

On the concept of hydraulically smooth wall

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ABSTRACT

A search for a precise definition of hydraulically smooth wall is carried out. It is argued that in the case of transitional flows such definition can be based on the onset of flow instabilities. Flow in a channel with distributed surface roughness is considered as a case study. Results of the linear stability analysis show that the presence of the roughness destabilizes the traveling-wave instability as well as introduces a new instability that manifests itself in the form of streamwise vortices. The critical conditions for the occurrence of both instabilities are given for different classes of roughness shape. It is shown that these conditions can be predicted with a reasonable accuracy in the case of an arbitrary (but Fourier transformable) roughness by considering only the leading Fourier mode (wavy-wall model). A segment in the parameter space where the roughness does not induce any instability regardless of its shape has been identified; this segment identifies conditions under which the rough wall behaves as a hydraulically smooth wall.

Key Words: flow instabilities, distributed surface-roughness, hydraulically smooth wall.

1. Introduction

Flows over rough walls have been studied since the early works of Hagen¹ and Darcy², which were focused on turbulent regimes. Reynolds³ was the first to pose the problem in the context of laminar-turbulent transition. While the questions studied, i.e., what kind of effects the presence of distributed surface roughness can induce in a flow and when a rough wall behaves as hydraulically smooth, are of fundamental importance their rational resolution is still lacking. Both questions are of considerable practical importance in several application areas, e.g., design of large Reynolds number laminar airfoils, small Reynolds number turbulent airfoils, compact heat exchangers, laminar electrostatic precipitators, etc. The original investigations involved measurements of turbulent flows in open channels and in pipes. Various possible roughness forms were classified using the concept of "equivalent roughness"³. Phenomenological effects of the "equivalent roughness" were summarized in the form of friction coefficient⁵⁻⁷. These and other similar investigations show that surface roughness contributes directly to the dynamics of turbulent flow only if the wall is hydraulically rough. The concept of hydraulic smoothness is very appealing; however, no precise criterion exists for predicting whether a given surface can be considered as being hydraulically smooth for flow conditions of interest. While the modelling concepts of this type have been continuously re-evaluated⁸⁻⁹, they failed so far to uncover the mechanisms that govern the complex, flow-condition-dependent interaction between the

roughness geometry and the moving fluid.

This presentation reviews the role played by distributed roughness in the laminar-turbulent transition process in shear layers. It is known that this process involves various instabilities that eventually lead to the fully turbulent state. The experimental evidence shows that the roughness contributes directly to the dynamics of the flow only if its amplitude is sufficiently large. A frequently used criterion for determination of the critical roughness size is that the roughness Reynolds number $Re_k = U_k k / \nu < 25$ ¹⁰, where k is the roughness height, U_k is the undisturbed velocity at height k and ν the kinematic viscosity. Such a criterion, however, does not address the issue of shape and distribution of the roughness.

There is a large body of experimental observations focused on the laminar-turbulent transition that provide phenomenological description of the flow response in the form of correlations between the height of the roughness, the flow conditions and the critical Reynolds number for certain classes of geometrical forms of the roughness¹¹⁻¹⁵. The range of applicability of these correlations is not certain because they are based on a limited experimental data and have been determined for, in essence, artificially created roughness forms. These correlations, nevertheless, form the basis of all roughness sensitive designs.

The surface roughness can be divided into three classes for the purposes of discussion, i.e., isolated

two-dimensional roughness, e.g., spanwise trip wire, isolated three-dimensional roughness and distributed roughness. The transition mechanisms for the first class of roughness is associated with inflectional separated velocity profiles and are considered understood at least on qualitative level¹⁶⁻¹⁸. The characteristic feature of the flow around an isolated, three-dimensional roughness element is the presence of the horseshoe vortex that generates streamwise vortices on the downstream side¹⁰. The transition mechanism is thought to be associated with the strong instabilities of inflectional shear layers set up by the streamwise vortices, similar to the case of Görtler instability¹⁹. The effects of distributed roughness are not understood¹⁰. Various experiments indicate that when the roughness is operative the departure from the laminar state is explosive²⁰⁻²¹. Theoretical attempts based on the roughness-induced distortion of velocity profile proved inconclusive²²⁻²⁴ similarly as did concepts based on the roughness-induced additional mixing²⁵. The spectral model of roughness shape²⁶ proved to be very powerful and holds a promise to uncover the mechanisms associated with the distributed roughness. Theoretical analysis of the two-dimensional traveling-wave instability²⁷ shows that the roughness is responsible for the reduction of the critical Reynolds number and the amount of reduction is in agreement with the experimental observations²⁸. Three-dimensional analyses of Couette flow over wavy-wall²⁹ and Poiseuille flow in a converging-diverging channel³⁰ show that surface corrugations are able to generate streamwise vortices. Surface roughness may also play a large role in the transition process through amplification of the transient growth mechanism³¹; however, this role remains to be substantiated.

The main objective of the analysis described in the next section is the determination of the role played by distributed surface roughness in the early stages of the transition process through the use of the linear stability theory. This analysis uses spectral models²⁶ where the roughness geometry is represented in terms of Fourier expansions. Determination of the effects of different geometries is reduced to scans of parameter space formed by the coefficients of such expansions. Use of stability theory provides a convenient tool for the identification of the conditions when the roughness is not hydraulically active; roughness that does not destabilize the flow modifies the flow in an insignificant manner and thus such wall may be considered as hydraulically smooth. The reader should note that the just proposed definition of hydraulic smoothness for transitional flows is different from the common albeit ill-defined smoothness for turbulent flows where the smoothness

implies turbulent friction independent of the roughness.

2. Outline of the analysis

We follow Ref.[32] and begin with the plane Poiseuille flow confined between flat rigid walls at $y=\pm 1$ and extending to infinity in the x -direction. Velocity and pressure fields in the form

$$\mathbf{V}_0(\mathbf{x}) = [u_0(y), 0] = (1 - y^2, 0), \quad p_0(\mathbf{x}) = -2x / \text{Re}, \quad (1)$$

describe the fluid motion, where the motion is directed towards the positive x -axis, $\mathbf{x}=(x, y)$, and the Reynolds number Re is based on the half-channel height and the maximum x -velocity. Assume that the lower wall is replaced by a corrugated wall whose location $y_L(x)$ is specified as

$$y_L(x) = -1 + \sum_{n=-\infty}^{n=\infty} S^{(n)} e^{in\alpha x} \quad (2)$$

where $S^{(n)} = S^{(-n)*}$ and star denotes the complex conjugate. The flow in the corrugated channel can be represented as

$$\mathbf{V}_2(\mathbf{x}) = [u_2(x, y), v_2(x, y)] = \mathbf{V}_0(\mathbf{x}) + \mathbf{V}_1(\mathbf{x}) = [u_0(y), 0] + [u_1(x, y), v_1(x, y)], \quad (3)$$

$$p_2(\mathbf{x}) = p_0(x) + p_1(x, y),$$

where \mathbf{V}_1 and p_1 are the velocity and pressure modifications owing to the presence of the corrugation. Substitution of the above representation of the flow quantities into the Navier-Stokes and continuity equations, introduction of stream function defined as $u_1 = \partial_y \Psi, v_1 = -\partial_x \Psi$, elimination of pressure and representation of the unknowns in the form of Fourier expansions

$$\Psi(x, y) = \sum_{n=-\infty}^{n=+\infty} \Phi^{(n)}(y) e^{in\alpha x} \quad (4)$$

where $\Phi^{(n)} = \Phi^{(-n)*}$, $f_u^{(n)} = f_u^{(-n)*}$, $f_v^{(n)} = f_v^{(-n)*}$, lead to

a system of nonlinear ordinary differential equations for the functions $\Phi^{(n)}$, $n \geq 0$, in the form

$$\left[D_n^2 - in\alpha \text{Re}(u_0 D_n - D_n^2 u_0) \right] \Phi^{(n)} - i\alpha \text{Re} \sum_{k=-\infty}^{k=+\infty} [k D \Phi^{(n-k)} D_k \Phi^{(k)} - (n-k) \Phi^{(n-k)} D_k D \Phi^{(k)}] = 0, \quad (5)$$

where $D = d/dy$, $D_n = D^2 - n^2 \alpha^2$. The boundary conditions at the channel walls are expressed in the form

$$u_0(y_L(x)) + u_1(x, y_L(x)) = 0, \quad v_1(x, y_L(x)) = 0, \quad (6a)$$

$$u_1(x, 1) = 0, \quad v_1(x, 1) = 0. \quad (6b)$$

The above formulation is closed with the fixed volume flux condition and the problem is solved numerically.

The linear stability analysis begins with the governing equations in the form of vorticity transport and continuity. Unsteady, three-dimensional disturbances are superimposed on the mean part in

the form

$$\boldsymbol{\omega} = \boldsymbol{\omega}_2(x, y) + \boldsymbol{\omega}_3(x, y, z, t), \quad \mathbf{V} = \mathbf{V}_2(x, y) + \mathbf{V}_3(x, y, z, t),$$

where subscripts 2 and 3 refer to the mean flow and the disturbance field, respectively. The above equation is substituted into the field equations, the mean part is subtracted and the equations are linearized. The resulting disturbance equations have the form

$$\begin{aligned} \partial \boldsymbol{\omega}_3 / \partial t + (\mathbf{V}_2 \cdot \nabla) \boldsymbol{\omega}_3 - (\boldsymbol{\omega}_3 \cdot \nabla) \mathbf{V}_2 + (\mathbf{V}_3 \cdot \nabla) \boldsymbol{\omega}_2 - \\ (\boldsymbol{\omega}_2 \cdot \nabla) \mathbf{V}_3 = \text{Re}^{-1} \nabla^2 \boldsymbol{\omega}_3, \\ \nabla \cdot \mathbf{V}_3 = 0, \quad \boldsymbol{\omega}_3 = \nabla \times \mathbf{V}_3 \end{aligned} \quad (7a-c)$$

and are subject to the homogeneous boundary conditions

$$\mathbf{V}_3(x, 1, z, t) = 0, \quad \mathbf{V}_3(x, y_L(x), z, t) = 0 \quad (7d)$$

where y_L is given by Eq. (2). The disturbance velocity vector is assumed in the form

$$\mathbf{v}_3(x, y, z, t) = \sum_{m=-\infty}^{m=+\infty} [g_u^{(m)}(y), g_v^{(m)}(y), g_w^{(m)}(y)] e^{i[(\delta+m\alpha)x + \beta z - \sigma t]} + CC \quad (8)$$

Substitution of (8) into (7) leads to an eigenvalue problem for (δ, β, σ) for the ordinary differential equations describing functions

$g_u^{(m)}, g_v^{(m)}, g_w^{(m)}$. The system of equations governing $g_u^{(m)}, g_v^{(m)}, g_w^{(m)}$ has the form

$$\begin{aligned} S^{(m)}(t_m g_w^{(m)} - \beta g_u^{(m)}) + C g_v^{(m)} = \\ i \text{Re} \sum_{n=-\infty}^{n=\infty} (W_u^{(m,n)} g_u^{(m-n)} + W_v^{(m,n)} g_v^{(m-n)} + W_w^{(m,n)} g_w^{(m-n)}) \\ T^{(m)} g_v^{(m)} = \\ - \text{Re} \sum_{n=-\infty}^{n=\infty} (B_u^{(m,n)} g_u^{(m-n)} + B_v^{(m,n)} g_v^{(m-n)} + B_w^{(m,n)} g_w^{(m-n)}) \\ i t_m g_u^{(m)} + D g_v^{(m)} + i \beta g_w^{(m)} = 0, \quad -\infty < m < \infty \end{aligned} \quad (9a-c)$$

where the explicit forms of the operators T, S, C, W, B are given in Ref.[32]. The boundary conditions have the form

$$\begin{aligned} g_u^{(m)}(1) = g_v^{(m)}(1) = g_w^{(m)}(1) = 0. \\ \sum_{m=-\infty}^{m=+\infty} [g_u^{(m)}(y_L), g_v^{(m)}(y_L), g_w^{(m)}(y_L)] = 0 \end{aligned} \quad (10)$$

Equations (9) with boundary conditions (10) have nontrivial solutions only for certain combinations of parameters δ, σ and β . The required dispersion relation has to be determined numerically. For the purposes of calculations, the problem is posed as an eigenvalue problem for σ and its solution is determined numerically.

3. Results and Discussion

Calculations have been carried out for the roughness in the form of sinusoidal wall (wavy wall model), wall with triangular indentations, wall with rectangular indentations and wall with "bump" 32

indentations. Unstable disturbances in the form of streamwise vortices and traveling waves have been identified in all cases. A critical Reynolds number, a critical disturbance wavenumber and a critical roughness wavenumber have been identified for each roughness amplitude S . Results displayed in Fig.1 show that it is possible to identify the maximum permissible roughness amplitude that does not induce any instability for flow conditions of interest regardless of the roughness shape.

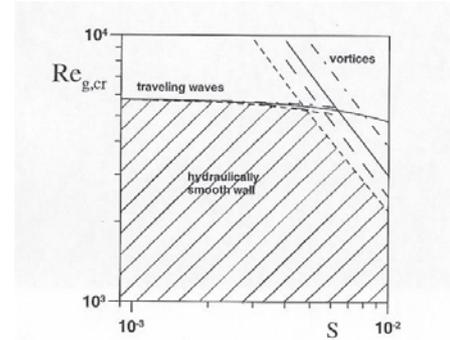


Figure 1. Variations of the global critical Reynolds number $Re_{g,cr}$ describing the traveling-wave instability and the vortex-like instability for the corrugated channel as a function of the corrugation amplitude S . The dash, dash-dot and continuous lines correspond to the corrugation in the form of rectangular grooves, triangular grooves and "sine-bump" grooves, respectively. The shaded area corresponds to the flow conditions that do not produce any instability for the corrugation geometries subject to this investigation.

4. Summary

The critical curves displayed in Fig. 1 demonstrate qualitative similarity of flow response for all corrugation geometries subject to this investigation. If the corrugation amplitude for a given shape and distribution (as defined by the corrugation wave number) and given flow conditions (as defined by the flow Reynolds number) is sufficiently small, such corrugation is able to induce only small modifications in the flow. When the size of the corrugation reaches critical conditions, it can induce large changes in the flow through various instability processes. We can therefore use the onset of any instability as the event that defines the conditions when the wall ceases to be hydrodynamically smooth.

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