The navigation for Formation flying of spacecraft

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Abstract: Japan Aerospace Exploration Agency (JAXA) is planning the next generation magnetosphere observation mission called "SCOPE." (cross-Scale Coupling in Plasma universE). SCOPE aims at observing the Earth's magnetotail with 5 satellites flying in formation to fully resolve the temporary and spatialdistribution of the magnetospheric phenomena in the near future. For this observation, the clock synchronization and relative distance measurement between the spacecrafts are essential. This paper describes introduced onboard relative ranging and clock synchronization systems and a robust and efficient approach for relative navigation and parameter estimation of spacecraft flying in formation ingeneral in the main topics. In the main focuses, the approach uses measurements from the optical data that provides a line of sight vector from the master spacecraft to the secondary satellite. State estimation for formation flying is achieved through an optimal observer design. Also, because the rotational and translational motions are coupled through the observation vectors, three approaches are suggested to separate both signals just for stability analysis.Simulation result indicate that the combined sensor/estimator approach provides accurate relative position estimate.

Keywords: Formation flying, Navigation, Orbit determination, Ranging system, Optical data.

1. INTRODUCTION

SCOPE aims at observing the Earth's magnetotail where the ions and electrons interact with each other, with 5 satellites flying information. To fully resolve the time-domain behaviour and spatial distribution of the magnetospheric phenomena, a simultaneous observation by spatially distributedelectro-magnetic instruments is essential.

SCOPE's orbit is a highly elliptical orbit with its apogee 30RE (200,000 km), perigee RE + 3000 km from the Earth center. SCOPE consists of one 450 kg mother satellite and four 90 kg daughter satellites, flying 5 to 5000 km apart from each other. The inter-satellite link is used for telemetry/command operation as well as ranging to determine the relative orbit of 5 satellites in a small distance, which cannot be resolved by the ground-based orbit determination. The relative ranging and clock synchronization is one of the essential technologies for this mission. One of the technology candidates to be implemented aboard spacecrafts is the onboard correlation capability. The onboard correlation for the wave and particle measurements requires each spacecraft to synchronize its clock to up to 1usec, and to use the relative distance between spacecrafts of the 10 m accuracy.

Other main focus in this paper, the navigation system described for formation flying applications comprises an optical sensor of a combined with specific light sources (beacons) in order to achieve a selective for the future spacecrft. It is assumed the sensor is made up of a position sensing system placed in the focal plane of a wide angle lens.

While the individual currents depend on the intensity of the light, their imbalances are weakly dependent on the intensity and are almost linearly proportional to the location of the centroid of the energy incident on the position sensing system. Calculating the current imbalances then yields two analog signals directly related to the coordinates locating the centroid of that beacon's energy distribution on the position sensing system, in a quasi-linear fashion, and therefore to the incident direction of this light on the wide-angle lens (which gives a line of sight vector). Because the beacons are offset from the mass center of the secondary satellite, the observed line of sight couples the rotational and the translational motion. The Kalman filter uses this raw information to update the position and the attitude equations without

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any discrimination about the nature of the signal. This approach is effective in most of the cases, but it is difficult for stability analysis because of the complexity of the system in hand and of the way that the Kalman gain is calculated. The approach presented in this paper is based on two special characteristics: the observer uses a constant gain for each parameter to be estimated (suboptimal filter) and the incoming signal is split according to the translational and the rotational dynamics. The use of constant gains avoids dealing with nonlinear time-varying systems, and the signal separation allows two independent plants where the stability analysis is feasible on each one using different approaches.

The organization of this paper proceeds as follows. First, the basic equations for the system are given. Next, the calibration procedure is shown. Then, the relative attitude equations are derived, followed by a presentation of the orbital equations of motion. The suggested methods for the signal separation process are then presented. Next, the observer design for relative attitude and position estimation is shown. Finally, simulation results for formation flying applications are presented.

2. SCOPE Formation Flying

To resolve the time and spatial distribution of ionic and electronic phenomena to the significant level, scientists require the spacecrafts' clocks should synchronize and the relative positions between the spacecrafts should be obtained throughout observations. The accuracy requirements are shown in Table.1. The requirements for scientific observations are as follows;

(1) The clocks must synchronize to the accuracy indicated in Tab.1 throughout the observation. The relative ranges (1-dimensional) should be obtained on board the spacecrafts with the accuracy indicated in Table.1, so that on-board data selection and correlation process can be executed.

(2) The 3-dimensional relative positions should be obtained with the accuracy indicated in Tab.1. As a real time determination is not required for this information, the positions are to be calculated on ground based on an offline analysis.

(3) As to the absolute precision of the mother satellite's clock (with reference to UTC) and the absolute positions with reference to the Earth center, only the ordinary level of accuracy is required.

2.1 Ranging and Clock Synchronization System

The range between mother and daughter satellite is measured by PN code signal-reproduction ranging, where the daughters loop-back the signal from mother satellite, and the mother compares the PN code phases of the forwarded signal with that of the returned signal to obtain the distance information. At the same time, the daughter reads the PN code from mother and synchronizes the internal clock, so that the clocks of mother and daughter tick at the same speed, as long as the daughters receive the signal from mother.

In addition to the mother-daughter link, inter-daughter link is prepared to measure the inter-daughter distance. Each daughter has a second receiver to receive the reverse link from another daughter, and has a capability of comparing the phase difference between this three-way PN code with its own clock.

2.2 SCOPE ranging system status

Though it still has areas violating the relative range requirement Tab1, this result implies that the clock synchronization compensating propagation delay, the most essential requirements, is satisfied. This in turn contributes to reduce the spatial and temporary scanning range for correlation, even when the link between spacecraft is restricted.

Table 1 Accuracy Requirements for Intersatellite Time Synchronization and Ranging

Mother-Doughter Distance	Time Synch. Accuracy	Ranging Accuracy
1 km	1 μsec	10 m
10 km	10 μsec	100 m
100 km	100 μsec	1 km
1000 km	1 msec	10 km
L km	L/1000 msec	L/100 km

3. Formation flying by using optical sensor observation

3.1 The equations

The mathematical models are presented in the context of the particular problem related to relative position and attitude estimation from line of sight observations. The angular velocity of the frame with respect to the β frame is represented by the physical vector $\omega_{\beta\alpha}$ (physical denotes that the vector is independent of the frame, whereas mathematical denotes the physical vector components expressed in some frame). The vector $\omega^{\gamma}_{\beta\alpha}$ is the mathematical vector made up of the components of $\omega_{\beta\alpha}$ taken in the γ frame. The derivative with respect to time is indicated by the operator p, where $p_\alpha\mathbf{R}$ is the rate of change of the vector $\mathbf{R}$ relative to the frame α and $p_\alpha\mathbf{R}^\alpha$ is the time derivative of the vector expressed in the α frame.

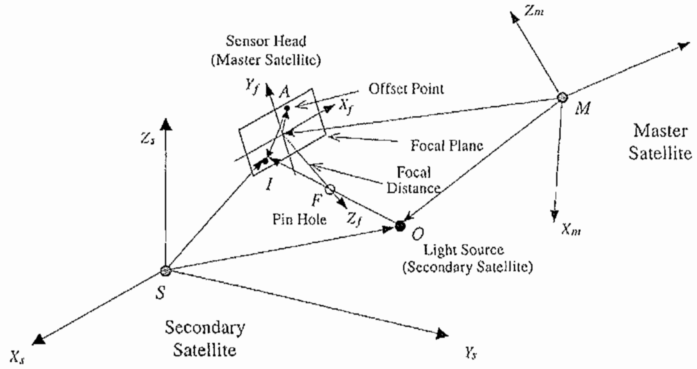

Fig. 1 Focal Plane observation

3.1.1 Observation equation

Figure 1 shows the focal plane measurement of the system for a master and secondary satellite system using one light source from a beacon. Three frames are used to describe the orientation and position of the master and secondary satellite. The first one, denoted by (X_s, Y_s, Z_s) is fixed on the secondary satellite, with the beacons firmly attached to the body of the satellite, and having known positions in the (X_s, Y_s, Z_s) frame.

We assume that this frame is centered at the mass center of this spacecraft, and is denoted using the superscript s on the mathematical vectors. Fig.1 Focal Plane observation The second reference system, denoted by (X_f, Y_f, Z_f) is fixed on the master satellite, where the focal plane of the system is located. We assume that the Z_f axis is along the boresight, passing through the input pin hole which is at a distance $Z_f = +f$ from the focal plane. The axes X_f and Y_f are arbitrary, but fixed in the sensor. This frame is denoted as the f frame. The third frame, denoted by (X_m, Y_m, Z_m) is fixed to the mass center of the master satellite. The position and orientation of this frame with respect to the focal frame is assumed to be known. The vectors for the master frame are identified with the superscript m.

The point S is the origin of the frame s. The point O is the location of each light beacon in the secondary satellite; normally there are several beacons to assure continuous tracking of the satellite and for redundancy. The point I is sometimes referred as the *image center* since it is the intersection of each light beam from the beacon with the focal plane, where position of I with respect to the focal reference system is used to form a line of sight observation. The point denoted as F in Figure 1 is the pinhole which is coincident with the sensor principal point. Three vectors are now defined: SO (the vector from the center S of the s frame to the beacon location O), SI (the vector from the the center S of the s frame to the image center I), and OI (the vector from the beacon location O to the image center I, with the constraint equation given by $OI = SI - SO$.

The orientation between the secondary and master frames is denoted by the (unknown) rotation matrix C^m_s which transforms a vector expressed in the secondary frame s to the primary frame m. The rotation matrix C^m_f between the focal and the master frames is known by ground calibration. Expressing the vectors $\vec{SI}$, $\vec{OI}$, and $\vec{SO}$, in frame components gives the following.

$$C^m_S\left(\vec{SI} - \vec{SO}\right)^S \equiv C^m_S\mathbf{v}^S \equiv \mathbf{v}^m \equiv \left(\vec{OI}\right)^m \tag{1}$$

$$\text{where } \mathbf{v}^S = \xi^{-1/2}\left[X_I - x_O, Y_I - y_O, Z_I - z_O\right]^T \text{ and } \xi \equiv \left(X_I - x_O\right)^2 + \left(Y_I - y_O\right)^2 + \left(Z_I - z_O\right)^2.$$

The quantity (Xo, Yo, Zo) represents the known beacon location, and (X_I, Y_I, Z_I) is the unknown position with respect to the secondary satellite. The measurements x_I and y_I in the focal frame can be expressed in unit vector form by

$$\mathbf{v}^f = \frac{1}{\sqrt{x_I^2 + y_I^2 + z_I^2}}\begin{bmatrix} x_I \\ y_I \\ z_I \end{bmatrix} \tag{2}$$

where f is the known focal distance. This unit vector in the master frame is expressed using the fixed rotation matrix between the sensor plane frame and the master satellite reference frame, with $\mathbf{v}^m = C^m_f\,\mathbf{v}^f$. A bias offset in the measurement is also accounted for in the model (denoted by A in Figure 1). The bias vector is a constant error vector induced by an unbalance of the horizontal and vertical gains in the focal plane detector relative to the particular coordinate system associated with the detector at calibration. Essentially this is the same offset between the "electrical center" (zero voltage imbalance) and the geometrical center associated with the optical boresight and sensor coordinate system. This vector is denoted by. $\mathbf{v}_\alpha$ and is normally referenced in the focal plane frame:

$$\mathbf{v}_{\alpha}^{m} = C_{f}^{m}\mathbf{v}_{\alpha}^{f} = C_{f}^{m}\begin{bmatrix} x_I \\ y_I \\ 0 \end{bmatrix} \tag{3}$$

Small separations between light beams reduces the discrimination of each beacon, which ultimately produces a dilution of precision in the position and attitude solution. A larger distance between the satellites also leads to a dilution of precision since the beacons ultimately approach angular co-location. If the relative position between satellites is known then only two non-colinear line of sight vectors are required to determine an attitude solution. In a similar fashion for the position navigation only problem, where the satellite is considered to be a "mass point" (in other words without attitude), two line of sight vectors are only required. A covariance analysis shows that when the relative position and attitude both are unknown then two line of sight vectors provide only one axis of attitude and one axis of position information. Furthermore, an observability analysis using two line of sight observations indicates that the beacon that is closest to the target provides the most attitude information.

The least position information, and the beacon that is farthest to the target provides the most position information but has the least attitude information. In order to find a deterministic solution for the position and velocity at least four vector observations are required.

3.1.2 Relative attitude

In this section the governing equations for the relative attitude kinematics between two bodies are reviewed. The kinematic equations presented here are derived using non-inertial reference frames, however only minor changes are required from the standard formulation. Starting from Eq. (1) and taking derivative of each vector with respect to the same frame in which they are expressed gives the following expressions

$$p\mathbf{v}^{m} = C_{S}^{m}p\mathbf{v}^{S} + pC_{S}^{m}\mathbf{v}^{S} = C_{S}^{p}\left(p\mathbf{v}^{S} + C_{m}^{S}pC_{S}^{m}\mathbf{v}^{S}\right) \tag{4}$$

The bias in Eq. (1) is considered to be a constant, so it's derivative is zero. The same expression in Eq. (4) can be derived by the application of the transport theorem, which yields the following expressions

$$p_{m}\mathbf{v} = p_{S}\mathbf{v} + \omega_{ms} \times \mathbf{v} \tag{5a}$$

$$p\mathbf{v}^{m} = C_{S}^{m}\left(p\mathbf{v}^{S} + \omega_{ms}^{S} \times \mathbf{v}^{S}\right) \tag{5b}$$

Both expressions, (4) and (5), must be equivalent. Setting these equations equal to each other yields the time rate of change of the attitude matrix, given by

$$C_{m}^{S}pC_{S}^{m} = \left[\omega_{ms} \times\right] \tag{6a}$$

$$pC_{S}^{m} = -\left[\omega_{sp}^{m} \times\right]C_{S}^{m} \tag{6b}$$

where [● ×] denotes the cross product matrix.

The relative attitude kinematics are described by the expression in Eq. (6) in terms of attitude matrix and theangular velocity between both frames.

We now write the expression in Eq. (6) in terms of the corresponding quaternions. Toward this end, the quaternion is expressed as, $\mathbf{q}_{m}^{s}\left[e^{t}\right.$ $\sin\frac{a}{2}$ $\cos\frac{a}{2}\left.\right]^{T}$ where e is the eigenaxis between both frames and α is the rotation angle measured from frame m to frame s. The quaternion is a vector with the same components in both the m and s frames, and can be expressed in any external frame as an arbitrary (i.e. general) vector. This has an advantage over the rotation matrix formulation, which is fixed to the reference system s and m in this case. An infinitesimal rotation is expressed in terms of the quaternion as $d\mathbf{q}_{m}^{s} = 1 + 1/2\ \omega_{sm}dt$, where dt is the time differential. Multiplying by the quaternion q_{m}^{s} and taking the first-order infinitesimal part, the following differential equation is given

$$p\mathbf{q}^s_m = \frac{1}{2}\mathbf{q}^s_m \otimes \omega^m_{sm} = \frac{1}{2}\begin{bmatrix} \varrho & \vdots & q_o I_{3\times3} + [\varrho\times] \\ \cdots & \vdots & \cdots\cdots\cdots\cdots \\ q_o & \vdots & -\varrho^T \end{bmatrix}\begin{bmatrix} \omega_{sm} \\ \cdots \\ 0 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} q_o\omega_{sm} + [\varrho\times]\omega_{sm} \\ \cdots\cdots\cdots\cdots \\ -\varrho^T \cdot \omega_{sm} \end{bmatrix} \tag{7}$$

wher th quaternion $\mathbf{q}^s_m$ is decomposed into a scalar and a vector part as $\mathbf{q}^s_m = \left[(Q^s_m)^T \; q_0\right]^T$, and $\left[Q\times\right]$ is the skew symmetric cross product matrix. Both the attitude matrix and quaternion formulations will be used in the definition of the observer feedback error, but the quaternion formulation is used in the actual implementation of the observer.

3.1.3 Relative navigation and dynamics

From basic orbit theory, s the equations of motion are written assuming that each satellite is referenced with respect to the same inertial frame. The vectors are described in the Fig. 2. The relative orbit is described by the difference between both vectors, $\mathbf{r} = \mathbf{R}_m - \mathbf{R}_s$. If the master satellite position vector is written as $\mathbf{R}_m = R_m[1\ 0\ 0]^T$, the expression can be simplified. The frame with this property is the *Local Vertical Local Horizontal* (LVLH) reference frame, which is widely used to reference Earth Pointing satellites. The vector $\mathbf{r}$ is decomposed in m frame components and takes the final expression given by

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -\mu \begin{bmatrix} \dfrac{R_m + x}{\|\mathbf{R}_m + \mathbf{r}\|^3} - \dfrac{R_m}{\|\mathbf{R}_m\|^3} \\ \dfrac{y}{\|\mathbf{R}_m + \mathbf{r}\|^3} \\ \dfrac{z}{\|\mathbf{R}_m + \mathbf{r}\|^3} \end{bmatrix} - \dot{\omega}\begin{bmatrix} -y \\ x \\ 0 \end{bmatrix} - \omega\begin{bmatrix} -2\dot{y} - \omega x \\ 2\dot{x} - \omega y \\ 0 \end{bmatrix} + \begin{bmatrix} \Delta a_x \\ \Delta a_y \\ \Delta a_z \end{bmatrix} \tag{8}$$

where $\Delta\alpha$ is the relative acceleration. The forcing part along the X axis component has the following structure:

$[f(\mathrm{R}_m + \mathrm{R}_{12}) - f(\mathbf{R}_m)]^T$ which is not robust from a numerical point of view. This expression is maintained for compactness and will be used in the observer analysis, but for practical implementations it is convenient to re-write it avoiding the subtraction of two large numbers. Eq. (8) expresses the dynamical model for relative navigation between the secondary satellite with respect to the master satellite.

We note that the number of master satellite orbit parameters computed on the ground and to be used in Eq. (8) is at most 3. For the general case, the magnitude R_m, the angular velocity ω, and angular acceleration $\dot{\omega}$ are just needed. For the special case involving circular orbits, only the position magnitude is necessary.

In the attitude problem, Euler's equation or the measured gyro outputs are the starting point for the derivation of the rotational dynamics equation to obtain the angular velocity between the inertial frame and a body frame. For the relative attitude problem, Euler's equation must be applied in a differential mode, similar in fashion as the orbit case. However, we seek an expression without an additional "third" frame (inertial one included), in addition to the m and the s frames, so that the system is independent of the extra reference frame's choice. In other words, the relative navigation and the relative attitude must be a function only of the definition of the master and secondary frames and completely independent of the particular choice of the inertial frame or any other frame other than m and s. This simple fact is common in control theory, where the error or its derivative is only defined by the current and the desired state independent of any other frame choice.

In the two body problem previously derived, the equation for r is very accurate because it is supported by well known models for almost all involved forces in hand, with any remaining small perturbation bounded. In the relative attitude dynamics the presence of internal torques, which are normally unmodeled with an unbounded time integral, plays an important role in the model equations. We assume that each satellite in the constellation has an attitude control subsystem able to maintain the desired satellite orientation inside of some allowable bound. The last hypothesis is a qualitative one. We assume that the measurements are available frequently enough to use simpler propagation models (to be derived) as a function of the sampling interval.

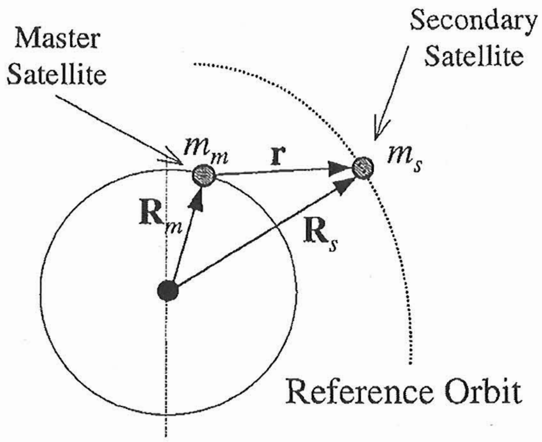

Fig. 2 Relative navigation

3.1.4 Signal separation

Due to the slow moving dynamics in orbital motion, the translational and rotational dynamics are almost independent (in fact there is a second order coupling thought the angular velocity which appears in both systems, but for this analysis we can assume a negligible coupling between both motions). This section presents three approaches for signal separation of the translational and rotational dynamics:

1. *Time Domain Approach*: This approach assumes that the contribution of the angular rotation is smaller than the translational motion contribution on the sensor output. In other words the distance between the beacon location and the secondary satellite mass center is much smaller than the distance between the mass center of both satellites.
2. *Spline Wavelets Filter*: This approach is a frequency band filter which can run in quasi-real time, and uses splines as the basis functions for which the incoming signal is represented.
3. *Standard Band Pass Filter*: This is the standard approach from basic signal processing theory and in general does not run in real time. It uses the different frequency scales between the rotational and translational motion to separate them via one or two band filters.

3.1.4 Observer design

In this section an observer is designed to estimate the relative attitude and angular velocity as well as the relative position and the linear velocity. The information matrix of the attitude and position estimation errors is explicitly calculated for two line of sight observations. The information matrix is divided into four partitions, where the two main diagonal elements correspond to the attitude and position information matrices that have the identical structure if each problem (i.e., attitude or position) was considered independent of each other. The off-diagonal partitions couple the attitude and the position errors. A diagonalization (i.e, a decoupling of the attitude and position) of the information matrix occurs only in very special cases (the presence of a deterministic solution for example). Therefore, the entire problem which includes both attitude and position estimation should be considered in the observer design. Toward this end, the signal is separated using the previous methods, and both observers (attitude and position) are designed independently.

The observer design treats the attitude portion by representing the residual (measurement minus estimate) error through a quaternion vector formulation, and treats the position portion of the residual in a straightforward position vector formulation. The angular error between the measured ($\mathbf{v}^m$) and the estimated ($\hat{\mathbf{v}}^m$) vectors in master frame can be visualized by a rotation axes normal to plane that contains both vectors. This axis ($\phi^m_{\hat{m}}$) can be interpreted as the vector part of the quaternion error, and the rotation angle between both vectors is the scalar part of the quaternion. The position error ($\mathbf{dz}$) is simple vector difference between the estimated and measured vectors.

Before continuing with this concept, the following matrix relation is first written

$$C^m_s = C^m_{\hat{m}} \hat{C}^m_s = \Delta C \hat{C}^m_s \tag{9}$$

where the estimated vector, matrix or frames are noted with the superscript ($\hat{\cdot}$), and $\hat{C}^m_{\hat{m}} \equiv \Delta C$. The rotation error matrix between the estimated and measured quantities can be written in terms of the quaternion as $\Delta C = I + 2q_0 + 2\,[Q\times]^2$. To simplify the notation this matrix is simply defined as $\Delta C \equiv (I + [\delta\times])^m_{\hat{m}}$. Eq. (1) can now be re-written as

$$\mathbf{v}^m = \left(I + [\delta\times]\right)^m_{\hat{m}} \hat{C}^m_S \hat{\mathbf{v}}^{\hat{S}} \tag{10}$$

where $\hat{\mathbf{v}}^{\hat{s}}$ is an estimated vector, which depends on only of the angular motion (after signal separation). Eq. (10) can be re-written in residual form as

$$\mathbf{v}^m - \hat{\mathbf{v}}^m = [\delta\times]^m_{\hat{m}} \hat{\mathbf{v}}^{\hat{m}} \tag{11}$$

Using the multiplicative property of the cross product matrix the right hand side of Eq. (11) can be expressed in a more convenient form as

$$\hat{\mathbf{v}}^m - \mathbf{v}^m = [\hat{\mathbf{v}}\times]^{\hat{m}} \left(\phi^m_{\hat{m}}\right)^{\hat{m}} \tag{12}$$

where the vector $\phi^m_{\hat{m}}$ is expressed as the vector part of a quaternion in any frame. As stated previously this is one advantage of using the quaternion parameterization over the rotation matrix in the obserber. The left hand side of Eq. (12) is denoted by $d\mathbf{z} \equiv \hat{\mathbf{v}}^m - \mathbf{v}^m$ for simplicity.

The number of measured line of sight vectors is generally greater than one, and the processing of this information can be done in the least square sense. Each estimated vector cross product is stacked into a matrix as

$$\hat{\mathbf{v}}_m = \begin{bmatrix} [\hat{\mathbf{v}}_1 \times] \\ \vdots \\ [\hat{\mathbf{v}}_N \times] \end{bmatrix} \tag{13}$$

In this case the pseudoinverse is computed using all available information. Therefore, the quaternion error is computed by

$$\hat{\mathbf{v}}_m^+ d\mathbf{z} = \phi_S^m \rightarrow [\delta Q, \delta q_0] \tag{14}$$

where $\hat{\mathbf{v}}_m^+$ is the pseudoinverse of $\hat{\mathbf{v}}_m$. The computation of the quaternion error is corrected, but the scalar part of the quaternion (δq_0) is assumed to always be equal to +1. However, the scheme presented in this section maintains all four elements of the quaternion error because the sign of the scalar part is used in the design of the observer.

The nonlinear is used for attitude estimation; however, two slight modifications are introduced. The first one incorporates an angular velocity model, and the second includes a model of a potential bias, represented by b m, in the quaternion differential equation to include any offset of the sensor, which may even be the computation of the focal distance.

The observers for the relative linear position and relative linear velocity are given by

$$\dot{\hat{\mathbf{r}}} = \hat{\mathbf{v}} - K_p d\mathbf{z} \tag{15}$$

$$\dot{\hat{\mathbf{v}}} = \mathbf{f}(\hat{\mathbf{r}}, \hat{\mathbf{v}}) - K_v d\mathbf{z} \tag{16}$$

where $\mathbf{f}(\cdot)$ is the right hand side of Eq. (8), $\mathbf{r}$ is the relative position vector, and $\mathbf{v}$ is the relative linear velocity vector. The minus signs in Eqs. (15) and (16) are due to the definition of $d\mathbf{z}$. The constant gains K_p and K_v are positive definite matrices (usually diagonal).

3.1.5 Simulation

The orbital elements used in the simulation of the master satellite are shown in Table 2.

A small initial condition perturbation of these elements is used to simulate the motion of the secondary satellite. The true inertia matrices of both satellites is given by

$$I_s = I_m = diag\ [100 \quad 120 \quad 140]\ (N\text{–}m\text{-}s^2)$$

In the observer the following inertia matrices are used:

$$I_s = I_m = \mathrm{diag}\ [110 \quad 115 \quad 150]\ (N\text{–}m\text{-}s^2)$$

The true relative initial angular velocity is constant, and given by

$$\omega = [0.07 \quad 0.05 \quad 0.03]^T\ (deg/sec)$$

Table 2 Orbital elements of the Master satellite

Semimajor axis	6900 km
Eccentricity	0.1 deg
Inclination	50 deg
Node right ascention	10 deg
Argument of perigee	10 deg
Mean anomaly	10 deg

The relative angular velocity trajectory is computed by integrating the following equation

$$\dot{\omega} = \lambda I_S^{-1} \omega \tag{17}$$

where Is is the true inertia and $\lambda = 0.02$. A noise of 0.01/3000 is assumed for each measurement on the focal plane. Four beacons are placed on the secondary satellite at a distance of 1 meter from the mass center along each coordinate axis. The fourth beacon is placed at $[1, 1, 1]^T$ in the secondary frame.

The observer described in the last section is implemented for state estimation from the line of sight measurements. The initial condition angular error is a rotation of about 15 degree along each of the coordinates axes. The initial angular velocity has 50 percent errors from Eq. (17). The initial position condition 10 percent from the true value and the initial linear velocity condition is 30 percent from the true value. The sampling rate is 4 Hz. The plots in Fig. 3-1 and 3-2 shows position and linear velocity errors each other for the estimator.

The relative distance along the X axis is almost three times the distance along the other two axes (around 94 meters against 30 meters).

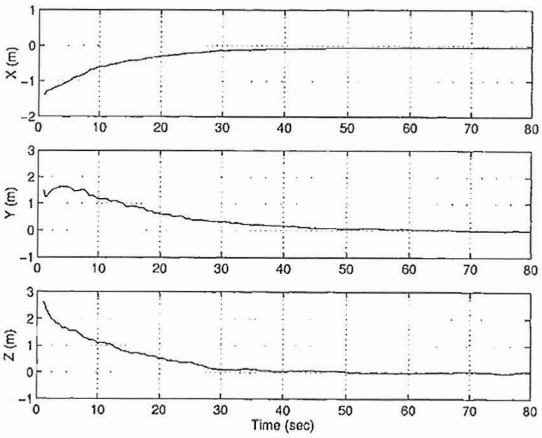

Fig. 3-1 Position error

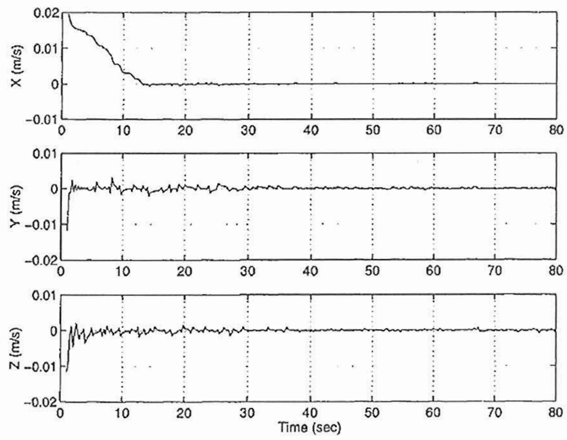

Fig. 3-2 Linear velocity error

This difference can be observed in the oscillation of the attitude error in roll, which is intuitively correct.

The roll angle error is within 0.3 degrees, and the pitch and yaw angles are within 0.05 degrees. The position errorin all three axis is within 1cm. Also, the velocities are well estimated using the observer.

4. Conclusion

The relative range determination and time synchronization system for formation flying of SCOPE mission was discussed first.

Next, a detailed algorithm and the result were discussed about the relative navigation by using optical data in the main topics of this paper. The optical sensor involving the beacons and position sensing technology in the focal plane has been introduced for formation flying applications. In order to achieve an accurate line of sight measurement from this sensor a calibration procedure has been shown. Experimental results indicate that the calibration provides accurate results. Also, an observer based system has been presented as an alternative to the extended Kalman filter for formation flying navigation of spacecraft. Simulation results have shown that accurate relative attitude and position estimation is possible.

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