SIMULTANEOU SHAPE AND FLIGHT TRAJECTORY OPTIMIZATION OF A SPACEPLANE USING PARALLEL OPTIMIZATION METHOD

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Key Words: Guidance and Control, Optimization Technique, Spaceplane, Parallel Computation

ABSTRACT

This paper presents a parallel optimization method to solve large-scale optimization problems and apply the method to the simultaneous vehicle design and flight trajectory optimization problems of a spaceplane. Generally, to obtain an accurate optimal solution, a large-scale problem has to be solved, and it takes much computing load and time. Therefore, the new method divides the problem into several sub-problems which can be solved in parallel. In the first half of the paper, we describe the way to decompose the problem and the fundamental algorithm to solve it. We introduce the conjunctive constraints among the sub-problems and define Lagrange multipliers for each conjunctive constrain. Each sub-problem can be solved in a parallel manner by updating both the Lagrange multipliers and design variables. In the latter half of the paper, the parallel optimization method is applied to obtain the optimal shape and optimal flight trajectory of a spaceplane.

INTRODUCTION

This paper considers an optimization design problem of a spaceplane in which design parameters of a vehicle and its assent trajectory are optimized simultaneously. A new parallel optimization technique is introduced to solve the large-scale simultaneous design. The method decomposes an original problem into multiple sub-problems by using the Lagrange variables which can be solve in parallel, and this process is iterated by updating Lagrange variables until converged solutions are obtained.

This method is applied to the design problem of a spaceplane which takes off horizontally, climbs up to a spacestation and returns to the earth airport with horizontal landing. The vehicle is intended to be fully reusable by installing the different type of propulsion system, e.g., air-breathing engines and a rocket engine. Since the mission requirements of this vehicle is highly severe for the present technological level, optimization design techniques must be introduced to obtain the most efficient design. It must be noted that the vehicle shape and its flight trajectory must be optimized simultaneously since these two designs have strong coupling.

The main emphasis of this paper is to develop the parallel multi-disciplinary optimization techniques, thus simplified models are used to estimate the aerodynamic and engine characteristics, and structural design is represented as structural weight estimation from statistical data. Flight trajectories are calculated by a point mass model where the angle of attack is selected as a control variable and the engine keeps the maximum thrust. In a total design problem, the design variables to be optimized are the design parameters in the vehicle design and the control parameters in the trajectory design. The former variables are the body length, the wing area, the size of engine, the body diameter and so on. The later variables are the time history of the angle of attack, the burn out time and the altitude, and the timing of engine selection.

As a numerical optimization method, we are using the Sequential Quadratic Programming which approximates a nonlinear optimization problem as a quadratic problem using derivative information of a Lagrange function. Whereas the optimal trajectory problem is a dynamic problem where the time history of control variable is a design variables and the time differential equations are the state equations to be satisfied as constrains in the optimization process, the direct collocation method can convert the dynamic optimization problem to a static optimization problems. Therefore, we can formulate this design problem as a conventional optimization problem. However, it should be noted that this problem has a large number of design variables in different disciplines. This is a main reason why we try to introduce a parallel optimization technique.

PARALLEL OPTIMIZATION METHOD

The parallel optimization method usually decomposes the original problem into sub-problems, and solves each sub-problem in a parallel manner. The program manager must be introduced to iterate this sequence and to update information in each sub-problem so as to obtain the total optimization solutions.

Dantzig-Wolfe originally presented decomposition algorithm[1] for linear programming problem. For nonlinear programming problem, two methods called model coordination method and goal coordination method which are reviewed by Kirsch[2] has been well accepted. In addition, a hybrid method[3] combining these two methods has been studied in recent years. However, these decomposition algorithms have not been wildly applied. Recently Braun et al. have proposed numerical methods called Collaborative Optimization[4] for more practical multidisciplinary optimization problems. Whereas those practical methods have robustness for practical design environment, they do not have strict mathematical background.

In our study, we try to develop mathematically well defined formulation which can be applied to practical

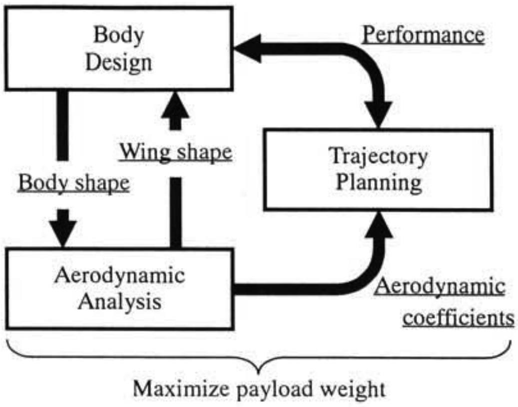

Fig. 2 Three technical fields

design problems with a large number of design variables. We consider the total model divided into three sub-system as shown in Fig.1. The total problem can be written as

$$\text{minimize}: f(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) \tag{1a}$$

$$\text{subject to: } \boldsymbol{g}_E(\boldsymbol{x}) \equiv \begin{bmatrix} \boldsymbol{g}_{E1}(\boldsymbol{x}_1) \\ \boldsymbol{g}_{E2}(\boldsymbol{x}_2) \\ \boldsymbol{g}_{E3}(\boldsymbol{x}_3) \end{bmatrix} = \boldsymbol{0} \tag{1b}$$

$$\boldsymbol{g}_I(\boldsymbol{x}) \equiv \begin{bmatrix} \boldsymbol{g}_{I1}(\boldsymbol{x}_1) \\ \boldsymbol{g}_{I2}(\boldsymbol{x}_2) \\ \boldsymbol{g}_{I3}(\boldsymbol{x}_3) \end{bmatrix} \le \boldsymbol{0} \tag{1c}$$

$$\boldsymbol{h}(x) \equiv \begin{bmatrix} \boldsymbol{h}_{12}(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ \boldsymbol{h}_{21}(\boldsymbol{x}_1, \boldsymbol{x}_2) \\ \boldsymbol{h}_{13}(\boldsymbol{x}_1, \boldsymbol{x}_3) \\ \boldsymbol{h}_{31}(\boldsymbol{x}_1, \boldsymbol{x}_3) \\ \boldsymbol{h}_{23}(\boldsymbol{x}_2, \boldsymbol{x}_3) \\ \boldsymbol{h}_{32}(\boldsymbol{x}_2, \boldsymbol{x}_3) \end{bmatrix} = \boldsymbol{0} \tag{1d}$$

where an equality constraint $\boldsymbol{h}_{ij}$ and $\boldsymbol{h}_{ij} = \boldsymbol{0}$ is respectively called a conjunctive function which connects two subsystems i and j.

In order to solve the sub-problem independently, conjunctive functions in connecting sub-problem and its Lagrange variables are incorporated in an objective function. This process can define the following sub-optimization problem.

$$\text{variable}: \boldsymbol{x}_i \tag{2a}$$

$$\text{minimize } f(\boldsymbol{x}_1, \boldsymbol{x}_2, \boldsymbol{x}_3) + \sum_{j=1(i \neq j)}^{3} \boldsymbol{v}_{ij}{}^{\mathrm{T}} \boldsymbol{h}_{ij}(\boldsymbol{x}_i, \boldsymbol{x}_j) \tag{2b}$$

$$\text{subject to } \boldsymbol{g}_{Ei}(\boldsymbol{x}_i) = 0 \tag{2c}$$

$$\boldsymbol{g}_{Ii}(\boldsymbol{x}_i) \le 0 \tag{2d}$$

$$\boldsymbol{h}_{ji}(\boldsymbol{x}_j, \boldsymbol{x}_i) = 0 \quad (j = 1, 2, 3 \text{ and } i \neq j) \tag{2e}$$

Note that the optimized variables in the sub-problem i are $\boldsymbol{x}_i$ and $\boldsymbol{v}_{ji}$, and the others are dealt with as constants those are updated in each iteration process.

A Fundamental algorithm to solve the sub-problems is summarized as follows:

(1) Determine proper initial solutions of all the variables $\boldsymbol{x}_i$ and Lagrange multipliers $\boldsymbol{v}_i$.
(2) Solve all sub-problems in parallel with optimization methods by which not only the variables but also the Lagrange multipliers can be computed, e.g. a sequential quadratic programming (SQP) method[5].
(3) Exchange the obtained variables and multipliers among the sub-problems as shown in Fig. 2, and return to (2).

SPACEPLANE DESIGN STUDY

In this section, a shape design and an ascent trajectory design a spaceplane is formulated. Figure 1 indicates three technical fields, body design field, aerodynamic analysis field and trajectory planning field. These three fields are not independent but have the interactions of design variables and design specifications, i.e., some variables of body shape, wing shape, performance and aerodynamic coefficients are used in each design field. Conjunctive conditions represent the equality constrains of the design variables and design specifications commonly used in each subsystem.

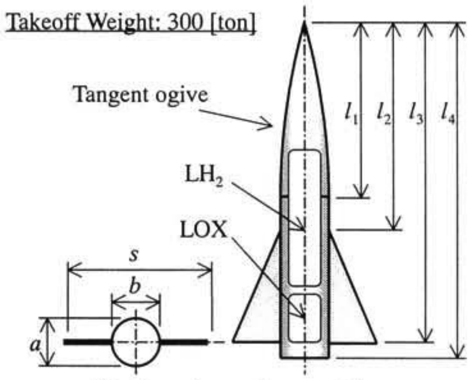

Fig. 1 Spaceplane model

(1) Body Design

The spaceplane shape model adopted in this paper is illustrated in Fig. 2. Takeoff Weight is assumed to be 300 ton and the body is composed of an elliptical cylinder body, a tangent ogive nose and a delta wing. The design variables in this fields are the geometrical data and design specifications, e.g., maximum dynamic pressure q_{max} ($\leq$ 100 kPa), and maximum load factor n_{max} ($\leq$4 G). Note that the tank volume of fuel compounded from liquid hydrogen (LH_2) and liquid oxygen (LOX) must be less than 70 % of the total body volume.

The vehicle weight is estimated by using WAATS[6] program in which some parameters are modified for a spaceplane, structural weight W_{STR} can be estimated from the body and wing size. Considering the fuel weight W_{fuel} obtained in the trajectory planning fields together, payload weight $W_{payload}$ is defined as

$$W_{payload} = 300 - W_{STR} - W_{fuel}\,[\text{ton}] \quad (3)$$

(2) Aerodynamic Analysis

The aerodynamic characteristics of the model are analytically computed by CRSFLW method in Ref. 7 and 8. Five sampling points are selected from low speed to hypersonic speed, where aerodynamic parameters related to lift coefficient and drag coefficient are calculated. Three variables, l_2, l_3 and s ($\geq b$), representing the wing size are decided in this field. The aerodynamic coefficients are used to compute trajectories in the following field and the wing size is needed to estimate the structural weight in the body design field.

(3) Trajectory Planning

The spaceplane takes off, rises and is accelerated by ATR (up to Mach 6), SCR (switched from ATR and useable to Mach 12) and ROC (useable with ATR and SCR at the same time). Then, after the engine is cut-off above 90 km, it zooms up to 400 km with no thrust in an elliptical orbit. Finally, it is put into a 400 km circular orbit at the apogee in the elliptical orbit (Fig. 3).

In the flight trajectory design, state variables are altitude h, velocity v, flight-path angle γ and weight m. A control variable is defined as the angle of attack α. Motion equations[8] of the spaceplane are expressed as

$$\frac{dh}{dt} = v\sin\gamma \quad (4a)$$

$$\frac{dv}{dt} = \frac{(T_{ATR} + T_{SCR} + T_{ROC})\cos\alpha - D}{m} - \frac{\mu \sin\gamma}{r^2} \quad (4b)$$

$$\frac{d\gamma}{dt} = \frac{(T_{ATR} + T_{SCR} + T_{ROC})\sin\alpha + L}{mv} + \left(\frac{v}{r} - \frac{\mu}{vr^2}\right)\cos\gamma \quad (4c)$$

$$\frac{dm}{dt} = -\left(\frac{T_{ATR}}{I_{SPATR}} + \frac{T_{SCR}}{I_{SPSCR}} + \frac{T_{ROC}}{I_{SPROC}}\right)\frac{1}{g_0} \quad (4d)$$

where μ is the gravity constant, g_0 is the gravity acceleration at the ground level, and D and L are the lift and drag respectively, which are computed by the aerodynamic coefficients obtained in the above field. T_{ATR}, T_{SCR} and T_{ROC} are the thrust of air-turboramjet (ATR) engine, scramjet (SCR) engine and rocket (ROC) engine, I_{SPATR}, I_{SPSCR} and I_{SPROC} are specific impulse of each engine, which are represented in Ref. 8.

Initial conditions at time t=0 are specified as

$$h(0) = 0\,[\text{km}] \quad (5a)$$

$$\gamma(0) = 0\,[\text{deg}] \quad (5b)$$

$$m(0) = 300\times 10^3\,[\text{kg}] \quad (5c)$$

$$L\cos\alpha + (T - D)\sin\alpha \geq m(0)g_0 \quad (5d)$$

$$v(0) \leq 150\,[\text{m/sec}] \quad (5e)$$

Terminal conditions at the engine cut-off time $t=t_f$ is expressed as

$$h(t_f) \geq 90\,[\text{km}] \quad (6a)$$

$$\gamma(t_f) \geq 0\,[\text{deg}] \quad (6b)$$

and the apogee altitude computed by the terminal states, $h(t_f)$, $\gamma(t_f)$ and $v(t_f)$, needs to be 400 km.

In addition, the following path constraints are defined.

$$h \geq 0\,[\text{km}] \quad (7a)$$

$$q \leq q_{max} \quad (7b)$$

$$\alpha \leq 20\,[\text{deg}] \quad (7c)$$

$$n \leq n_{max} \quad (7d)$$

where q is dynamic pressure and n is load factor.

It should be noted that the motion equations change discontinuously since the operating engines are switched according to the flight conditions. Therefor the trajectory planning field is subdivided into four stages, that is, ATR, SCR SCR+ROC and ROC stage, which provide four sub-problems.

(4) Optimization Process

The flight trajectory can be optimized by solving the conventional optimal control problem in which an objective function is minimization of required fuels. In this study, the state variables and constraints are discretized to 200 elements, and the state equations (Eqs. 4) are approximated as equality constrains by using the collocation method[8]. These procedures transform an optimal control problem into nonlinear programming problem with static variables.

The trajectory optimization problem cane be integrated with vehicle design problems in which the accent trajectory, the vehicle shape and the engine size are optimized simultaneously. The objective function in the integrated problem is maximization of the payload in Eq. (3). The parallel optimization method manages each sub-problem, i.e., the body design, the aerodynamic analysis and the trajectory planning which is divided into four problems according the engine type.

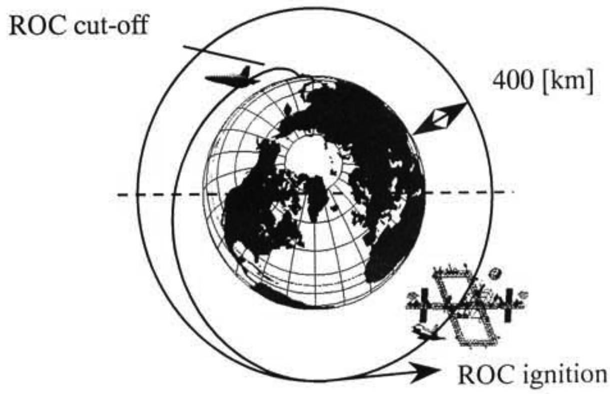

Fig.3 Flight Trajectory

NUMERICAL RESULTS

The numerical solutions are shown in Fig. 4–6 and Table 1. The maximized payload weight is negative value, -13.75 ton, and the spaceplane cannot reach the orbit even without the payload. This indicates that the present technological level cannot launch the spaceplane to the spacestation and that the weight reduction more than 5 % is required to realize it.

Figure 4 and table 1 show the optimized wing area and the intake area of ATR are very small, and the intake area of SCR is 0 m^2, which means that SCR is unnecessary in this study. It can be considered that the wing area and ATR are respectively the limit size in order to take off and fly the vehicle against the aerodynamic drag, and that the volume of LH_2 is reduced because SCR isn't used.

CONCLUSIONS

The new parallel optimization method for a large-scale system design with a huge number of variables and constraint conditions is proposed in this paper. This method divides the problem into some small optimization sub-problems based on subsystems constituting the system, which are solved in parallel. This method is successfully applied to a shape and ascent trajectory optimization for a spaceplane. While the obtained payload is minus, the effectiveness and the need of multidisciplinary optimization are demonstrated.

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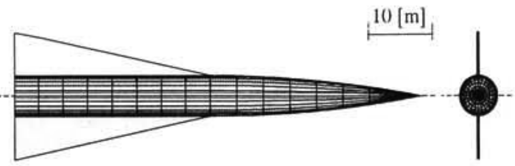

Fig.3 Optimal shape

Table 1 Characteristics of optimal spaceplane

Characteristics		Optimal values
Body length	l_4 [m]	63.48
Body height	a [m]	6.00
Body width	b [m]	6.36
Wing span	s [m]	10.02
Intake area of ATR	S_{ATR} [m^2]	12.59
Intake area of SCR	S_{SCR} [m^2]	0.00
Thrust of ROC	T_{ROC} [ton]	226.6
Max. thrust	T_{max} [ton]	226.6
Max. dynamic pressure	q_{max} [kPa]	100.0
Max. load factor	n_{max}[G]	3.82
Payload weight	$W_{payload}$ [ton]	-13.75

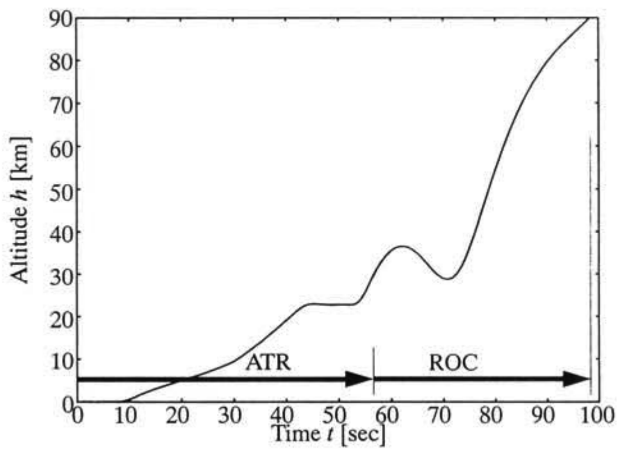

Fig.4 Time history of altitude

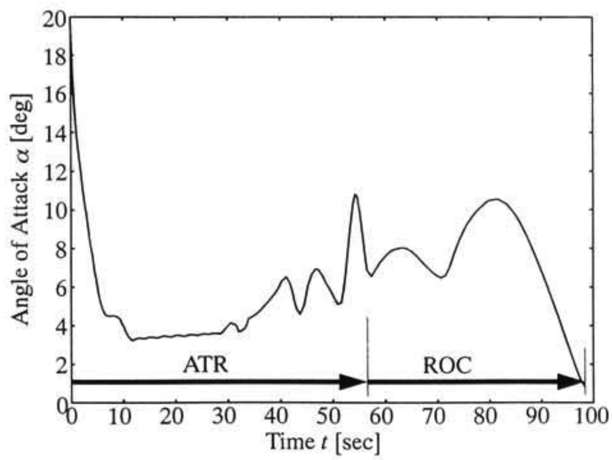

Fig. 3 Time history of angle of attack