A Numerical Study on the Receptivity of Supersonic Laminar Boundary Layer

Shoji Sakaue and Michio Nishioka

Department of Aerospace Engineering, Osaka Prefecture University, Sakai Osaka, Japan.

The motivation of the present study is to obtain a better understanding of supersonic boundary layer receptivity. To investigate the generating process of T-S wave, direct numerical simulations are made using the TVD scheme. In order to introduce the external disturbance in the basic boundary layer flow, we allow the streamwise velocity on a narrow strip of the wall to oscillate in time with an angular frequency ω. Results of the present simulations show that the intensity of the excited T-S wave is proportional to that of T-S wavenumber spectrum contained in the external disturbance. Also shown is that for supersonic boundary layer flow most of external disturbance energy is simultaneously radiated as sound wave to the outer flow.

1. Introduction

Laminar flow control is one of the most important aerodynamic subjects for the development of future supersonic transport airplane. Indeed, the reduction of skin friction drag heavily relies on our knowledge of how to prevent the turbulent transition and to maintain the supersonic boundary layer in laminar state.

The stability theory is a powerful tool for predicting the early stage of boundary-layer transition. Also important for the prediction and control of the transition is the so-called receptivity process, by which external disturbances are internalized as small amplitude fluctuation (like T-S waves). Without knowledge of the flow process and the necessary condition for generating T-S waves, we cannot accurately predict the onset of the actual transition in various disturbance environments. For incompressible flow, Nishioka and Morkovin (1986) clarified that external disturbances should cause x-dependent Stokes layer which contains the wave number components corresponding to the viscosity-conditioned T-S waves to be generated. Even under this condition, for supersonic boundary layers, the generation of T-S waves might be less effective due to the possible, simultaneous radiation of sound waves, depending on the type of external disturbance.

In the present study, in order to examine the point above mentioned and to obtain a better understandings of the generation process of T-S waves, we investigate the development of T-S wave from localized wall disturbances, through a direct numerical simulation of a supersonic flat-plate boundary layer.

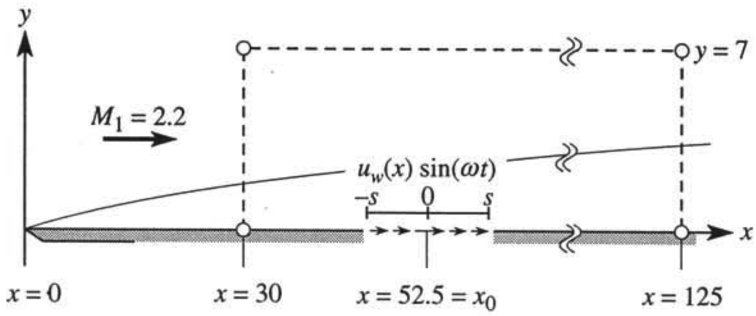

Figure 1 Schematic diagram of computational domain.

2. Numerical scheme

We consider a two-dimensional boundary layer along flat plate placed in supersonic uniform flow at Mach number $M_1 = 2.2$, as illustrated in Figure 1. The origin of the coordinates is set on the plate leading edge and x-axis is in the direction of the freestream and y-axis is normal to the wall. The boundaries of the computational domain are placed at $x = 30$ and $x = 125$, and $y = 0$ and $y = 7$. The Navier-Stokes equations are non-dimensionalized with freestream density ρ_1, sound speed c_1, viscosity μ_1 and reference length L, where the computational Reynolds number,

$$Re = \frac{\rho_1 c_1 L}{\mu_1} \tag{1}$$

is 4500. The Reynolds number based on the displacement thickness δ_1 and the freestream velocity U_1

$$Re_{\delta_1} = \frac{\rho_1 U_1 \delta_1}{\mu_1} \tag{2}$$

is from 2196 to 4482, and the non-dimensional displacement thickness is $\delta_1 = 0.2934$ at $x = 52.5$. The computational domain is descretized with the 760 and 240 grid points in x and y directions, respectively. Thus, the boundary layer is descretized with 42 grid points (at $x = 30$) and 80 grids (at $x = 125$).

The Navier-Stokes equations are solved numerically by the third-order upwind TVD scheme (Chakravarty and Osher) for the convection terms and the second-order central difference scheme for the other spatial derivatives. For the time advancement, the second-order explicit Euler scheme is used.

In order to introduce Stokes layer in the basic boundary layer flow as an external disturbance, we allow the streamwise velocity component to have non-zero value on a narrow strip of the wall and to oscillate in time with the angular frequency ω.

$$u = u_w(x)\sin(\omega t)\,, \quad u_w(x) = \begin{cases} A_w\left(\cos^2(\pi(x - x_0)/2s)\right) & \text{for } |x - x_0| \le s \\ 0 & \text{for } |x - x_0| > s \end{cases} \tag{3}$$

The origin of the disturbance (x_0) is placed at x = 52.5 (see figure 1). The disturbance thus introduced would produce a local fluctuation field. Furthermore, it is quite easy to control the intensity and streamwise scale of the external disturbance by changing A_w and s in eq. (3), independently.

The angular frequency ω and the amplitude A_w of the external disturbance are set to be 1.2469 and $0.03U_1$, respectively. The disturbance introduced is weak enough to expect a linear-stability behavior in the fluctuation development. To verify the excited wave, we carried out stability calculations on the basis of the mean velocity profile at various x-position assuming parallel flow. The complex wave number α (= $\alpha_r + \mathbf{i}\ \alpha_i$) of the T-S wave with ω = 1.2469 is 0.9099 - $\mathbf{i}$ 0.0132 (at $x = x_0$). Therefore the wavelength $2\pi/\alpha_r$ is 6.91.

3. Results

First of all, we would like to give an example of the simulation for the streamwise scale 2s = 6.0. Figure 2 shows the streamwise development of the structure of the excited wave by showing the y-distributions of amplitude and phase for density, u- and v-velocity, pressure and temperature fluctuations at various x-positions, and compared with the stability theory. Over the disturbance source on the wall (x = 52.5), the structure of the disturbance is not unlike that of the Stokes layer on the oscillating plate. About two wavelength downstream of the disturbance source (x = 65.0), the excited wave structure develops into that of the T-S mode. Figure 3 plots the mean u-fluctuation amplitude ($\tilde{u}_{mean}$) and phase at the wall against x. Here, $\tilde{u}_{mean}$ is defined as

$$\tilde{u}_{mean} = \left[\left\{\int_0^{3\delta} \tilde{u}^2(y)\, dy\right\} / \, 3\delta\right]^{1/2} \qquad (4)$$

where δ is the boundary layer thickness. For comparison, solid lines show the wave behavior predicted from the stability calculations. For $x \geq 65.0$, both the amplitude and phase vary downstream according to the linear stability theory. As seen from these figure, the T-S wave is surely generated by the present wall disturbance.

In the present numerical simulations, it is expected from the receptivity consideration that the intensity of the exited T-S wave is dependent on the characteristic scale of the external disturbance, $2s$. Figure 4 illustrates the maximum (rms) amplitude of the u-fluctuation plotted against $2s$. As seen from the figure, the maximum intensity is obtained for the scale $2s$ around 6.0, which is comparable with the wavelength of T-S wave. Figure 5 plots maximum u-fluctuation against $F(\alpha_{TS})$, where $F(\alpha)$ is the Fourier component of disturbance source $u_w(x)$,

$$F(\alpha) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} u_w(x)\, \mathrm{e}^{-\mathbf{i}\,\alpha x}\, dx = \frac{A_w}{4\pi}\left(\frac{2}{\alpha} + \frac{s}{\pi - s\alpha} - \frac{s}{\pi + s\alpha}\right)\sin(s\alpha)\ , \qquad (5)$$

and α_{TS} is the wave number of T-S mode, α_{TS} = 0.9099. As seen from the figure, the u-fluctuation is completely correlated with $F(\alpha_{TS})$. In addition, we verified that in all the cases the u-fluctuation grows with an amplification rate $-\alpha_i$, which is in good agreement with the corresponding eigenvalue from the stability theory as plotted in figure 6. Therefore the amplitude of the excited T-S wave is expressed as

$$\tilde{u}(x) = C\,F(\alpha_{TS})\exp\left[-\int_{x_{TS}}^{x}\alpha_i(\xi)\,d\xi\right] \quad \text{for} \quad x \geq x_{TS} \tag{6}$$

Here, x_{TS} is about 80 and the coupling coefficient C is found to be about 0.04 for the present simulation. This coupling coefficient is about 1/5 times smaller than the incompressible flow case (Asai and Nishioka, 1993). This is an important conclusion from the present study.

Figure 7 shows pressure and vorticity fluctuations for the case of $2s$ = 6.0. As seen from the figure, the wall disturbance radiates pressure fluctuation to freestream along Mach lines. This fluctuation have no vorticity, so that it can be identified as a sound wave. The amplitude of this wave is about ten times lager than that of the T-S waves excited. In supersonic boundary layer, the external disturbance radiates much energy as sound waves, so that the amplitude of excited T-S wave is much smaller than the corresponding incompressible flow case.

The relationship (6) gives the necessary condition for the excitation of the T-S wave. The intensity of the excited wave is proportional to that of the T-S wavenumber spectrum contained in the external disturbance. When the scale (s) of the external disturbance tends to infinity, $F(\alpha_{TS})$ tends to zero and no T-S wave is excited. In that case, only the Stokes layer is induced on the oscillating flat plate. In other words, unless the external disturbance contains the wavenumber spectrum of the T-S wave, the T-S wave can never be excited.

Acknowledgments

This work is in part supported by a Grant-in Aid for Scientific Research (No. 08455465) from the Ministry of Education, Science and Culture, Japan.

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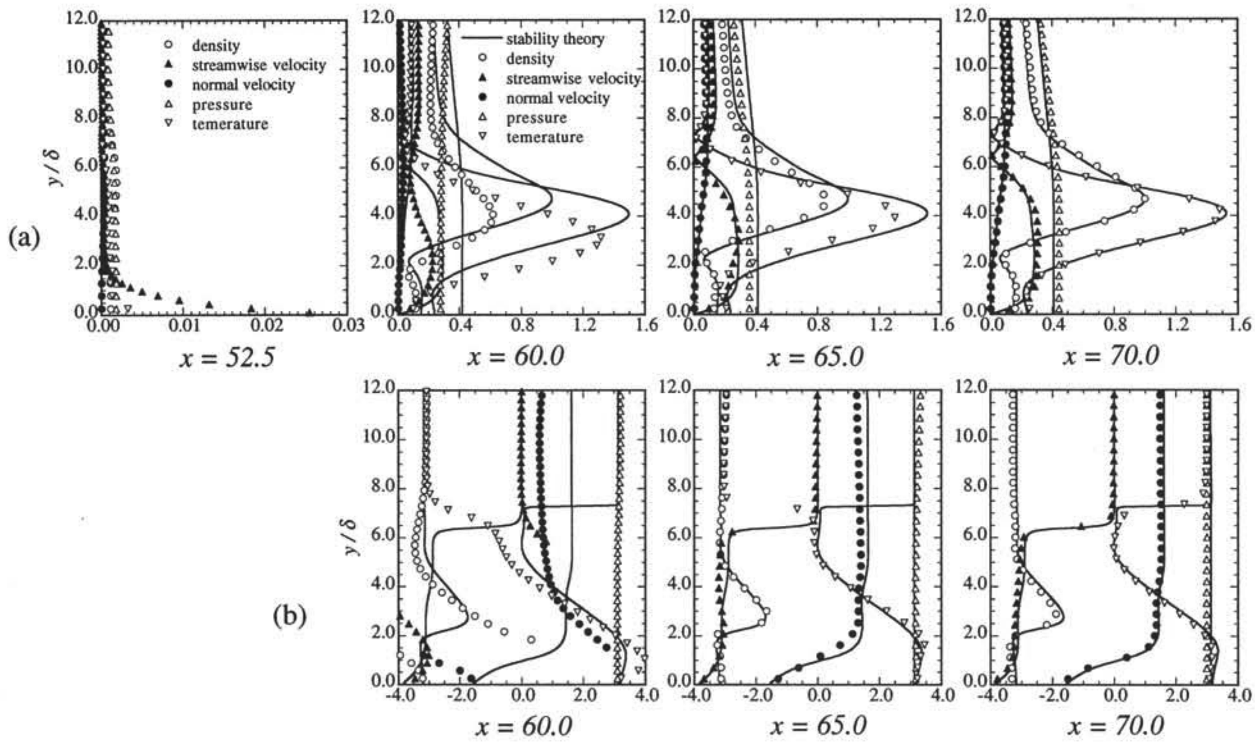

Figure 2 Streamwise development of the T-S wave illustrated for 2s = 6.0.
(a) wave amplitude, (b) phase.

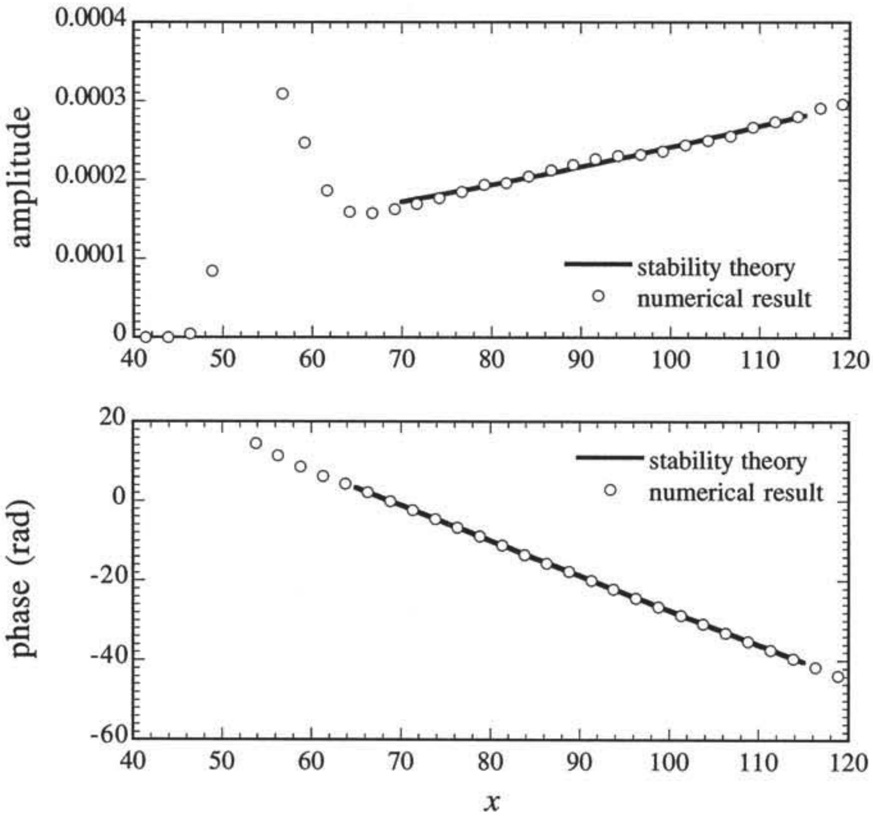

Figure 3 The comparison on streamwise variations of amplitude and phase between stability theory and simulation results (2s = 6.0).

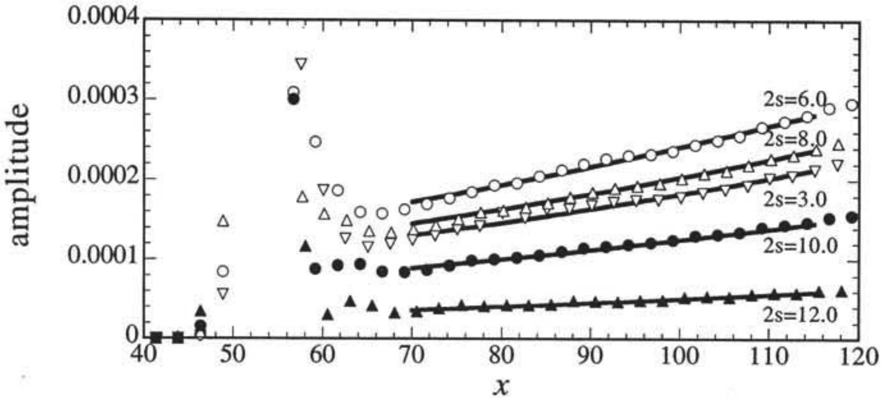

Figure 4 Streamwise development of u-fluctuations for various $2s$.

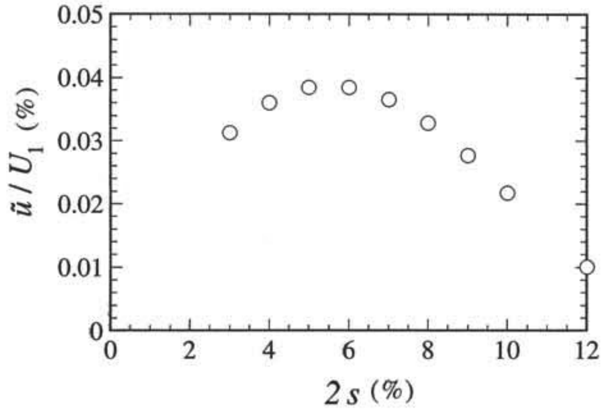

Figure 5 Relationship between the scale of the external disturbance and the excited T-S mode ($x = 80$).

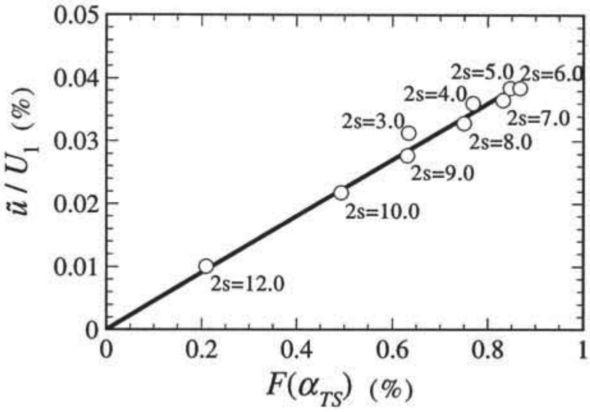

Figure 6 Relationship between the intensity of the external disturbance and the excited T-S mode ($x = 80$).

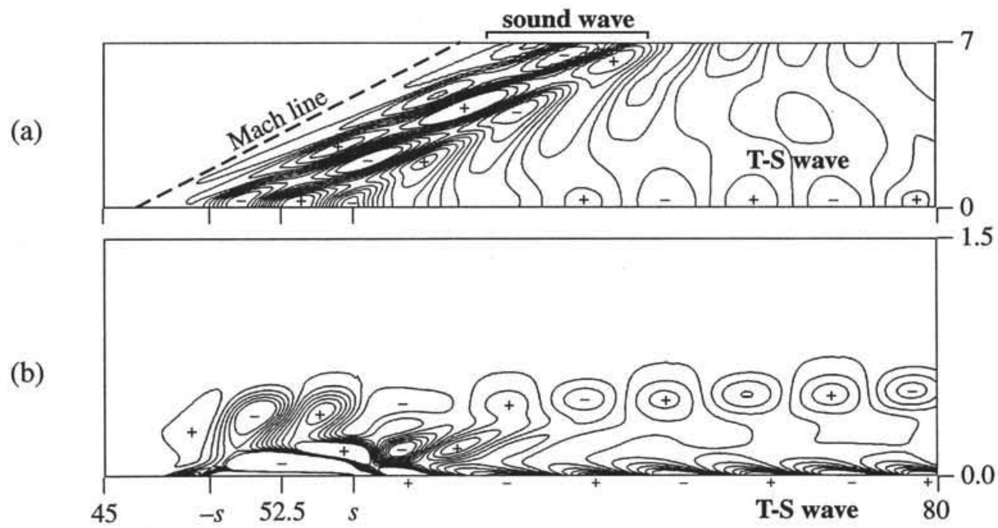

Figure 7 Instantaneous flow field for $2s = 6.0$:
(a)pressure flucutation contours,
(b)vorticity fluctuation contours.