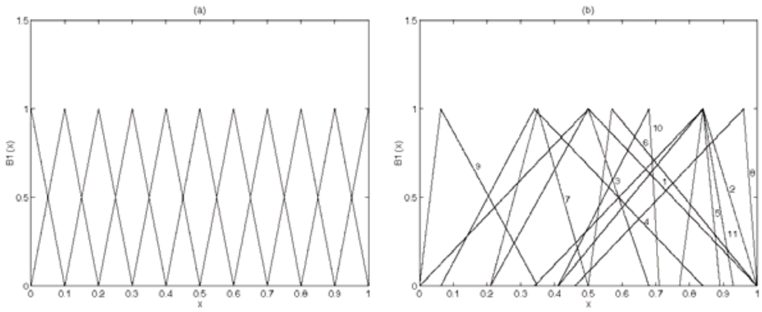


Figure 4.2 Basis functions distributions for the one-dimensional Poisson problem: (a) standard uniform and (b) optimal.

Approach #2

[A. J. Meade et al., *Communications in Numerical Methods in Engineering*, **13** (1997)]

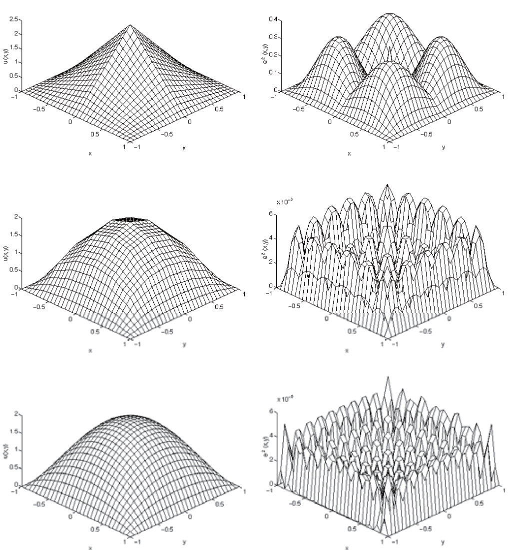


Figure 4.4 Two-dimensional Poisson's equation: (left) uniform grid finite element solutions and (right) associated squared error distributions for (top to bottom) $N = 9$, 49, and 441 bilinear Lagrangian shape functions.

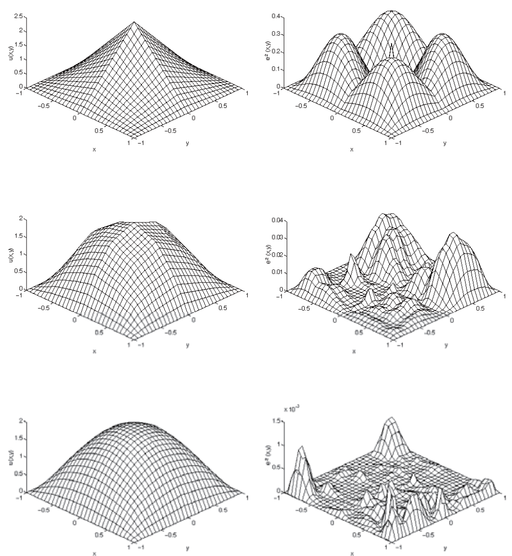
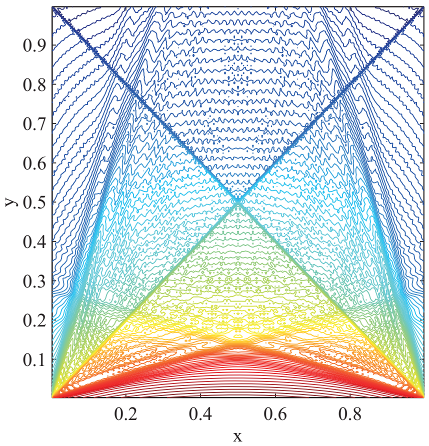


Figure 4.3 Two-dimensional Poisson's equation: (left) proposed method solutions and (right) associated squared error distributions for (top to bottom) $N = 1$, 10, and 100 optimal basis functions.

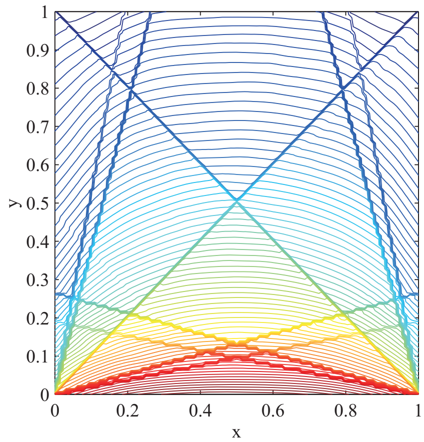
Approach #2

[D.L. Thomson et al. *International Journal of Thermal Sciences,* **40** (6) (2001)]

Discrete Ordinate Method (DOM) Thermal Radiation Problem



Non-dim. temp. approx. using 40401 finite volume bases



Non-dim. temp. approx. using 21 exp. bases by meshless method

Approach #2

The solution for *discrete* data sets is the nonlinear minimization of

$$1 - \frac{(\mathbf{R_{n-1}} \cdot \Phi_\mathbf{n})^2}{(\Phi_\mathbf{n} \cdot \Phi_\mathbf{n})(\mathbf{R_{n-1}} \cdot \mathbf{R_{n-1}})} \quad \text{with} \quad c_n = \frac{(\mathbf{R_{n-1}} \cdot \Phi_\mathbf{n})}{(\Phi_\mathbf{n} \cdot \Phi_\mathbf{n})},$$

The parameters c_n, σ_n, and x_n are solved for the n^{th} basis function sequentially.

The iterative process continues until either a pre-determined tolerance is reached

$$\left(\text{i.e., } \max\left|\mathbf{R_n}\right| \le \tau \text{ or } \mathbf{R_n} \cdot \mathbf{R_n} \le s\frac{\tau}{3}\right) \text{ or } n = s.$$

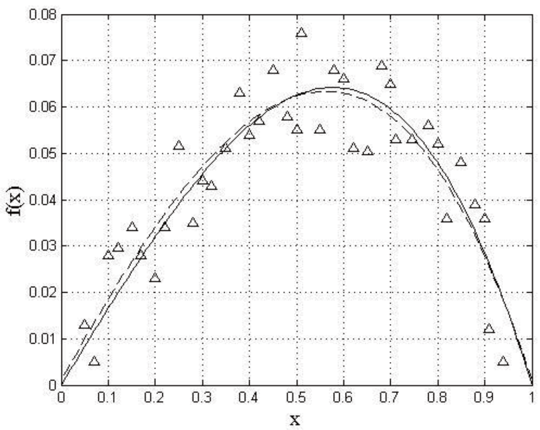
A conservative estimation of the convergence rate gives: $\mathbf{R_n} \cdot \mathbf{R_n} \le C_1 \exp\left[-C_2 n\right]$

This approach has been used successfully with RBF in the approximation and analysis of high-dimensional scattered data problems.

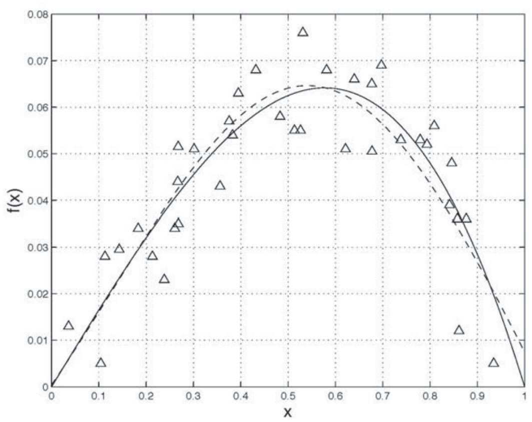
Approach #2 Results

Use of Scattered Data Approximation in Side-Stepping Λ

$$u(x)=\frac{(x^3+x)}{6}\ ,\quad u_0(x)=\frac{(x^3+x)}{6a}\ ,\quad u_a(x)=u_0(x)+\sum_{i=1}^{N}\phi_i(x)c_i$$



Interpolation of data with noise (±0.014) with a = 1.3.
Dashed curve – regularized ANN model with 1 RBF using SFA



Interpolation with noise (±0.05 and ±0.014) and a = 4.
Dashed curve – regularized ANN model with 1 RBF using SFA

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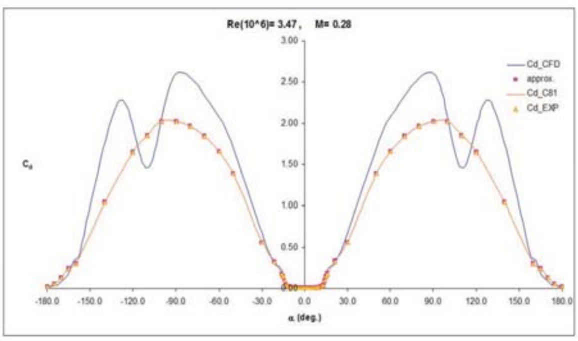
Approach #2 - Results

Regression / Fusion for An Airfoil Table

[J. Navarrete and A. J. Meade, AIAA 2004-0952]

We have used this approach to combine experimental airfoil coefficient data with numerical data in an effort to construct airfoil performance tables (C81 Tables) given limited data sets.

NACA 0012 AIRFOIL TESTCASE



C_d vs. α for NACA 0012 Test Run 29 with $\varepsilon=0$.
$Re=3.47\times10^6$, $M=0.28$.

Mechanical Engineering and Material Science Department

RICE University

Approach #2 - Results

Regression / Fusion of CFD Codes for Design

[D. Toal et al., *AIAA J.*, **46** 5 (2008)]

The use of scattered data approximation in side-stepping Λ has also been quite popular in computational aerospace design where it has been used in the construction of models by the fusion of multiple CFD solvers.

Fig. 1 Overview of the benchmark procedure.

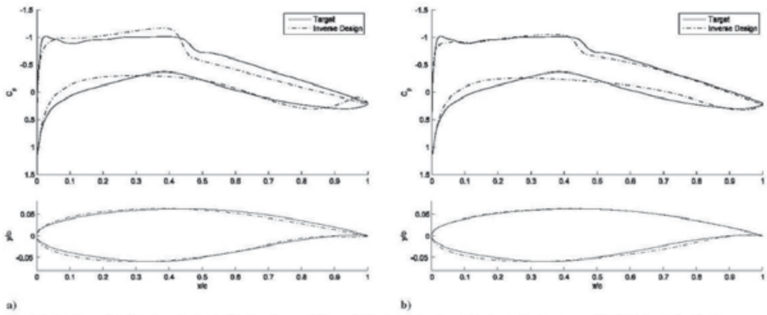


Fig. 7 Best airfoil design obtained with the nine-variable optimization using the a) 75-simulation budget and b) 150-simulation budget.

Approach #3

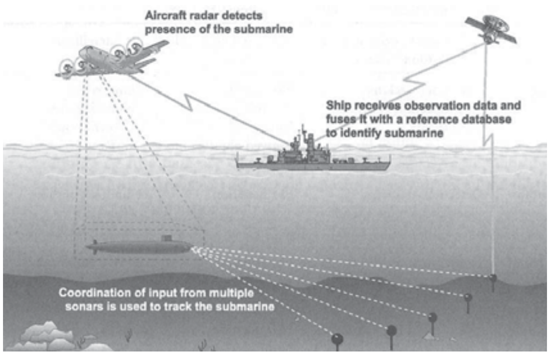
GTR questions:

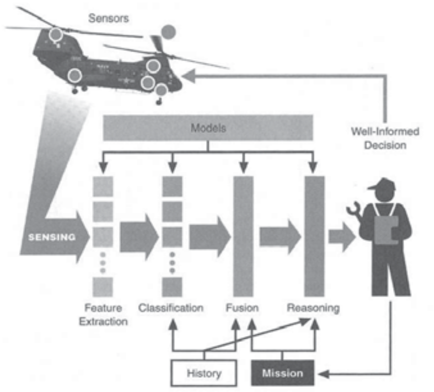
- The Green's functions for practical models are usually unavailable. Is it possible to use existing numerical solutions in the literature for fusion?
- Is it possible to find a value for Λ that satisfies $s\frac{\tau^2}{3}$ without the trouble of solving for $u_a(x)$ explicitly?
- Rather than using a single value for Λ is it possible to use a distributed one?
- Is the full data set required to accomplish data-model fusion?
- Is it possible to do all of this with minimal user interaction?
- Can we accelerate through experiments with incomplete mathematical models or no model at all (Black-Box modeling)?
- Can we reduce uncertainty in EFD and CFD?

Approach #3

Bayesian estimation is by far the most popular technique in empirical information fusion (Black Box problems, i.e., data fusion without mathematical models). This approach has been used in everything from defense related decision codes to condition-based maintenance.

However, some practical difficulties do arise in a Bayesian framework in setting the *a priori* probabilities since noninformative priors can cause erroreous bias to further reasoning.





Approach #3

I am a strong supporter of using scattered data approximation for aerospace problems since:

(a) it is a more straightforward numerical technique

(b) we already have an idea of the reliability of the sensors used in measurements

(c) setting the *a priori* probabilities are replaced with simply mapping the inputs to the known outputs.

(d) we showed in a previous paper that for a set basis function, this method should give us the best approximation.

We have applied our SFA algorithm to a Black-Box problem, a with proxy variable, in accelerating through an experiment.

Approach #3 - Results

Identification / Classification of Naval Rotorcraft Recovery

[A. Srivastava et al. *Proc. of the 2007 Infotech@Aerospace Conference, 2007.*]

Physical experiments, especially flight tests, can be very expensive and tedious.

Design of launch/recovery envelope for a U.S. Navy helicopter requires:

4-5 days of ship-board flights

4 pilots, 2 aircrew, 4 test engineers, 5 maintenance personnel

Manuvering the ship to simulate various sea conditions

Hundreds of thousands of US$

Figures: HH-60H and U.S. Navy Amphibious Assault Ship

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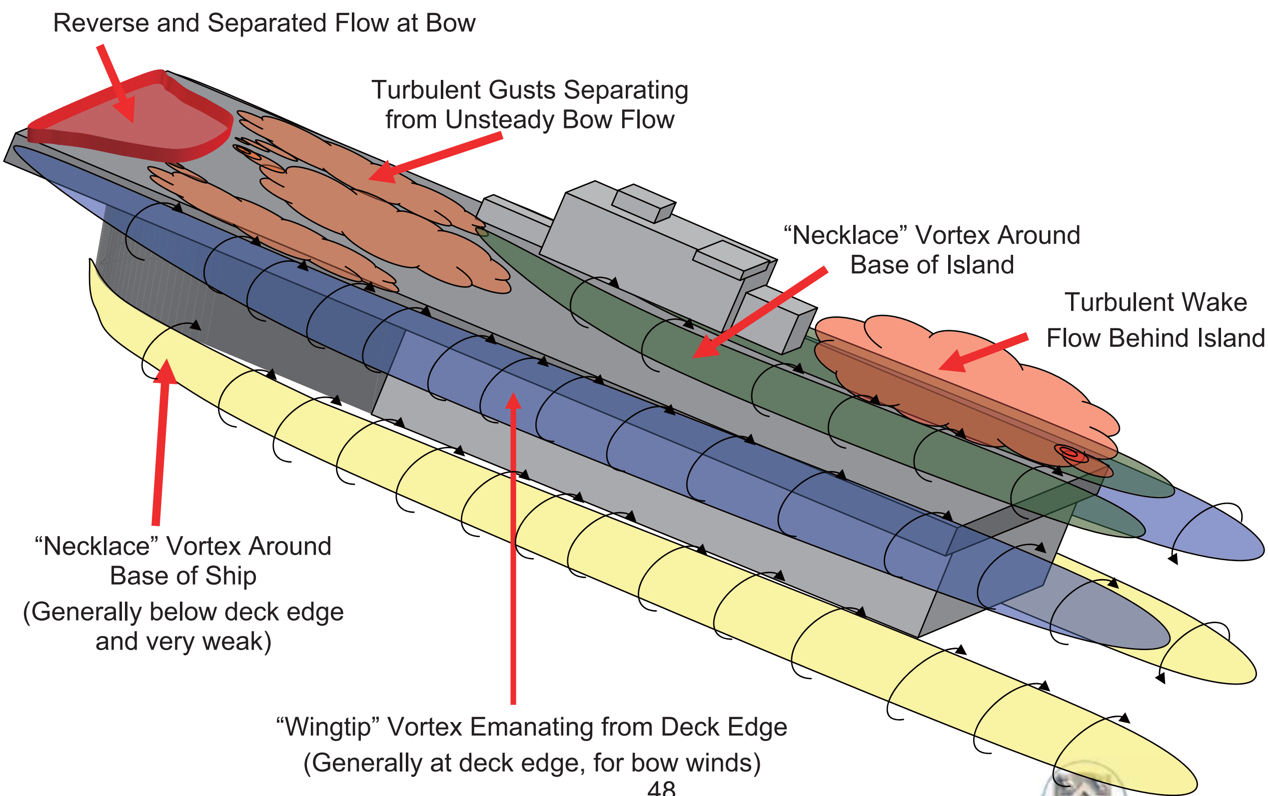
Approach #3 - Results



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Approach #3 - Results

Effort to replace the standard two-dimensional launch/recovery envelope. The SFA approach is used to construct a response surface that approximates the quality ratings of several HH-60H command pilots after recovery from Navy amphibious carriers.

369 data sets
13 dimensions
4 classes

PRS # Pilot Effort	Rating Description
1 Slight	No problems; minimal pilot effort required to conduct consistently safe shipboard evolutions under these conditions.
2 Moderate	Consistently safe shipboard evolutions possible under these conditions. These points define fleet limits recommended by NAWCAD Pax River.
3 Maximum	Evolutions successfully conducted only through maximum effort of experienced test pilots using proven test methods under controlled test conditions. Loss of aircraft or ship system likely to raise effort beyond capabilities of average fleet pilot.
4 Unsatisfactory	Pilot effort and/or controllability reach critical levels. Repeated safe evolutions by experienced test pilots are not probable, even under controlled test conditions.

Table: Pilot Rating Scale (PRS)

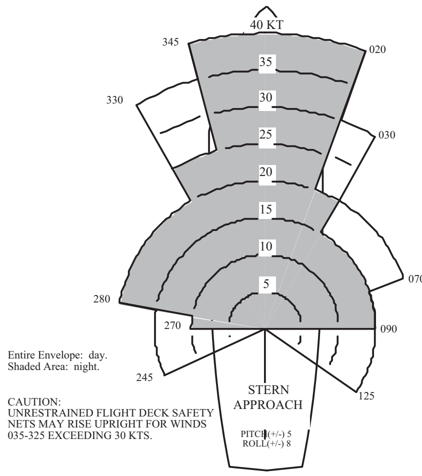

Figure: HH-60H Operational Recovery Envelope

Approach #3 - Results

This example was used to demonstrate that Black-Box fusion can be used for accelerating experiments and reducing uncertainty in experiments. This was thought to be a more rigorous test since there are no known models for the launch and recovery problem to fuse with the data and we are using a proxy variable.

We used DoE tools to select a small subset of the test matrix.
We imaged that the results from this first day of tests were then used to map the PRS to the inputs.

Input Index	Abbreviation	Definition (Units)
1	Ship Type	USN Ship Type: DD 967 (1), DDG 61 (2) DD 971 (3), DD976 (4)
2	WOD Spd	Wind Over Deck Speed. Relative wind speed (kts).
3	WOD Dir	Wind Over Deck Direction. Relative wind direction (degrees)
4	Long CG	Longitudinal CG station of helicopter. Length aft of datum (in).
5	Wfuel	Weight of fuel aboard helicopter (lb).
6	GW	Gross Weight of helicopter (lb).
7	Qavg	Average hover torque required during evolution (%).
8	Qmax	Maximum hover torque required during evolution (%).
9	Pitch	Pitch angle of ship during evolution (degrees)
10	Roll	Roll angle of ship during evolution (degrees)
11	OAT	Outside Air Temperature (degrees)
12	Hp	Pressure altitude (ft).
13	Hd	Density altitude (ft).

Table: Classification Model Inputs

Approach #3 - Results

The resulting approximation was then used to predict the results of the next subset. Errors in predictions were used to update the PRS model. Because the SFA is a numerical technique with a known convergence rate, we can use this convergence rate to conservatively estimate how many more data points are necessary for the surrogate to reach some desired accuracy.

Applying sensitivity analysis to the PRS model we can also get an idea of which inputs are relevant to the test and eliminate those that have little effect on the PRS.

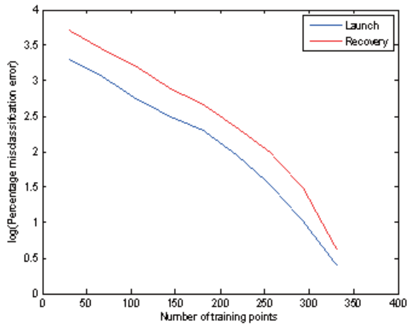

Figure: Decrease in error equivalent to the theoretical convergence rate.

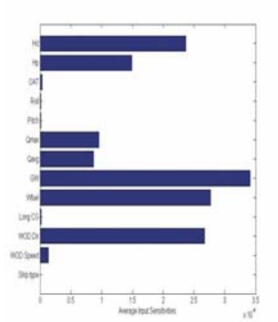

Figure: Average input sensitivities in the prediction of pilot ratings.

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Approach #3 - Results

This convergence rate for black-box modeling also means that our prediction accuracy increases in a known manner, thereby reducing the uncertainty as more data is added to our approximation.

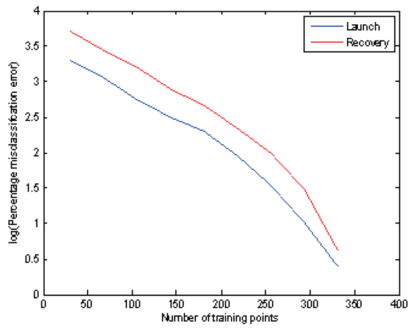

Figure: Decrease in error equivalent to the theoretical convergence rate.

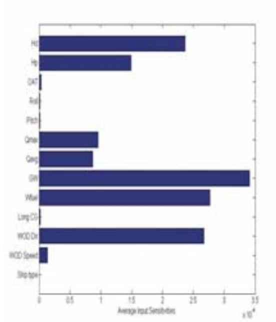

Figure: Average input sensitivities in the prediction of pilot ratings.

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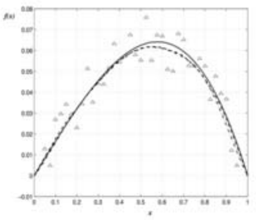
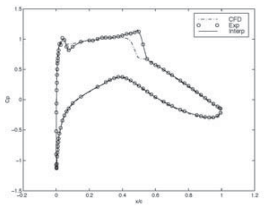
Conclusions & Future Work

It has been demonstrated that through the use of regularization, scattered data approximation, and meshless methods it is possible to seamlessly fuse mathematical models, computational methods, and experimental data. Possible benefits include:

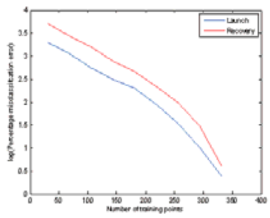
- Noise filtering of data (reducing the error bars)
- Filling in the blanks in experiments (including reducing uncertainty in EFD)
- Accelerating CFD solutions using data (including reducing uncertainty in CFD)
- Accelerating through test matrices (steering and predicting the amount of data needed)

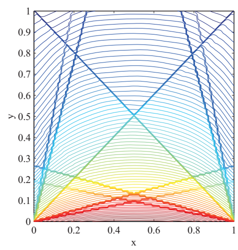


(a)



Comparison of Pressure Coefficient Distribution for the RAE 2822 Airfoil at $M_\infty = 0.725$, $\alpha_{num} = 2.10$, $Re = 6.5 \times 10^6$





Conclusions & Future Work

The Generalized Tikhonov Regularization framework shows promise as a way to merge theory, experimental observations, and computational fluid dynamics in a deterministic manner.

Further investigation of the method and the applications are required:

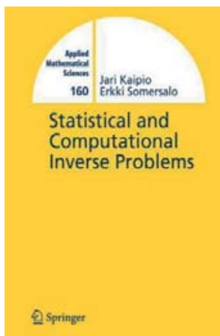
- Perform meshfree solution of $u_a(x)$ with the *solution* of $u_{CFD}(x)$ combined with $u_e(x_i)$. This would produce meshless and data-driven computational mechanics solvers.

- Investigate the method in designing better experiments and accelerating through them.

- Investigate the method with aerodynamic proxy data.

Suggested reading :
Keane & Nair

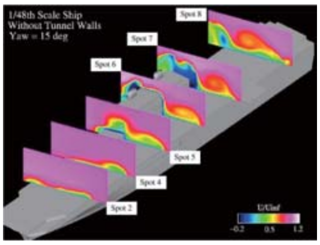
and Kapio & Somersalo :

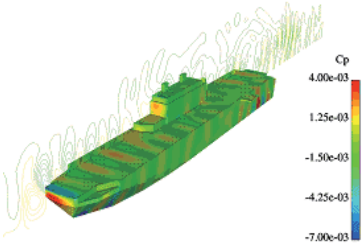


Conclusions & Future Work

I would like to see us as a community, using whatever tools become suitable, to routinely drive and merge experiments with various CFD solvers and have this become a regular part of our student's education and research thinking.

In addition to archiving our CFD results in journals, these models can then be used as international databases that can be further refined with advances in CFD and EFD. As a result, work does not have to be needlessly reproduced and uncertainty can be reduced in future experiments.





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Conclusions & Future Work

Questions?

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