Large-Eddy Simulation of Transition in Wall-Bounded Flow

P. Schlatter*, S. Stolz**, and L. Kleiser

Institute of Fluid Dynamics, ETH Zurich, Switzerland,
* KTH Mechanics, Stockholm, Sweden,
** Philip Morris Research & Development, Neuchâtel, Switzerland

ABSTRACT

Laminar-turbulent transition is a crucial phenomenon appearing in a variety of industrial applications. However the involved physical mechanisms as well as methods for reliable and accurate prediction of transition are still a matter of active research. In the present contribution, we give a brief overview on recent advances in the simulation and prediction of transitional and turbulent wall-bounded shear flows. The focus is on large-eddy simulations (LES), which differ from direct numerical simulations (DNS) by resolving only the large-scale, energy-carrying vortices of the fluid flow, whereas the fine-scale fluid oscillations, assumed to be more homogeneous, are treated by a subgrid-scale (SGS) model. The application of LES to flows of technical interest is promising and LES is getting more and more applied to practical problems. The main reason for this is that LES provides an increased accuracy compared to solutions of the (statistical) Reynolds-averaged Navier-Stokes equations (RANS), while requiring only a fraction of the computational cost of a corresponding fully-resolved DNS. Nevertheless, LES of practical transitional and turbulent flows still require massive computational resources and the use of large-scale computer facilities.

Key Words: Large-Eddy Simulation, Deconvolution Modelling, Wall Turbulence, Transition

1. Laminar-Turbulent Transition

Fluid flows are important in many technical applications of today's industrial world. The knowledge of the local fluid state, i.e. laminar or turbulent, is of major importance, since for instance drag and mixing significantly differ between the ordered laminar flow and the chaotic turbulent motion. Applications include e.g. flows along wings, intermittent flows around turbine blades and in combustion engines. The laminar-turbulent transition process and specifically its triggering mechanisms are not fully understood even nowadays. A summary of developments in transition research is given in the review article by Kachanov (1994) and in the monograph by Schmid & Henningson (2001).

A schematic overview of laminar-turbulent transition is given in Fig. 1 (taken from the LES presented in Schlatter, 2005) for the canonical case of plane incompressible channel flow excited by Tollmien- Schlichting (TS) waves (natural transition). The fluid flows along the plate until at a certain downstream position the laminar flow becomes unstable giving rise to two-dimensional wave disturbances. These spanwise rollers rapidly evolve into three- dimensional perturbations of triangular shape (Λ-vortices), which in turn tend to break down into localised turbulent spots through the formation of pronounced hairpin vortices. The spots grow and merge to form a fully turbulent flow.

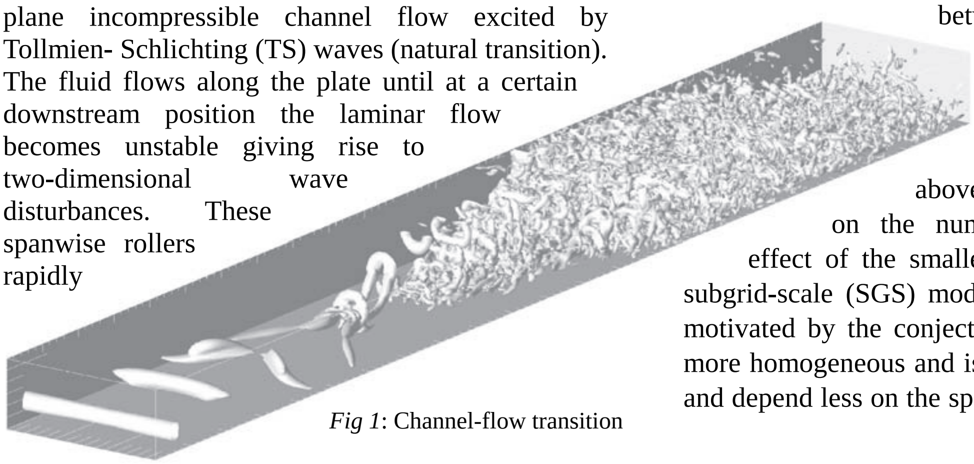

Fig 1: Channel-flow transition

2. Numerical Simulation: LES

The fully resolved numerical solution of the Navier-Stokes equations is extremely expensive even for moderate Reynolds numbers *Re* since the required CPU time roughly scales as Re^3. Practical high Reynolds-number calculations thus need to be performed using simplified turbulence models. Commonly used methods include the Reynolds-averaged Navier-Stokes equations (RANS) in which the mean flow is computed with statistical turbulence models. A technique with a level of generality in between DNS and RANS is the large-eddy simulation (LES). In an LES, only eddies (turbulent vortices) above a certain size are resolved on the numerical grid, whereas the effect of the smaller scales is modelled by a subgrid-scale (SGS) model. The scale separation is motivated by the conjecture that smaller eddies are more homogeneous and isotropic than the large ones and depend less on the specific flow situation. For an

LES thus only a fraction of the computational cost compared to a fully resolved DNS (typically of order 0.1-1%) is required.

The success of an LES is essentially dependent on the quality of the underlying subgrid scale (SGS) model, but also on the applied numerical discretisation scheme (its order and accuracy). However, the latter point has only recently been put into active consideration (Chow and Moin, 2003). The most common SGS model is the Smagorinsky (1963) model, based on the eddy-viscosity assumption. A major generalisation of SGS modelling was achieved by Germano et al. (1991) who proposed an algorithm which allows for dynamically adjusting coefficients of SGS models. A different class of SGS models has been introduced by Bardina et al. (1980) based on the scale-similarity assumption. Considerable research effort has recently been devoted to the development of SGS models of velocity estimation or deconvolution type, see e.g. the review by Domaradzki and Adams (2002). General reviews about different strategies for LES and SGS modelling are given in Lesieur and Métais (1996), Meneveau and Katz (2000) and Piomelli (2001) as well as in the recent text books by Sagaut (2005), Geurts (2003) and Lesieur et al. (2005).

3. LES of Laminar-Turbulent Transition

In transitional flows one is typically dealing with stability problems where small initial disturbances with energies many orders of magnitude smaller than the energy of the steady base flow are amplified and may finally evolve into turbulent fluctuations. Moreover, the spatial and temporal evolution of various wave disturbances and their nonlinear interaction needs to be computed accurately over many disturbance cycles. An SGS model suitable for transition should be able to deal equally well with laminar, various stages of transitional and turbulent flow states. The model should leave the laminar base flow unaffected and only be effective when nonlinear interactions between the resolved and non-resolved scales become important. The initial laminar flow and the following growth of the instability waves is often sufficiently resolved even on a coarse LES grid.

While a number of applications of different SGS models to turbulent flows have been analysed, the application to transitional flows has become an active field of research only recently. An example of the difficulty of transitional flows is that the classical Smagorinsky model is too dissipative and usually, in addition to distorting laminar flows, relaminarises transitional flows. Several improvements have been proposed, e.g., by Piomelli et al. (1990), Voke and Yang (1995) and Germano et al. (1991) with the dynamic model. Several extended and more robust versions of the dynamic model have been proposed, e.g. the Lagrangian dynamic SGS model (Meneveau et al., 1996) or the localisation model (Ghosal et al., 1995). A slightly different approach was followed by Ducros et al. (1996) with the filtered structure function (FSF) model. A high-pass filter is used to decrease the influence of large scales in the calculation of the SGS terms. As a consequence, the model influence is reduced in regions where the mean-flow shear dominates over the turbulent shear, e.g. in the vicinity of walls or in laminar regions. Related models include the filtered Smagorinsky model (Sagaut et al., 2000) and also the dynamic mixed-scale model (Sagaut, 1996). Another way to avoid model contributions in laminar flow was followed by Vreman (2004) and subsequently Park et al. (2006) by constructing the SGS stress tensor such that it vanishes in undisturbed flow. The variational multiscale (VMS) method by Hughes et al. (2000), providing an explicit scale separation between the large and small scales based on disjunct spectral filters has, e.g., been used for simulating bypass transition along a flat plate (Calo, 2004).

As to the work of our group, in Schlatter (2005), results obtained using LES of transitional and turbulent incompressible channel flow are presented. These simulations have been performed using spectral methods in which numerical errors (differentiation, aliasing) are small. Various classical and newly devised SGS closures have been implemented and evaluated, including the approximate deconvolution model (ADM, Stolz and Adams, 1999), the relaxation-term model (ADM-RT) (Stolz and Adams, 2003 and Schlatter et al., 2004), and the new class of high-pass filtered (HPF) eddy-viscosity models (Stolz et al. 2005, Schlatter et al., 2005a and Stolz et al., 2005, 2007). These models are discussed briefly in the following.

In Schlatter et al. (2004), in addition to the original ADM algorithm, new variants have been examined. In particular an SGS model (ADM-RT model) with direct relaxation regularisation of the velocities based on a 3D high-pass filtering of the computational quantities is investigated. This model is related to the spectral vanishing viscosity (SVV) approach (Karamanos and Karniadakis, 2000). The appropriate definition of the relaxation term causes the model contributions to vanish during the initial stage of transition and, approximately, in the viscous sublayer close to walls.

The application of the HPF eddy-viscosity models to transitional flows was presented in Stolz et al. (2005), see also Vreman (2003). The HPF formulation is related to the VMS by computing the SGS terms on a highpass-filtered velocity field, thereby, with suitable filters, ignoring mean shear. Detailed analysis of the energy budget including the SGS terms revealed that the contribution to the mean SGS dissipation is nearly zero for the HPF models, while it is a significant part of the SGS dissipation for other models (Schlatter et al., 2005a). Moreover, unlike the classical eddy-viscosity models, the HPF models are able to predict backscatter. It has been shown that in channel flow that locations with intense backscatter are closely related to low-speed turbulent streaks in both LES and filtered DNS data.

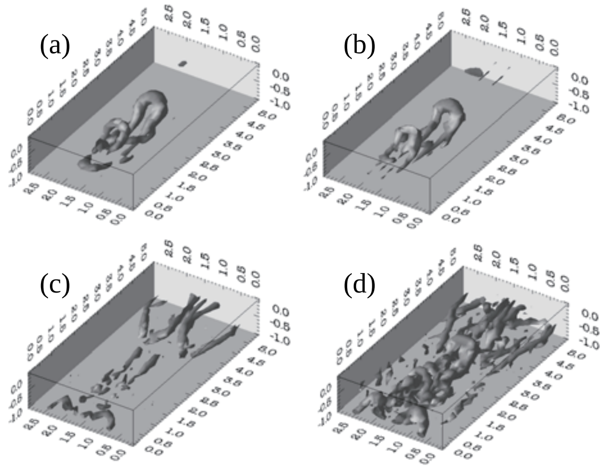

Fig. 3: Comparison of the prediction of transitional structures using different SGS models: (a) fully-resolved DNS, (b) ADM-RT, (c) dynamic Smagorinsky model, (d) no-model LES (coarse-grid DNS). The box contains only 32x32x33 grid points (from Schlatter et al., 2005b).

The above references demonstrate that, e.g. for the model problem of temporal transition in channel flow, averaged integral flow quantities like the skin friction Reynolds number Re_τ or the shape factor H_{12} can be predicted reasonably well by LES even on coarse meshes (see also Meyers and Sagaut, 2007). However, for a reliable LES in particular applied to transitional flows, it is equally important to faithfully represent the physically dominant transitional flow mechanisms and their 3D vortical structures such as the formation of Λ and hairpin vortices. A successful SGS model needs to predict those structures well even at low resolution, as demonstrated by Schlatter et al. (2005b), Schlatter et al. (2006) and Stolz et al. (2007). A comparison of various SGS models and their performance to predict transitional structures is shown in Fig. 2 for temporal channel-flow transition. When considering integral quantities only (e.g. skin friction) major differences between the predictions could not be established (Schlatter et al., 2004). The flow structures however have been found to be fairly different. In particular, the no-model LES and the standard dynamic Smagorinsky model fail to predict a distinct roll-up of the shear layers, and additionally spurious structures appear which lead to premature breakdown to turbulent flow. On the other hand, the high-order relaxation in the ADM-RT model closely follows the evolution of the exact (DNS) data.

In Schlatter et al. (2006), different SGS models have been tested and compared in both the temporal and the spatial transition simulation approach. Fig. 3 shows a series of visualisations taken from a spatial LES using the ADM-RT model during classical K-type transition clearly showing the relevant series of break-ups of the distorted vortical structures eventually leading to a turbulent flow.

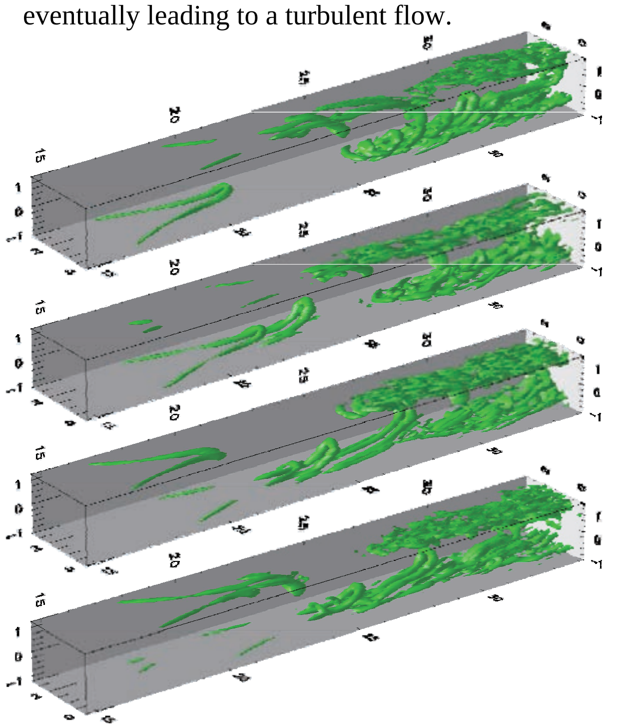

Fig. 2: Sequence (top to bottom) of vortical structures during spatial K-type transition using the ADM-RT model with only 32 grid points in the wall-normal and spanwise direction (Schlatter et al., 2006).

Compressible supersonic boundary-layer transition has recently been considered by Stolz et al. (2007). Compressible flows differ in various aspects from incompressible one: Not only is the type of equations changed to hyperbolic, giving the possibility of shock waves, but also the applied numerical methods are different. Whereas the above results used spectral methods, for the compressible case finite differences were employed. It is important to test modelling approaches also for compressible transition and turbulence. The results in Stolz et al. (2007) show that using both ADM and the HPF model accurate approximate statistics (velocity profiles, skin friction etc.) are found. In addition ADM was found to be capable to predict instantaneous (flow structures) and at significantly reduced resolution.

At present, research in LES follows various directions. On the one hand, improved and new SGS models are developed; existing models are also applied to more complex flow cases with good results. In that respect, LES has matured to a research tool to predict e.g. complex transitional scenarios, see the recent application to bypass transition and control mechanisms (Schlatter et al. 2007a,b). On the other hand, methods to actually quantify the errors of LES, e.g. induced by the lower resolution, but also by the discretisation scheme (Geurts, 2006) are considered. Solution-adaptive grid-refinement methods are currently being developed which could allow more reliable (and efficient) results for complex flow cases (Hoffman, 2006).

4. Summary

The results obtained for transitional wall-bounded flows using various SGS models show that it is in fact possible to accurately simulate transition using LES on relatively coarse grids. However, the performance of the various models examined is considerably different with respect to an accurate prediction of e.g. the transition location and the characteristic transitional flow structures.

By examining instantaneous flow fields from LES of channel flow transition, additional distinct differences between the SGS models can be established. Some models which are based on high-pass filtering, e.g. ADM, ADM-RT and also the HPF eddy-viscosity models, are able to provide a realistic description of the flow structures up to the point of breakdown. In addition, the HPF eddy-viscosity models can be easily implemented in particular as an alternative to classical fixed-coefficient eddy-viscosity models, whilst performing significantly better than their non-highpass-filtered counterparts.

To conclude, LES using advanced SGS models are able to faithfully simulate flows which contain intermittent laminar, turbulent and transitional regions.

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