A Transition Prediction Model in Supersonic Flow

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ABSTRACT

A simple model was proposed to predict the transition region of compressible flow. An algebraic type of RNG model was employed with modified expressions for dissipation rate and length-scale. A 2D flat plate flow with no pressure gradient was calculated to validate the model. The results showed favorable agreement with theoretical values, which suggests its capability to predict the transition cases for simple flow.

1. Introduction

In some downstream region, a boundary layer flow is subjected to transition from laminar to turbulent state, due to instability produced within a shear layer. So far, several instability modes have been identified, such as Tollmien-Schlichting, cross-flow, Görtler and Mack's second mode, where viscous instability by Tollmien-Schlichting and cross-flow inviscid instability with three-dimensional flow due to crosswise pressure difference are two most important modes regarding SST. Here in this study, only the Tollmien-Schlichting mode is taken into account.

The objective of this research was to predict the location of transition and to examine its characteristic features, especially compressibility effects. To elucidate the fundamental aspects of model's capabilities, 2D compressible boundary layer over a flat plate was chosen. Furthermore, as simple a method as possible is favored to capture the transition phenomena. The Renormalization Group (RNG) approach proposed by Yakhot and Orszag[1)] was selected as a baseline. Some modifications were made to this original model, using Prandtl's mixing-length theory with new expressions of dissipation rate and length-scale, which were developed by Sakya and Nakamura[2)]. The modified version is of zero-equation type, that is, the model is algebraic. This avoids the need to calculate partial differential equations for turbulent properties, which remarkably simplifies the procedure.

2. RNG Model Based on Mixing-Length Theory

The philosophy of the RNG turbulence model is to include the effects of the small scales of turbulent motion into the expression for turbulent viscosity[1)]. The effective viscosity ν_e defined as the sum of the laminar and turbulent viscosities ($\nu_e=\nu+\nu_t$) is given in the following expression.

$$\nu_e = \nu\left\{1 + H\left(\frac{a}{\nu^3}\frac{\varepsilon}{\Lambda^4} - C\right)\right\}^{1/3} \tag{1}$$

where H is the heaviside function serving as a switch that models the onset of transition from purely laminar to turbulent flow, depending on the sign of its argument (see Eq.(2)). When the argument is negative, the flow is fully laminar.

$$H(x)=x \quad \text{if } x\geq 0.0 \qquad \text{and} \qquad H(x)=0 \quad \text{if } x<0.0 \tag{2}$$

Λ is the wave number given in Eq.(3), and a and C are some constants given by experimental fit (a=0.12, C=75).

$$\Lambda = \frac{2\pi}{L} \tag{3}$$

The dissipation is made equal to the production, assuming that the turbulent flow is in the equilibrium condition.

$$\varepsilon = \nu_t S_r \tag{4}$$

where

$$S_r = \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\frac{\partial u_k}{\partial x_k}\delta_{ij} \right) \frac{\partial u_i}{\partial x_j} \tag{5}$$

The terms inside the heaviside function can be expressed in terms of a mixing length by Prandtl, where the turbulent viscosity is expressed by the mixing length and the shear rate of averaged velocity, as shown in Eq.(6). This gives a RNG model based on the mixing-length theory.

$$\nu_t = L_m^2 S_r^{1/2} \tag{6}$$

$$\nu_e = \nu \left[1 + H\left\{ \frac{\varepsilon L_m^4}{\nu^3} - C \right\} \right]^{1/3} \tag{7}$$

3. Sakya-Nakamura's Modified RNG Model

There are a number of possibilities to express the dissipation rate ε and the length scale L_m in Eq.(7), so as to appropriately represent the flow characteristics. The following expressions were proposed in this study[2)].

$$\varepsilon = \frac{u_\tau^3}{\kappa y} D_\varepsilon \tag{8}$$

$$\frac{L_m}{\delta} = \frac{0.2\eta(1-0.46\eta)}{0.45+\eta} p^+ \tag{9}$$

where u_τ is the frictional velocity, D_ε is a damping function which serves to make the value of ε finite (=0.1 in this case) at wall (y=0), κ=0.4 (von Karman constant), and y is the distance from wall. The length scale L_m in Eq.(9) was obtained from normalized experimental fit for Klebanoff's data. This consists of two terms : a function of η and pressure gradient p^+. The cases presented here assume that there is no pressure gradient in the flow direction, which means p^+=1. Substituting Eqs.(8) and (9) into Eq.(7) gives a modified effective viscosity as

$$\left(\frac{\nu_e}{\nu} \right)^3 = 1 + H\left(\frac{u_\tau^3}{\kappa y} \frac{L_m^4}{\nu^3} D_\varepsilon \gamma_R - C \right) \tag{10}$$

The additional term γ_R included here means an intermittency function, which serves to reduce the strength of turbulent viscosity near the boundary layer edge.

4. Test Case of 2D Compressible Flat Plate Flow

In order to validate the present model, a simple boundary layer flow over a flat plate was calculated by using it. The governing equations are shown in Eqs.(11) and (12), which were derived using the Falkner-Skan transformation.

The momentum equation is

$$(bf'')'+m_1 ff''=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right) \tag{11}$$

and the energy equation is

$$(eg'+df'f'')'+m_1 fg'=x\left(f'\frac{\partial g}{\partial x}-g'\frac{\partial f}{\partial x}\right) \tag{12}$$

where

$$f'=\frac{u}{u_e},\quad g=\frac{H}{H_e} \tag{13}$$

$$b=C\left(1+\nu_t^+\right),\quad e=\frac{C}{\Pr}\left(1+\nu_t^+\frac{\Pr}{\Pr_t}\right),\quad d=\frac{Cu_e^2}{H_e}\left[1-\frac{1}{\Pr}+\nu_t^+\left(1-\frac{1}{\Pr_t}\right)\right]$$

$$\nu_t^+=\frac{\nu_t}{\nu},\quad C=\frac{\rho\mu}{\rho_e\mu_e},\quad c=\frac{\rho_e}{\rho},\quad m_1=0.5$$

The equations take almost the same form as the incompressible ones, owing to the mass-average or Favre average. Therefore, in this study we employed the incompressible RNG model by substituting it into the appropriate terms of the governing equations. If the flow Mach number is not so high at supersonic regime, this quasi-compressible model is expected to work well. In particular this is thought to be reasonably appropriate for the case of bounded flows. Thus the RNG model shown in Eq.(10) was used to calculate the turbulent viscosity ν_t contained in Eq.(13).

5. Results and Discussion

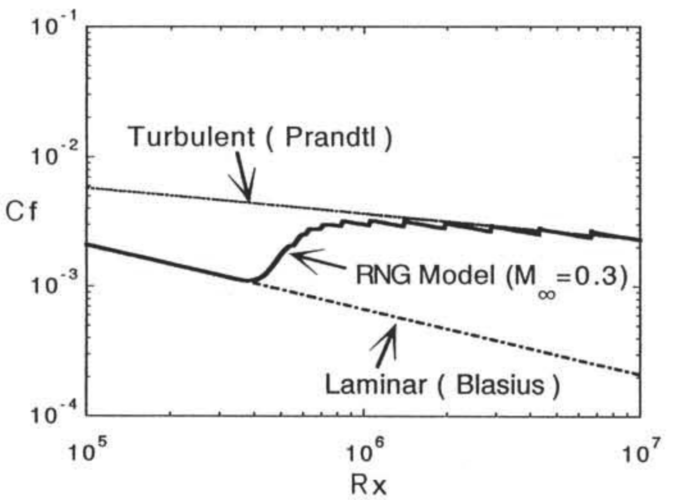

Fig. 1 Local skin-friction coefficient in low Mach number

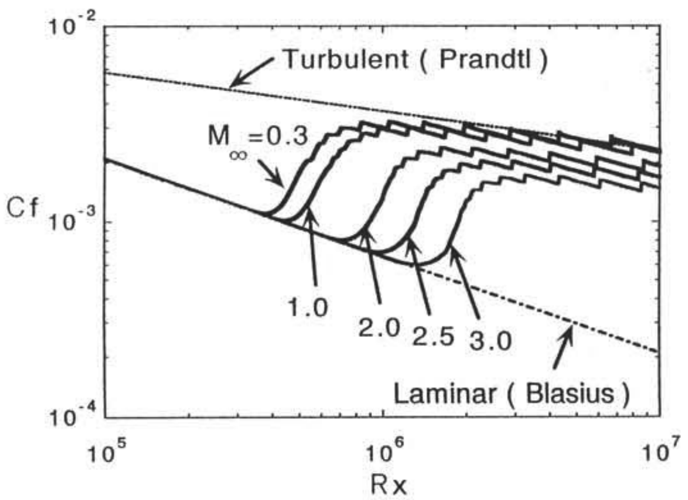

Fig. 2 Local skin-friction coefficient for various Mach numbers

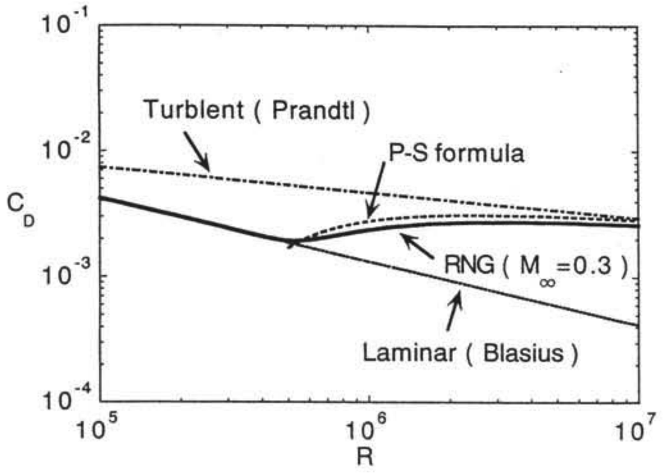

Fig. 3 Total skin-friction coefficient in low Mach number

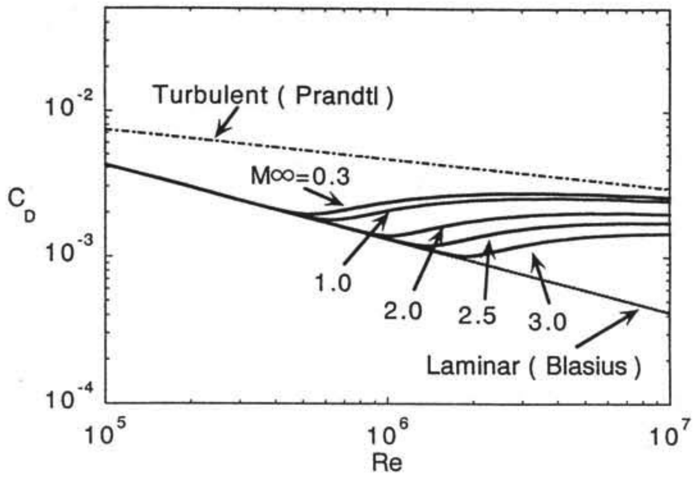

Fig. 4 Total skin-friction coefficient for various Mach numbers

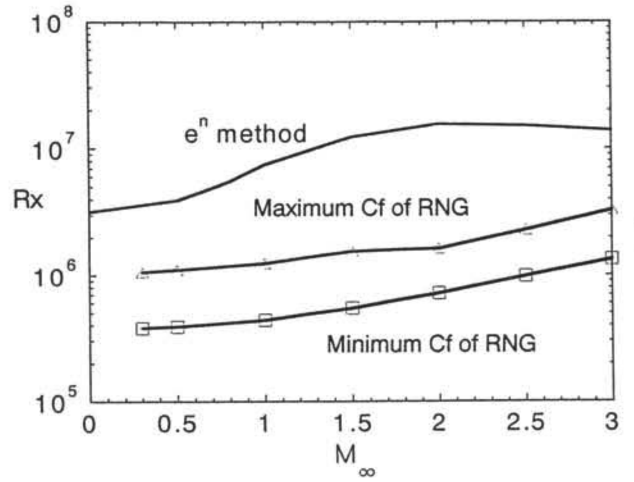

Fig. 5 Transition Reynolds number

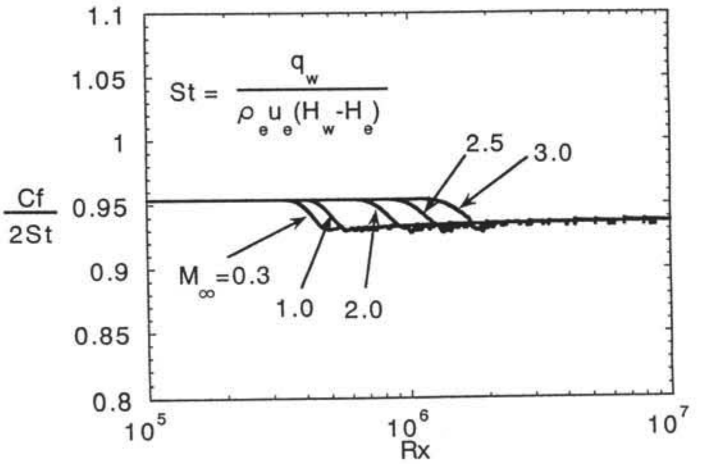

Fig. 6 Reynolds analogy factor

Sudden changes in the values of local and total skin-friction coefficients were considered to indicate the onset of transition. By comparing the coefficients with theoretical values by Blassius and Prandtl, as shown in Figs. 1 to 4, it can be deduced that the laminar to turbulent transitional characteristics of the flow can be captured quite reasonably. For the low compressible case, the transition location is in close agreement with the theoretical value by Prandtl-Schlichting for incompressible flow (see Fig. 3).

The general trend in compressible flow can be seen in Figs. 2 and 4, where increase in Mach number shifts the transition location in the downstream direction. The effects of compressibility are summarized in Fig. 5, where the result is compared with that of e^n method based on linear stability theory. The transition region is depicted as a band between the minimum and maximum values of local skin friction. It is seen that the predicted transition region is upstream of that of the linear theory, which is quite natural, due to the difference in the assumed level of disturbance in the upstream flow. In the linear theory, the disturbance is assumed to be very small, which leads to a high value of transitional Reynolds number. On the other hand, the current prediction model employs values derived from experimental data. For example, the constant C in Eq.(10) is indirectly related with the magnitude of disturbance level. Hence it can contain a more realistic level of disturbance, so that the transition occurs more upstream than the prediction by the linear stability theory.

The local skin friction obtained here shows a staircase appearance in the transition region, as can be found in Figs. 1 and 2, which seems to be the coupling effect between the RNG model and the boundary layer equations. This should be mended as a future work.

Additionally, the heat transfer at the wall is shown in Fig. 6 in the form of the familiar concept of Reynolds analogy. The ratio of the skin friction to the heat transfer gives respective constant values in the laminar and turbulent regions, which also confirms the validity of the present model. Even in this result the effect of compressibility can be clearly seen.

6. Concluding Remarks

An algebraic RNG model was examined to see the effects of compressibility, and the results are summarized as follows.

- A simple RNG model proposed here can predict the laminar, turbulent and transition characteristics even in compressible flow.
- Increase in Mach number shifts the transition location downstream. For example, $Re_{tr} = 3.8 \times 10^5$ at $M_\infty = 0.3$, and $Re_{tr} = 1.3 \times 10^6$ at $M_\infty = 3.0$.
- The transition predicted with the present model occurs more upstream than that of e^n method based on the linear stability theory.
- Increase in Mach number does not change the respective values of constants in the Reynolds analogy in laminar and turbulent regions

References

1. Yakhot, V. and Orszag, S.A., Renormalization Group Analysis of Turbulence, 1, Basic Theory, J. of Scientific Computing, 1, 1, 1986, pp. 3-51.
2. Nakamura, Y. and Sakya, A.E, Capturing of Transition by the RNG Based Algebraic Turbulence Model, Computers and Fluids, 24, 8, 1995, pp. 909-918.