Minimization of Unsteady Thermal Deformation by Using Laminated Structures under the Stress Restrictions

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Abstract

Laminated beam structures are designed in order to minimize thermal deformations in steady or unsteady temperature field. To suppress thermal deformation, composite materials that has negative longitudinal coefficient of thermal expansion are layered with material that has positive coefficient of thermal expansion (CTE). Assuming steady temperature fields, beam with no strain at central axis and no curvature can be designed. In unsteady temperature fields, it is possible to suppress thermal deformations while lowering thermal stresses. If it takes a long time until the temperature distribution get steady, the beam should be designed with considering unsteady temperature distributions because the beam that was designed with considering only steady temperature distribution could have large thermal deformations in its transitional period. For suppressing thermal deformations, interlaminar shearing stress, and interlaminar moment, materials with large Young's Modulus, CTE, and thermal conductivity are effective to use for low temperature side, oppositely, materials with small Young's Modulus, CTE, and thermal conductivity are effective to use for high temperature side.

1. Introduction

Many space structural components experience a non-uniform temperature variation because of solar radiant heating. Through-thickness temperature variation of thin structures may cause thermal deformations composed of in-plane expansion and out-of-plane curvature.

Nowadays, not only aluminum alloys but also carbon fiber composites are often used for space structures because of their lightweight, high strength and small coefficient of thermal expansion (CTE). Even if its CTE is small, however, large through-thickness temperature gradient in the structure may cause thermal deformations and that could lead undesirable problems to structures like space antennas because such kind of structure has to keep high accuracy of dimension.

By the way, there are some composite materials that possess a negative axial CTE and high stiffness. With the availability of such kind of materials, composite laminae that has negative axial CTE may be made. By laminating these composite laminae with other laminae that has positive CTE, thermal deformations can be suppressed.

Whetherhold and Wang investigated the way to eliminate both in-plane expansion and out-of-plane curvature of symmetric laminated beam or eliminate out-of-plane curvature while matching in-plane expansion in a desired value in steady temperature distributions[1][2].

In this paper, the methods to suppress thermal deformations by using asymmetric laminated beam in steady and unsteady temperature variations are investigated.

2. Analysis model

Figure 1 shows the model of this study. It is an asymmetric three-layer laminated beam. The upper side is kept in high temperature and the bottom side is kept in low temperature.

Where,

E : Young's Modulus

α : Coefficient of thermal expansion, CTE

λ : Thermal conductivity

c : Specific heat

ρ : Density

T : Temperature

h : Thickness

Thickness ratios ϕ_1 and ϕ_3 are respectively defined as follows.

$$\phi_1 = \frac{h_1}{h_2},\ \phi_3 = \frac{h_3}{h_2} \tag{1}$$

In this study T = 0 [°C] is defined as the reference temperature in which materials have no thermal deformation.

Table 1 shows the material properties used in this study. For composite materials, CTE means longitudinal CTE.

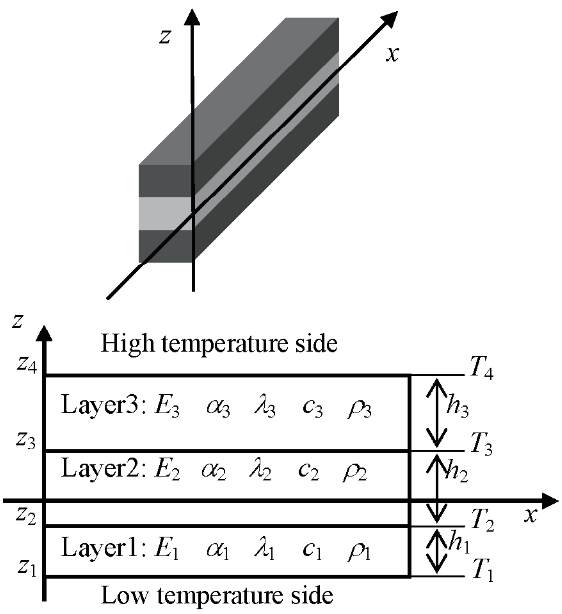


Fig. 1. Analysis Model

3. Designing laminated beam in steady temperature distribution

3.1. Temperature distribution

The temperature distributions are assumed linear for each layer, so they are represented as below.

$$T(z)=\begin{cases} T_1+\dfrac{T_2-T_1}{h_1}(z-z_1) & \text{for Layer 1} \\ T_2+\dfrac{T_3-T_2}{h_2}(z-z_2) & \text{for Layer 2} \\ T_3+\dfrac{T_4-T_3}{h_3}(z-z_3) & \text{for Layer 3} \end{cases} \tag{2}$$

Since the heat flux which pass through each layer is constant,

$$\begin{aligned} \frac{\lambda_1}{h_1}(T_2-T_1)&=\frac{\lambda_2}{h_2}(T_3-T_2)=\frac{\lambda_3}{h_3}(T_4-T_3) \\ &=\frac{T_4-T_1}{h_1/\lambda_1+h_2/\lambda_2+h_3/\lambda_3} \end{aligned} \tag{3}$$

From these equations, T_2 and T_3 are represented respectively as,

$$\begin{aligned} T_2&=T_1+\frac{(T_4-T_1)\phi_1}{\phi_1+(1+\phi_3/r_3)r_1}, \\ T_3&=T_4-\frac{(T_4-T_1)\phi_3}{\phi_3+(1+\phi_1/r_1)r_3} \end{aligned} \tag{4}$$

Here,

$$r_1=\frac{\lambda_1}{\lambda_2},\ r_3=\frac{\lambda_3}{\lambda_2} \tag{5}$$

This means that temperature distribution depends on only bottom side temperature T_1 and upper side temperature T_4.

3.2. Thermal stress and thermal deformations

Thermal stress for any z is given as below.

$$\sigma_x(z)=E(z)\left[\varepsilon_x^0+\kappa_x z-\alpha(z)T(z)\right] \tag{6}$$

Using this equation and condition of equilibrium, thermal deformations are given as follows.

$$\begin{Bmatrix} \varepsilon_x^0 \\ \kappa_x \end{Bmatrix}=\begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}\begin{Bmatrix} N_x^T \\ M_x^T \end{Bmatrix} \tag{7}$$

Here,

ε_x^0 : strain at the central axis of the beam

κ_x : curvature

N_x^T : thermal force

M_x^T : thermal moment

A : in - plane stiffness

B : coupling stiffness

D : out - of - plane stiffness

3.3. Optimization

3.3.1. Optimization theory

In this study, the barrier method is used for constraint conditions of parameters and the penalty method is used for constraint conditions of other functions. In the conjugate gradient method, unless the Hessian matrix of objective function is positive definite, descent direction vector dose not face toward

Table 1 Material Properties

Material	Young's Modulus E (GPa)	CTE (longitudinal) α (μ/°C)	Thermal conductivity λ (W/mK)	Specific heat c (J/kgK)	Density ρ (kg/m^3)
ASGr/Ep	138	-0.3	0.71	1.37	1.40
Kevlar/Ep	76	-4.0	0.16	1.10	1.60
P100Gr/Ep	480	-1.22	2.0	0.95	1.80
Aluminum	69	24	180	0.90	2.70

descent direction but saddle point of the objective function. Objective function in this study is composed of strain, curvature, and stress, and Hessian matrix of these are not always positive definite. So optimization problem is solved with the steepest descent method when descent direction vector does not face toward descent direction in the conjugate gradient method.
Three types of optimization were calculated. In all of them temperature at upper surface and bottom surface are given, and material properties for each layer are also given.

3.3.2. Minimizing thermal deformations

The purpose of the first optimization is to minimize both strain and curvature under the constraint that interlaminar stress must be under the desired value $\Delta\sigma_{max}$ [MPa]. Design parameters are thickness ratio ϕ_1 and ϕ_3. Table 2 shows the result of this optimization.

3.3.3. Minimizing interlaminar stress

The purpose of this optimization is to minimize interlaminar stress under the constraint that strain must be under the desired value ε_{max} [μ] and curvature is 0. Design parameters are also ϕ_1 and ϕ_2. The result is shown in table 3.

3.3.4. Eliminating both strain and curvature

In the previous optimization, if ε_{max} is set to 0, the beam with no strain and no curvature is designed. Table 4 shows some examples of the result. When the bottom side temperature T_1 is 0 [°C], thickness ratios in which strain and curvature become 0 are independent of the upper side temperature T_4. However, when T_1 is 50.0 [°C], thickness ratios depend on T_4. So, thickness ratio depends on both surface temperatures except the case T_1 is 0 [°C]. When T_1 is 0 [°C], interlaminar stresses $\Delta\sigma_1$ and $\Delta\sigma_2$ are proportional to T_4

Figure 2 shows the stress distribution and the temperature distribution of the beam [P100Gr/Ep -Al-Kevlar/Ep] with thickness ratios that eliminates both strain and curvature under the thermal boundary condition, T_1 is 0 [°C] and T_4 is 50[°C]. or 100 [°C]. Temperature in an aluminum layer is almost constant and composite material layers have large temperature gradients.

Table 2. Minimizing thermal deformations, T_1 = 0 [°C], T_4 = 100 [°C]

(a): Layer1-2-3 = Kevlar/Ep-Al- Kevlar/Ep

$\Delta\sigma_{max}$ [MPa]	ϕ_1	ϕ_3	ε [μ]	κ[μ/m]	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
100	2.879	2.771	-3.05	-52.07	100.00	99.67
50	0.951	2.866	-43.96	-144.85	50.00	49.03
30	0.453	2.653	-71.22	-170.22	30.00	28.86
10	0.086	1.843	-95.89	-195.74	10.00	8.72

(b): Layer1-2-3=P100Gr/Ep-Al-ASGr/Ep

$\Delta\sigma_{max}$ [MPa]	ϕ_1	ϕ_3	ε [μ]	κ[μ/m]	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
100	3.749	4.817	0.00	0.00	48.50	36.84
50	3.849	4.758	0.25	0.22	49.99	37.97
30	1.585	4.095	-0.51	-4.38	30.00	20.82
10	0.145	3.239	-7.14	-11.22	10.00	3.17

(c): Layer1-2-3=P100Gr/Ep-Al- Kevlar/Ep

$\Delta\sigma_{max}$ [MPa]	ϕ_1	ϕ_3	ε [μ]	κ[μ/m]	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
100	2.515	0.922	0.00	0.00	40.13	35.25
50	2.515	0.922	0.00	0.00	40.13	35.25
30	1.350	0.720	0.44	-8.04	30.00	25.71
10	0.590	0.839	-57.59	-84.71	10.00	10.00

Table 3. Minimizing interlaminar stress, $\kappa = 0$ [μ/m]

(a): Layer1-2-3 = Kevlar/Ep-Al- Kevlar/Ep

ε_{max} [μ]	T_1 [°C]	T_4 [°C]	ϕ_1	ϕ_3	ε [μ]	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
10.0	0.0	100.0	4.569	2.296	9.93	130.50	130.53
20.0	0.0	100.0	4.243	2.199	19.96	129.22	129.25
30.0	0.0	100.0	3.957	2.108	29.89	128.06	128.09
10.0	0.0	200.0	4.746	2.346	9.98	262.37	262.42
10.0	50.0	100.0	3.037	2.408	10.00	152.72	152.74

(b): Layer1-2-3 = P100Gr/Ep-Al-ASGr/Ep

ε_{max} [μ]	T_1 [°C]	T_4 [°C]	ϕ_1	ϕ_3	ε [μ]	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
10.0	0.0	100.0	1.856	2.896	9.99	45.61	32.31
20.0	0.0	100.0	1.126	1.977	19.98	45.86	30.17
30.0	0.0	100.0	1.124	1.974	20.01	45.86	30.16
10.0	0.0	200.0	2.548	3.663	10.00	92.83	68.17
10.0	50.0	100.0	2.762	10.722	-10.00	117.36	91.32

(c): Layer1-2-3 = P100Gr/Ep-Al- Kevlar/Ep

ε_{max} [μ]	T_1 [°C]	T_4 [°C]	ϕ_1	ϕ_3	ε [μ]	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
10.0	0.0	100.0	1.145	0.614	10.00	33.14	25.70
20.0	0.0	100.0	0.616	0.433	20.00	31.09	20.49
30.0	0.0	100.0	0.361	0.315	30.00	31.11	17.13
10.0	0.0	200.0	1.651	0.748	10.00	71.35	59.26
10.0	50.0	100.0	1.834	1.888	-10.00	116.05	105.04

Table 4. Eliminating strain and curvature, $\varepsilon = 0$ [μ], $\kappa = 0$ [μ/m]

(a): Layer1-2-3 = Kevlar/Ep-Al- Kevlar/Ep

T_1 [°C]	T_4 [°C]	ϕ_1	ϕ_3	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
0.0	50.0	4.936	2.397	66.0	66.0
0.0	100.0	4.936	2.397	131.9	131.9
0.0	200.0	4.936	2.397	263.8	263.8
50.0	100.0	3.169	2.495	152.8	152.8
50.0	150.0	3.492	2.439	213.4	213.4

(b): Layer1-2-3 = P100Gr/Ep-Al-ASGr/Ep

T_1 [°C]	T_4 [°C]	ϕ_1	ϕ_3	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
0.0	50.0	3.749	4.816	24.2	18.4
0.0	100.0	3.749	4.816	48.5	36.8
0.0	200.0	3.749	4.816	97.0	73.7
50.0	100.0	2.192	7.331	122.8	93.1
50.0	150.0	2.251	6.406	136.9	103.8

(c): Layer1-2-3 = P100Gr/Ep-Al- Kevlar/Ep

T_1 [°C]	T_4 [°C]	ϕ_1	ϕ_3	$\Delta\sigma_1$ [MPa]	$\Delta\sigma_2$ [MPa]
0.0	50.0	2.515	0.922	20.1	17.6
0.0	100.0	2.515	0.922	40.1	35.2
0.0	200.0	2.515	0.922	80.3	70.5
50.0	100.0	1.564	1.759	119.5	104.5
50.0	150.0	1.564	1.470	129.7	113.5

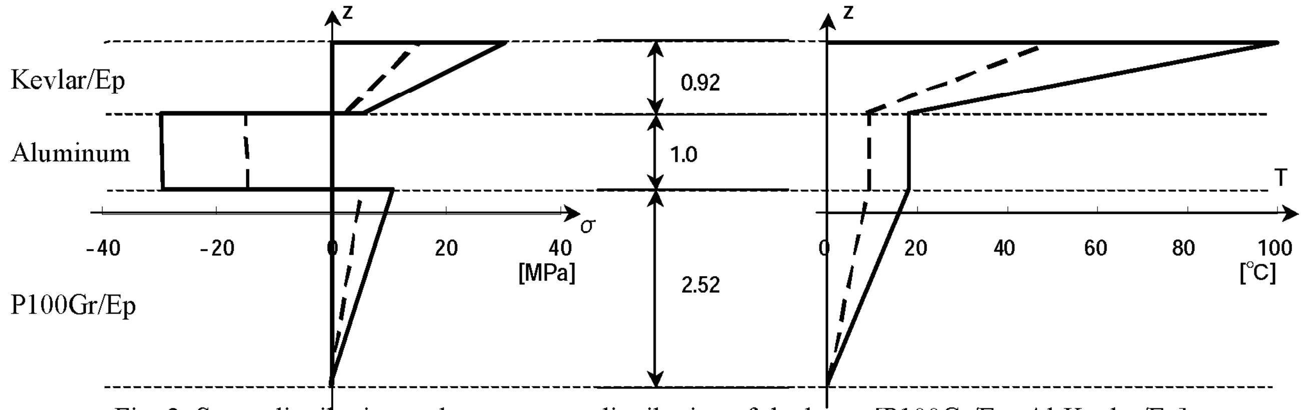


Fig. 2. Stress distribution and temperature distribution of the beam [P100Gr/Ep -Al-Kevlar/Ep] with thickness ratios that eliminates both strain and curvature under the thermal boundary condition, T_1 is 0 [°C] and T_4 is 50 [°C] or 100 [°C].

3.3.5. Matching thermal deformations in desired value

Figure 3 shows the normalized strain and curvature of the beam [P100Gr/Ep-Al-Kevlar/Ep] for thickness ratios from 0 to 10 under the thermal boundary condition, T_1 is 0 [°C] and T_4 is 100 [°C]. Thickness ratios with strain $\varepsilon = \xi$ and curvature $\kappa = \zeta$ are found out at the intersection of two curves, $\varepsilon = \eta$ and $\kappa = \zeta$. However, the solution for these requests does not always exist. For example, the curve for ε = -0.1 and the curve for $\kappa = 0$ have no intersection in first quadrant. So, requests for thermal deformations are not always satisfied.

4. Designing laminated beam in unsteady temperature distribution

Assuming that temperature distribution is steady, optimal laminated beam can be designed to satisfy a variety of requests for deformations or stress. In application to space structures, however, temperature distribution is not always steady. So, the laminated beam structure, some times, need to be designed with considering unsteady temperature distributions.

4.1. Unsteady temperature distributions

In this section, the beam is assumed to be heated uniformly. Then, a temperature distribution is calculated from Crank-Nicolson method based on one-dimensional heat conduction equation. Fig. 4 shows the thermal boundary conditions. The initial temperature in whole beam is 0 [°C]. And at time 0 [s], the upper side temperature increases to 100 [°C], T_4= 100[°C], and is kept constant after that. The bottom side temperature is kept 0 [°C], T_1 = 0[°C]

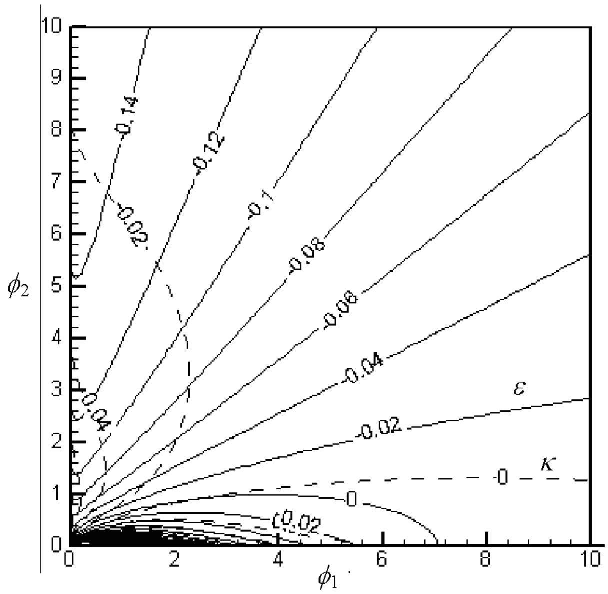


Fig. 3. Normalized strain and curvature of the beam [P100Gr/Ep -Al-Kevlar/Ep] T_1 = 0 [°C], T_4 = 100 [°C].

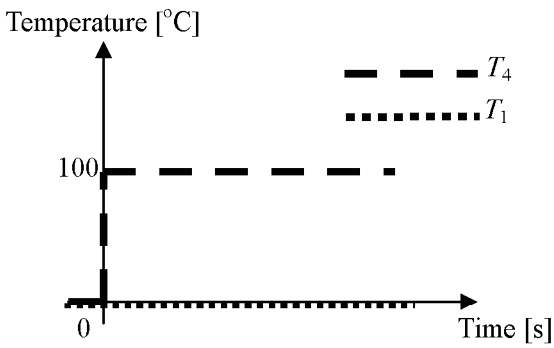


Fig.4. Surface Temperature

4.2. Interlaminar shearing stress and moment

To avoid delamination, interlaminar stress was considered as constraint in chapter 3. In this chapter, however, interlaminar moment and interlaminar shearing stress are considered as constraints to avoid delaminations of mode-1 and mode-2 at free edge because delamination is easy to happen at the free edge, Fig. 5. Fig. 6 is the image of interlaminar moment M_z and shearing stress F_{xz}. These are respectively defined as below.

$$M_z(z_i) = \int_{z_i}^{z_{i+1}} \sigma_x(z)(z - z_i)\mathrm{d}z \tag{8}$$

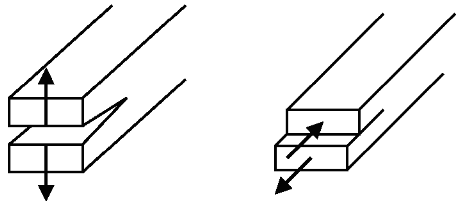
$$F_{xz}(z_i) = -\int_{z_i}^{z_{i+1}} \sigma_x(z)\mathrm{d}z \tag{9}$$

4.3. Optimization

Thermal deformations are represented by curvature and strain at the central axis of the beam (κ_x and ε_x^0). They are calculated with classical lamination theory and objective function consists of them.

$$f(x) = \max_{0\le t\le t_f}\left\{c_1\left[\varepsilon_x^0(t)/\varepsilon_0\right]^2 + c_2\left[\kappa_x(t)/\kappa_0\right]^2\right\} \tag{10}$$

Here, ε_0 and κ_0 are, respectively, strain at the central axis and curvature of the aluminum beam in the same condition. c_1 and c_2 are weighting factors. This equation means that the objective function is the maximum value of the weighted square sum of normalized strain and curvature from time 0 [s] to termination time t_f. And the purpose of this optimization is to find optimum design parameters such as thickness ratio and material properties that minimize objective function.



(a): Mode-1 (b): Mode-2

Fig. 5. 2 modes of delamination

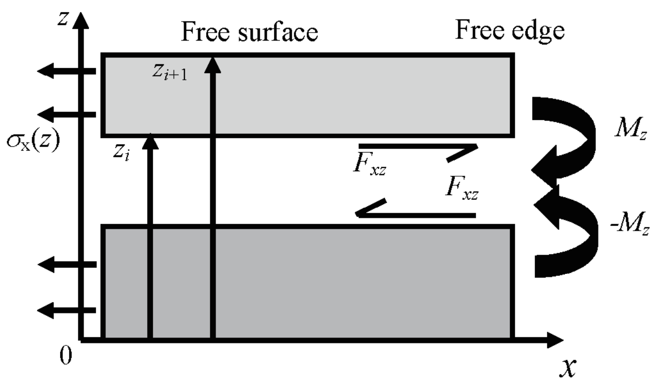


Fig. 6. Interlaminar shearing stress and interlaminar moment

Optimization procedure is as follows, Fig. 7. At first, objective function, constraint conditions, design parameters, and thermal boundary conditions are set for initialization. Then, unsteady temperature distributions; temperature distributions for each time instant, are calculated by Crank-Nicolson method. Thermal deformations and thermal stresses for each time instant are calculated based on those temperature distributions. If the objective function satisfies the at-end condition, it will be an optimum solution. If not, the appropriate design parameters are chosen by using descend method and return to the calculation of temperature distributions.

4.3.1. Optimization about Material Properties

The purpose of this optimization is to find out the best material properties of the first and the third layers that minimize thermal deformation. The material in the second layer is assumed to be aluminum.
Design parameters are ratio of thickness and material properties of the first and the third layers. Table.5 shows optimization results. The strain and the curvature of an Aluminum beam in the same condition are $\varepsilon_0 = 1200\times10^{-6}$, $\kappa_0 = 48000\times10^{-6}$/m, respectively.

The material with small $|E\alpha|$ and λ is effective on the high temperature side. This has large temperature gradients and is not easy to transform at the high temperature. Oppositely, the material with large $|E\alpha|$ and λ is effective on the low temperature side. This has small temperature gradients and is easy to transform at the low temperature.

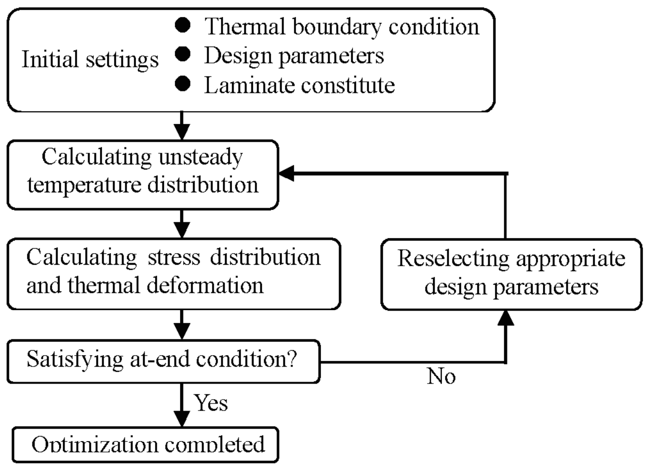


Fig. 7. Optimizing Procedure

Table.5. Optimization results about material properties

ϕ_1	E_1 (GPa)	α_1 ($\times 10^{-6}$/K)	λ_1 (W/mK)	$c_1\rho_1$ (MPa/K)	ε_{max} ($\times 10^{-6}$)	$f(x)$ ($\times 10^{-6}$)	$\lvert F_{xz}\rvert_{max}$ (kN/m)
0.741	500	-3.50	2.50	800	2.16	3.57	23.7
ϕ_3	E_3 (GPa)	α_3 ($\times 10^{-6}$/K)	λ_3 (W/mK)	$c_3\rho_3$ (MPa/K)	κ_{max} ($\times 10^{-6}$/m)	$\Delta\sigma_{max}$ (MPa)	$\lvert M_z\rvert_{max}$ (kNm/m)
1.04	107	-0.10	0.050	800	82.0	5.29	0.200

Table.6. Optimization results using actual materials

Layer1-2-3	ϕ_1	ϕ_3	ε_{max} ($\times 10^{-6}$)	κ_{max} ($\times 10^{-6}$/m)	$f(x)$ ($\times 10^{-6}$)	$\lvert F_{xz}\rvert_{max}$ (kN/m)	$\lvert M_z\rvert_{max}$ (kNm/m)	$\Delta\sigma_{max}$ (MPa)
Kevlar/Ep-Al-Kevlar/Ep	10.0	2.46	55.2	2990	3930	254	8.29	157
P100Gr/Ep-Al-ASGr/Ep	7.29	5.73	5.13	294	41.6	131	5.17	67.0
P100Gr/Ep-Al-Kevlar/Ep	8.89	1.13	12.7	520	118	186	7.39	82.3

4.3.2. Optimization Using Actual Materials

The optimization using actual materials shown in Table 1 is carried out. In this time, laminate constitution is given and only thickness ratios are the design parameters. Constraints for interlaminar shearing stress and moment are not considered.

Table.6 shows the optimization result. The laminate constitution [P100Gr/Ep-Al-ASGr/Ep] reduces thermal deformations most. This is because that laminate constitution has the closest material tendency to the tendency that was proved in the previous optimization. So optimization about material properties will be useful for deciding laminate constitution in designing process.

4.3.3. Constraints for interlaminar shearing stress and moment

For the laminate constitution [P100Gr/Ep-Al-ASGr /Ep], optimization about thickness ratios is carried out with considering constraints for inter laminar shear and moment in the same temperature condition. Interlaminar shearing stress and moment are restricted not to exceed 80% of the maximum values of the previous optimization, Table 6, $|F_{xz}|_{max} < 0.8\times131$ [kN/m], $|M_z|_{max} < 0.8\times5.17$[kNm/m]. Fig. 7 shows the time history of the objective functions. Line A shows the time history of objective function without considering constraints for interlaminar shearing stress and moment, and line B shows that with considering them. Instead of decreasing interlaminar shearing stress and moment, thermal deformations increase.

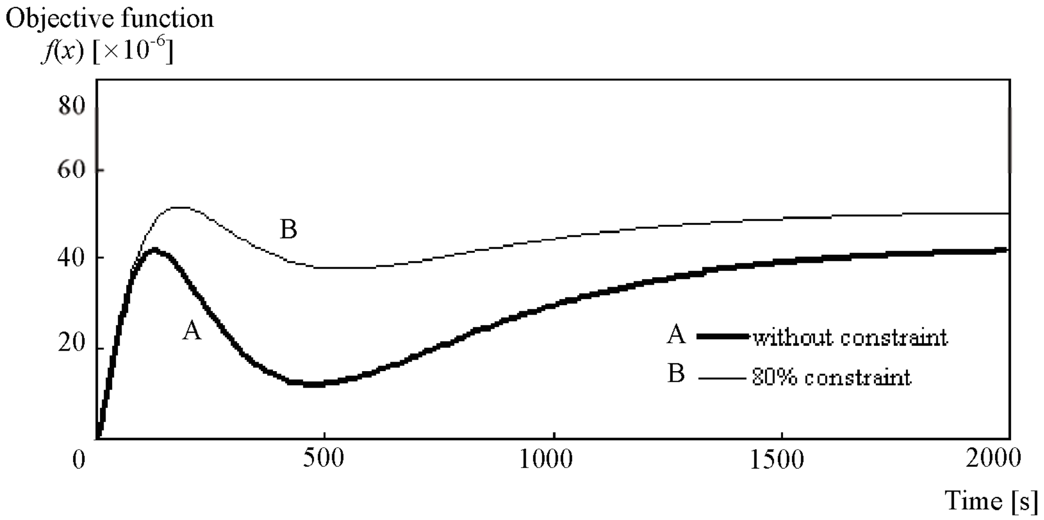


Fig. 7. Time histories of objective functions of the beam [P100Gr/Ep-Al-ASGr/Ep]
A: Optimization without any constraints for interlaminar shearing stress or moment
B: Optimization with constraints for interlaminar shearing stress and moment

4.3.4. Discussion on considering steady temperature distributions or unsteady ones

Optimum thickness ratios for laminate constitution [P100Gr/Ep-Al-ASGr/Ep] in steady temperature distribution, the bottom side temperature is 0 [°C], upper side temperature is 100 [°C], have been found out in section 3.3.4.

Line C in Fig. 8 shows how the time history of objective function changes if the beam with those thickness ratios experiences the unsteady temperature distribution shown in Fig. 4.

Comparing line C with the line A in Fig. 7; which is a time history of objective function for [P100Gr/Ep-Al-ASGr/Ep] in unsteady temperature distribution, line C is much larger than line A in transitional period. Since objective function represents thermal deformations, this means the beam, which was designed with considering steady temperature distribution, can be deformed much in transitional period. Oppositely, the beam, which was designed with considering unsteady temperature distribution, can suppress transitional thermal deformations, however deformations will remain after temperature distribution gets steady.

Conclusions

This paper showed that thermal deformation of the beam can be suppressed by using laminated structure composed of materials with negative longitudinal coefficient of thermal expansion (CTE) and materials with positive CTE. If temperature distribution is steady, thermal deformations; strain of the central axis and curvature, can be eliminated by laminating such materials in appropriate thickness ratios. It is impossible to eliminate both thermal deformations and interlaminar stress but it is possible to suppress both of them simultaneously. Even if temperature distribution is unsteady, thermal deformations can be suppressed while lowering thermal stresses by laminating material with small $|E\alpha|$ and λ on the high temperature side and material with large $|E\alpha|$ and λ on the low temperature side. Beam structure should be designed with considering unsteady temperature distribution if that structure experiences temperature changes because beam structure designed with considering steady temperature distribution could be deformed much when it experiences temperature changes.

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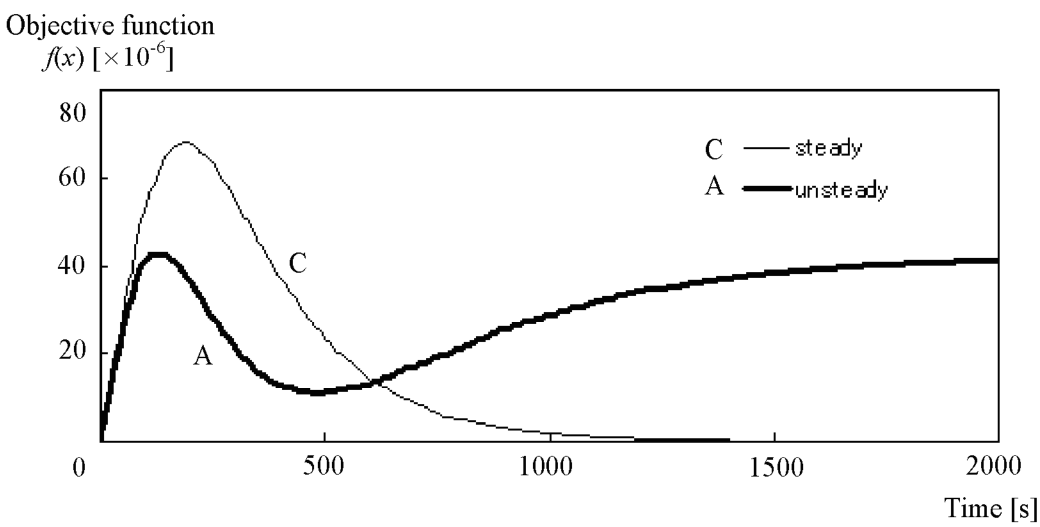

Fig. 8. Time histories of objective functions of the beam [P100Gr/Ep-Al-ASGr/Ep]
A: Considering unsteady temperature distributions
C: Considering steady temperature distributions