Numerical Simulation of Magneto Plasma Sail by using 3D Hybrid Code

Yoshihiro Kajimura, Kenji Noda, Hideki Nakashima

Department of Advanced Energy Engineering Science
Interdisciplinary Graduate school of Engineering Sciences, Kyushu University, Japan
kajimura@aees.kyushu-u.ac.jp

We have been studying the Magneto Plasma Sail (MPS) by using 3D hybrid numerical simulation code. There are two key issues of MPS research, one is the possibility of magnetic inflation, and the other is the analysis of the interaction between the solar wind and the artificially generated magnetic field. We simulated the interaction between the solar wind and dipole magnetic field in pure-Magsail and compared with the drag coefficient obtained by using other hybrid code [1]. And we simulated the magnetic inflation by injected plasma. The simulation model is shown in Fig.1. Plasma is injected from the region located in 1.0 [m] from the center of the coil. Plasma is injected in radial direction with $N=10^{20}[m^{-3}]$ and v=4.0[km/s]. Next we simulated the interaction between the solar wind and inflated magnetic field by the plasma injection. The simulation geometry considered here is illustrated in Figure 2. The initial magnetic field is produced at the center of the system as dipole configuration. The solar wind flows only in the positive Z direction with 5×10^{6} $[m^{-3}]$ density and 400 [km/s] velocity. The Ar plasma with 5×10^{9} $[m^{-3}]$ density and 20 [km/s] velocity is injected isotropically. We will show these simulation results and list up the difficulties of numerical simulation of MPS in each theme. And finally I will make a brief summary and talk about the Future plan.

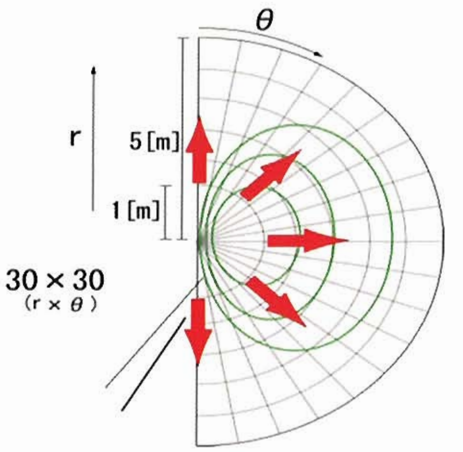


Figure.1 Simulation Model (inflation)

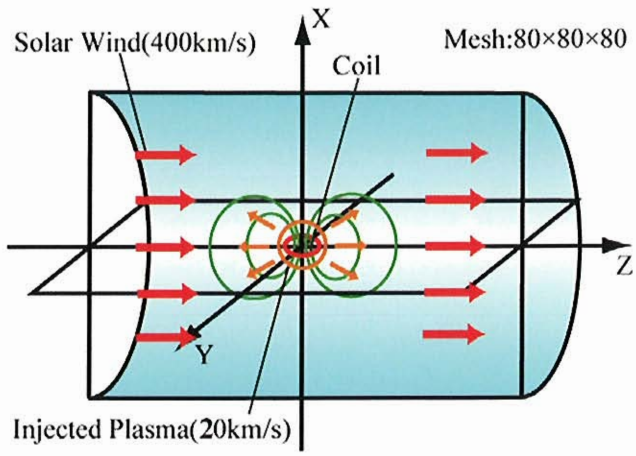


Figure 2. Simulation model (whole simulation)

[1] K. Fujita, "Particle Simulation of Moderately-Sized Magnetic Sails" Journal of Space Technology and Science, Vol.20, No.2, pp.26-31, 2005.

Numerical Simulation of Magneto Plasma Sail by using 3D hybrid code

JAXA/JEDI Workshop
Oct 2nd–3rd ,2006

Yoshihiro Kajimura, Kenji Noda, Hideki Nakashima

Department of Advanced Energy Engineering Science,
Interdisciplinary Graduate School of Engineering Sciences,
Kyushu University, Japan

I-EggS Interdisciplinary Graduate School of Engineering Sciences

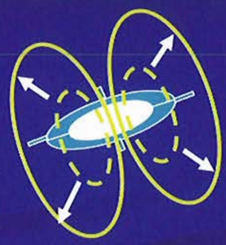
Department of Advanced Energy Engineering Science

Outline

- Key theme of MPS research
- Fundamental parameters of MPS
- Our resent simulation results of MPS
- The difficulty of MPS simulation (using hybrid code)
- Summary and Future plans

Key themes of MPS research

- Possibility of magnetic inflation by plasma injection
  - beta_0>1 , Magnetic Re >>1
  - How? Where? can we inject plasma for inflation effectively.
  - How far? is magnetic field inflated.

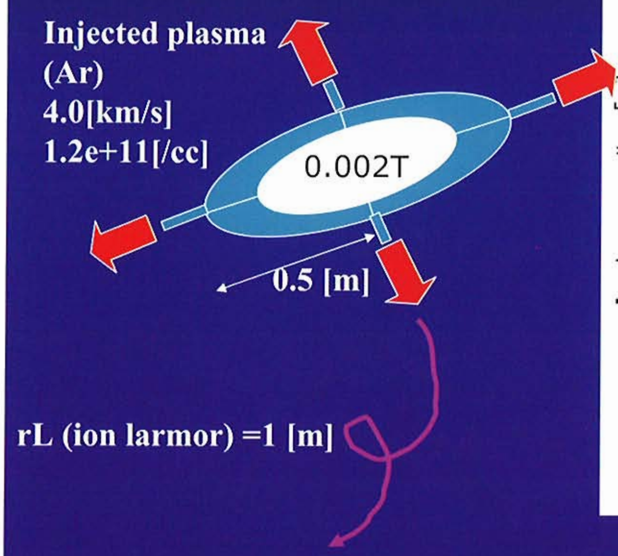


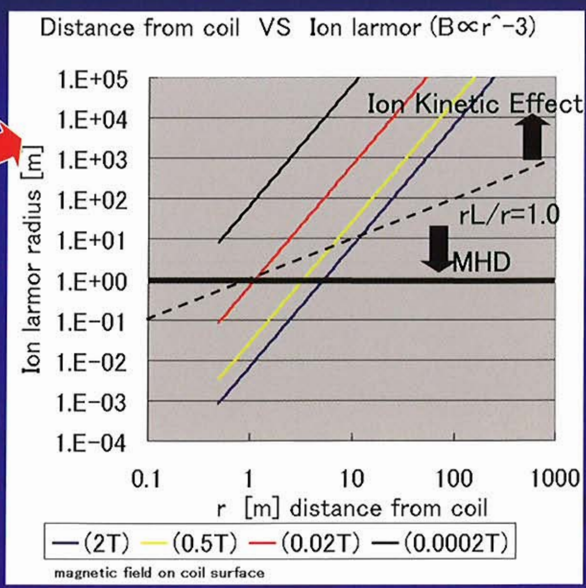
- Mechanism of the thrust transformation from the solar wind through the inflated magnetic field.
  - Is it possible to inflate the magnetic field in the solar wind?

http://www.ess.washington.edu/Space/M2P2/ by Winglee

Fundamental parameters of MPS

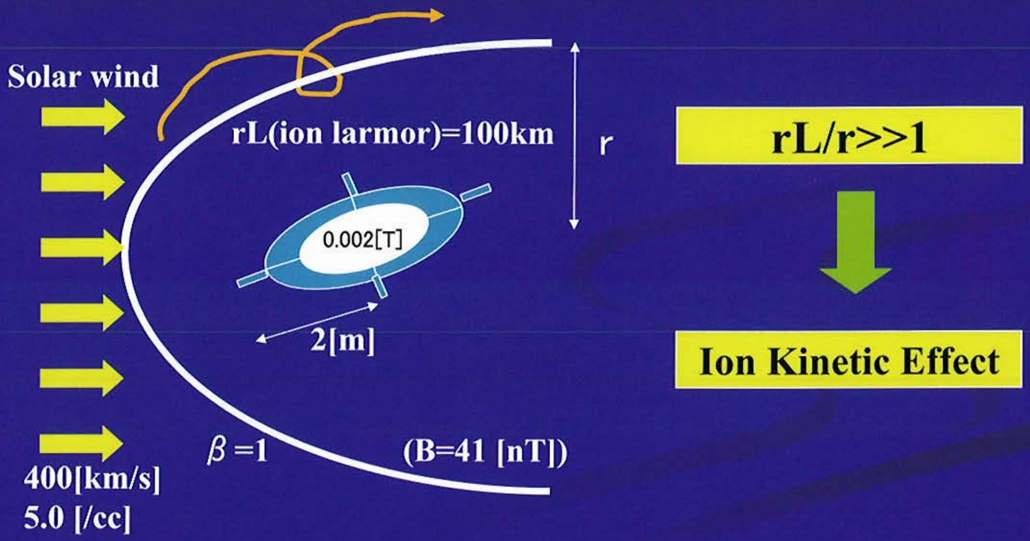
- Magnetic inflation





Fundamental parameters of MPS

- Interaction between the magnetic field and solar wind



General method of numerical simulation for plasma

- MHD
  - Plasma is treats as a fluid
  - The ion kinetic effect is ignored (rL<< L)
- Hybrid
  - Ions are treated as particles, electrons as a fluid
  - The ion kinetic effect is taken into account. (rL≧L)
- Full particle
  - Ions and electrons are treated as particles
  - The debye length must be used as the mesh size:
  - Injected plasma ([0.8 μm]) 、solar wind ([10m])

In the MPS study,
Ion kinetic simulations (hybrid simulation) are required

Our 3D Hybrid Code

- Basic equations (e: electron, i: ion)

Ion momentum equation

$$m_i \frac{d\mathbf{v}_i}{dt} = Ze(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \qquad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

Electron momentum equation

$$n_e m_e \frac{d\mathbf{v}_e}{dt} = -en_e(\mathbf{E} + v_e \times \mathbf{B}) - \nabla P_e$$

Ampere's law

$$\nabla \times \mathbf{B}_p = \mu_0(\mathbf{J}_e + \mathbf{J}_i)$$

Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Current density

$$\mathbf{J}_e = -en_e\mathbf{v}_e, \mathbf{J}_i = en_i\mathbf{v}_i$$

Position of ions $x_i^{n+1} = x_i^n + \mathbf{v}_i^{n+1/2}\Delta t$

Velocity of ions $\mathbf{v}_i^{n+1/2} = \mathbf{A}^{-1}\mathbf{S}$

Electric field (Plasma Region)

$$\mathbf{E}^n = \frac{1}{n_i}\left\{\frac{1}{\mu_0 Z_e}(\nabla \times \mathbf{B}_p^n) \times \mathbf{B}^n - \frac{1}{Ze}\mathbf{J}_i^n \times \mathbf{B}^n - \frac{T_e}{e}\nabla n_i^n\right\}$$

Electric field (vacuum region)

$$\nabla^2 \mathbf{E} = 0$$

CFL condition $\Delta t < \frac{\Delta x}{V_A}$

Numerical stability condition $\omega_{ci}\Delta t < 0.2$

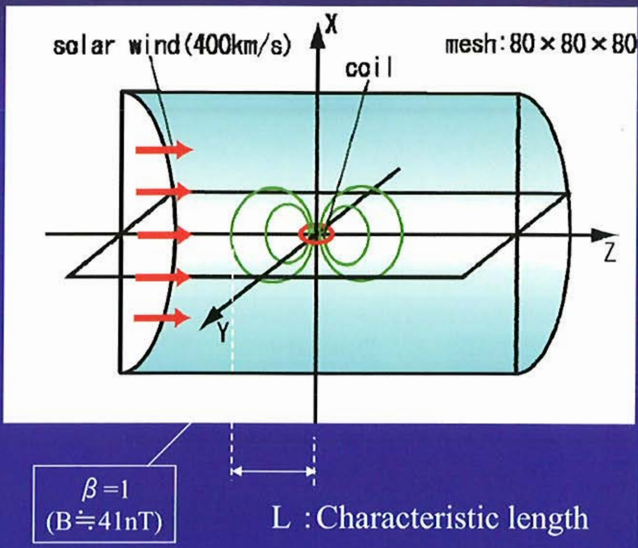
Electron energy equation is not used.
(T_e = const)

Our resent study of MPS

- Plasma flow around Pure Magsail
- Magnetic inflation
- Interaction between the inflated magnetic field and solar wind (whole simulation)

1. Brief results
2. Difficulty in each simulation

Plasma flow around Pure Magsail



Ion : Hydrogen
Ion velocity : 400km/s
Ion number density : 5×10^{6}m^{-3}
Super particle number : 2milion
In each cell : 15

Drag coefficient

$$C_d = \frac{F}{\frac{1}{2}\rho V^2 S}$$

F: Thrust [N] (calc from momentum)
ρ : density of solar wind
V : velocity of the solar wind
S: The cross section of the magnetosphere = πL^2 [m^2]

Simulation Parameters

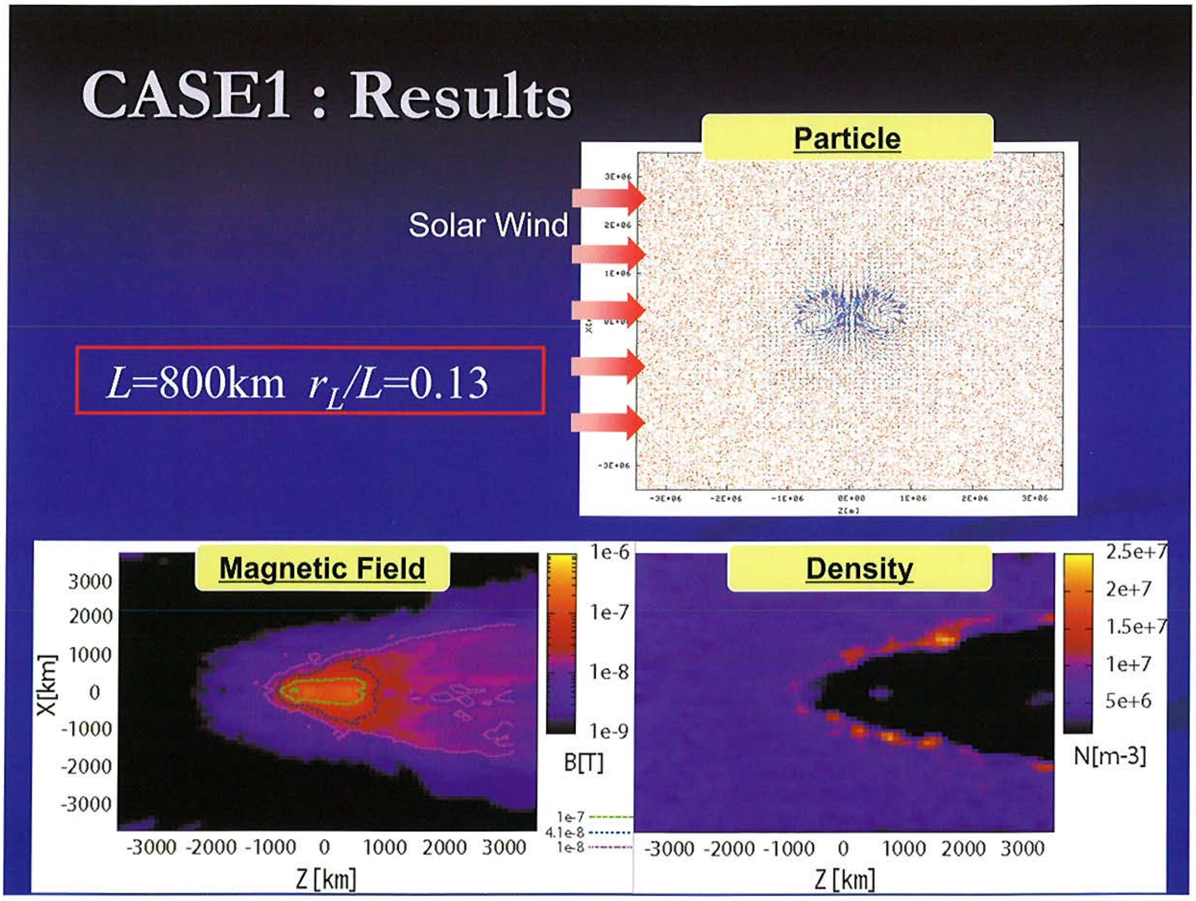
MHD Region

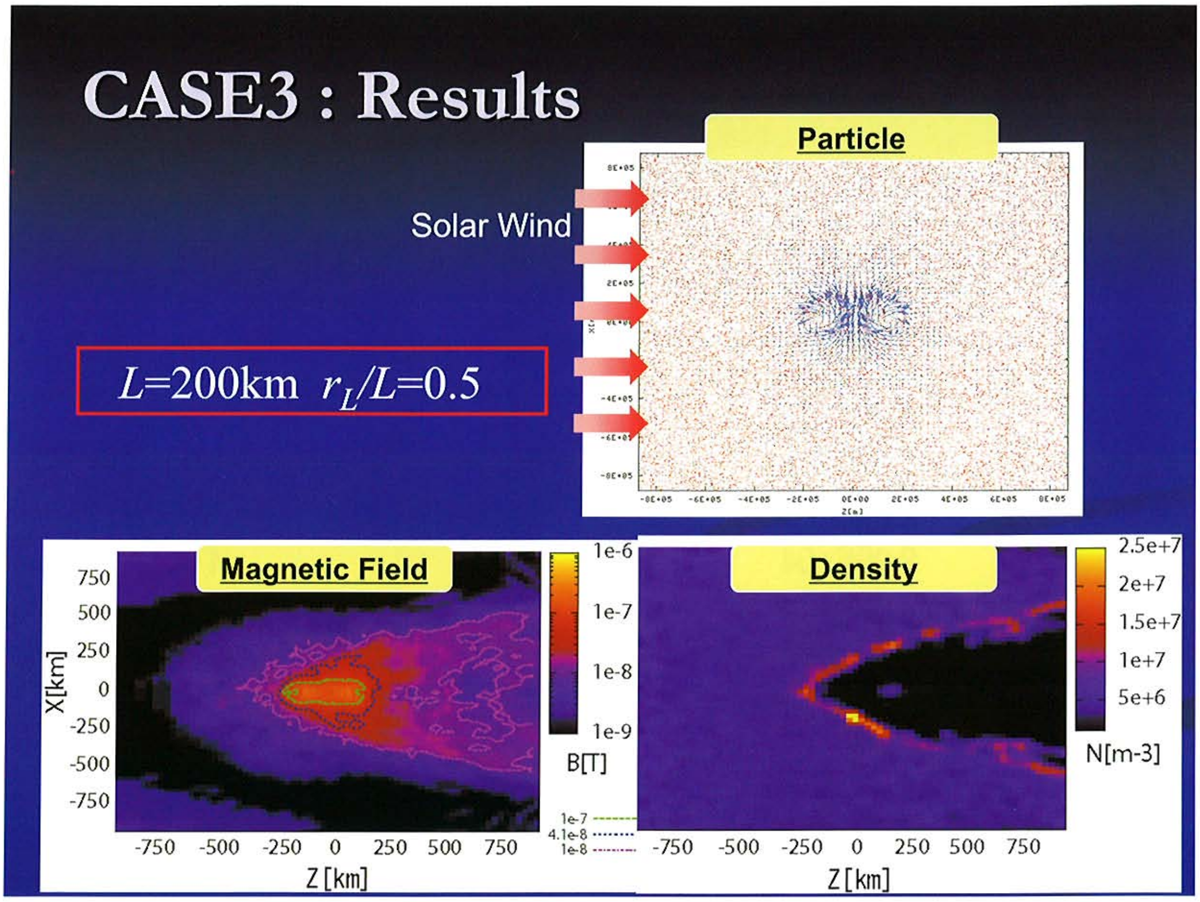
case	L(m)	r_L/L	$\delta/\Delta x$	System Size(m)
1	8.00E+05	0.13	1	4.00E+06
2	4.00E+05	0.25	2	2.00E+06
3	2.00E+05	0.50	4	1.00E+06
4	8.00E+04	1.25	10	4.00E+05
5	4.00E+04	2.50	20	2.00E+05

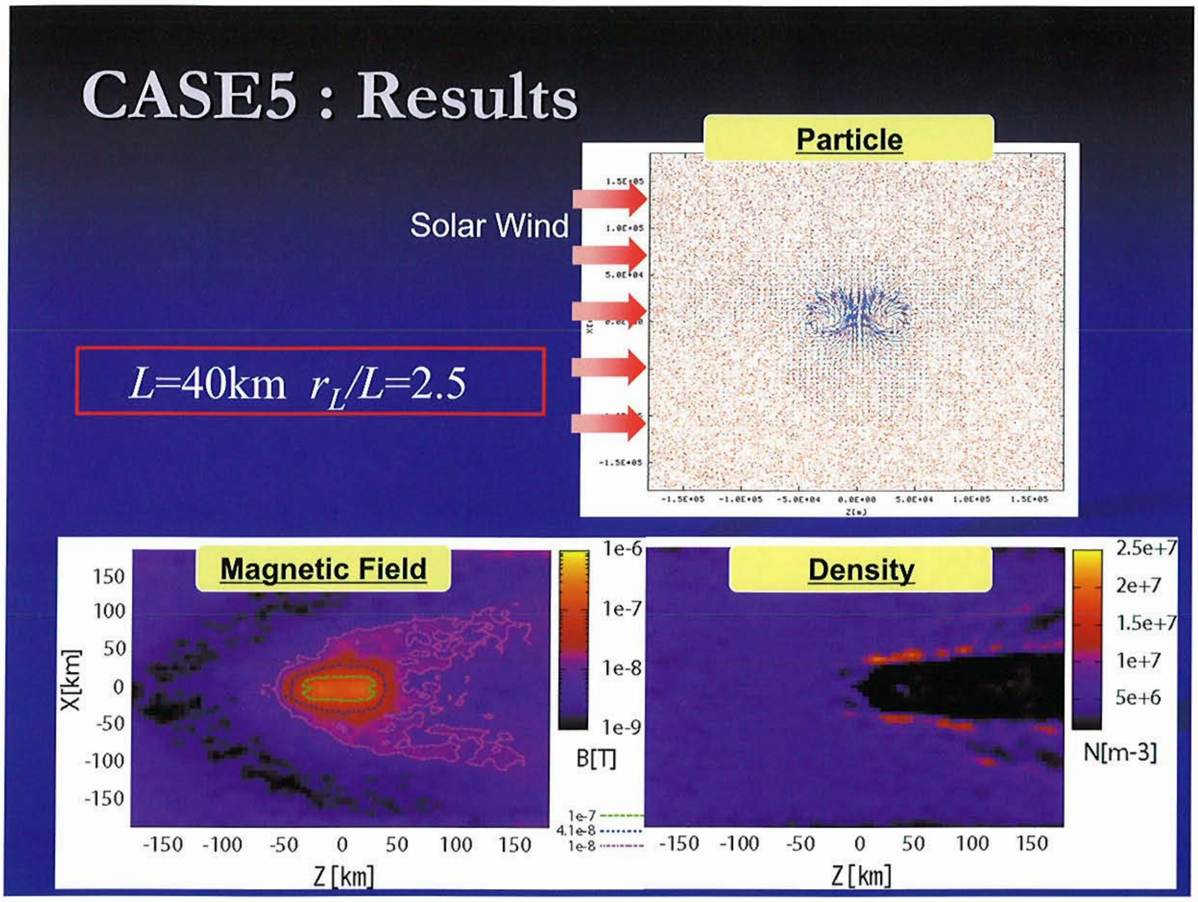
δ: Skin depth

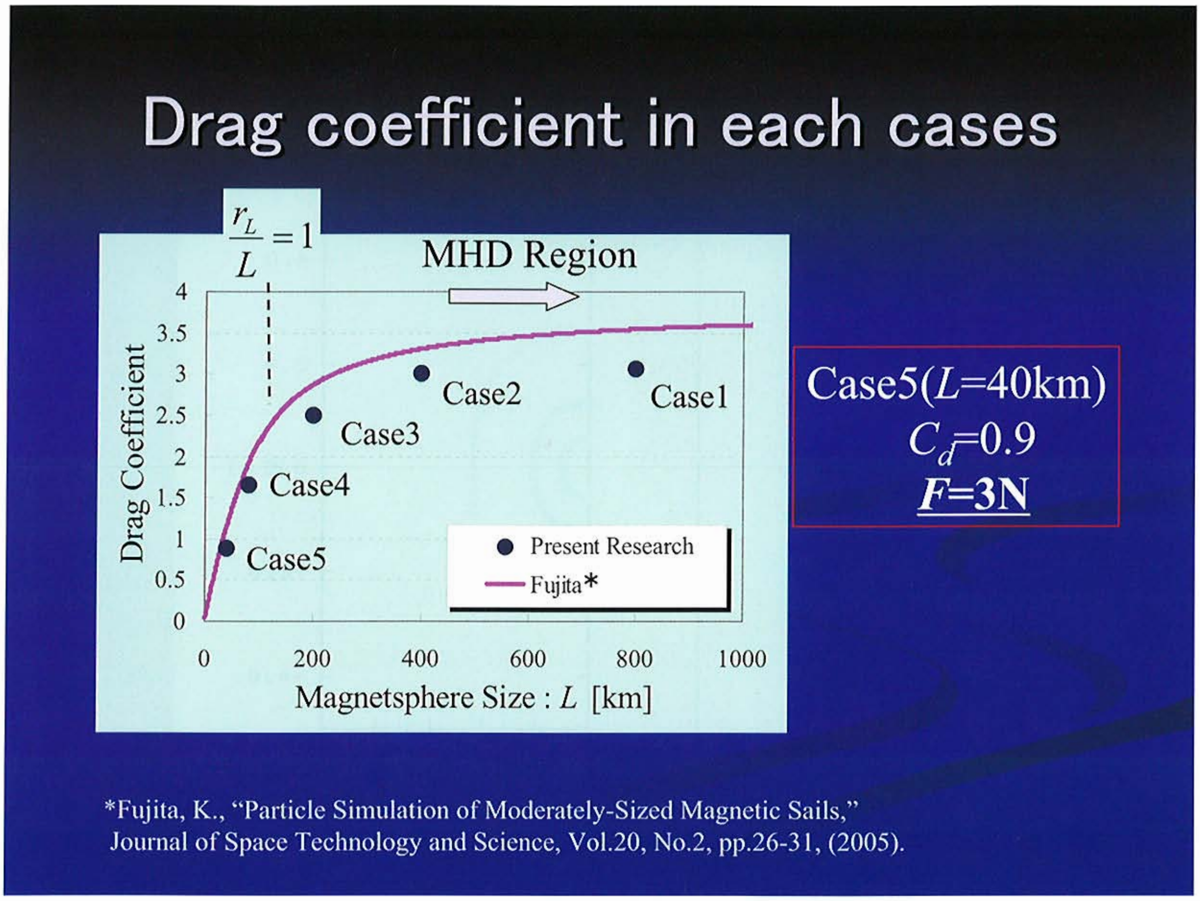
target size of MPS

Non MHD Region


CASE1 : Results
Particle
Solar Wind
L=800km r_L/L=0.13
Magnetic Field
X[km]
3000
2000
1000
0
-1000
-2000
-3000
-3000 -2000 -1000 0 1000 2000 3000
Z [km]
1e-6
1e-7
1e-8
1e-9
B[T]
1e-7
4.1e-8
1e-8
Density
2.5e+7
2e+7
1.5e+7
1e+7
5e+6
N[m-3]
-3000 -2000 -1000 0 1000 2000 3000
Z [km]

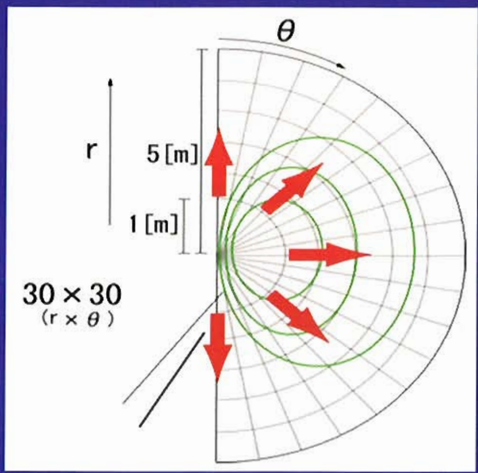

CASE3 : Results
Particle
Solar Wind
L=200km r_L/L=0.5
Magnetic Field
X[km]
750
500
250
0
-250
-500
-750
-750 -500 -250 0 250 500 750
Z [km]
1e-6
1e-7
1e-8
1e-9
B[T]
1e-7
4.1e-8
1e-8
Density
2.5e+7
2e+7
1.5e+7
1e+7
5e+6
N[m-3]
-750 -500 -250 0 250 500 750
Z [km]


CASE5 : Results
Particle
Solar Wind
L=40km r_L/L=2.5
Magnetic Field
Density
150
100
50
0
-50
-100
-150
X[km]
-150 -100 -50 0 50 100 150
Z[km]
1e-6
1e-7
1e-8
1e-9
B[T]
1e-7
4.1e-8
1e-8
2.5e+7
2e+7
1.5e+7
1e+7
5e+6
N[m-3]


Drag coefficient in each cases
r_L/L = 1
MHD Region
Drag Coefficient
4
3.5
3
2.5
2
1.5
1
0.5
0
Case1
Case2
Case3
Case4
Case5
Present Research
Fujita*
0 200 400 600 800 1000
Magnetsphere Size : L [km]
Case5(L=40km)
C_d=0.9
F=3N
*Fujita, K., "Particle Simulation of Moderately-Sized Magnetic Sails,"
Journal of Space Technology and Science, Vol.20, No.2, pp.26-31, (2005).

Magnetic inflation

- 2D spherical coordinates grid

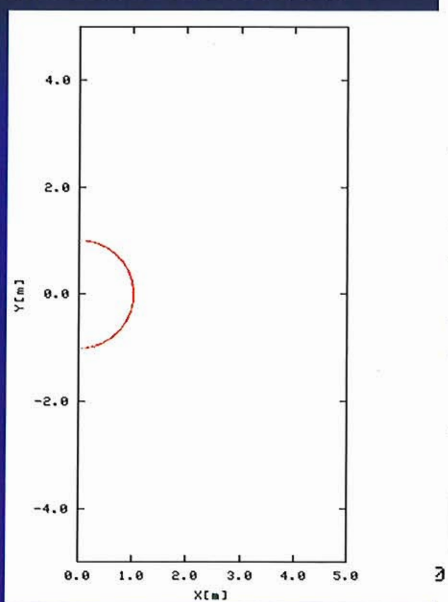


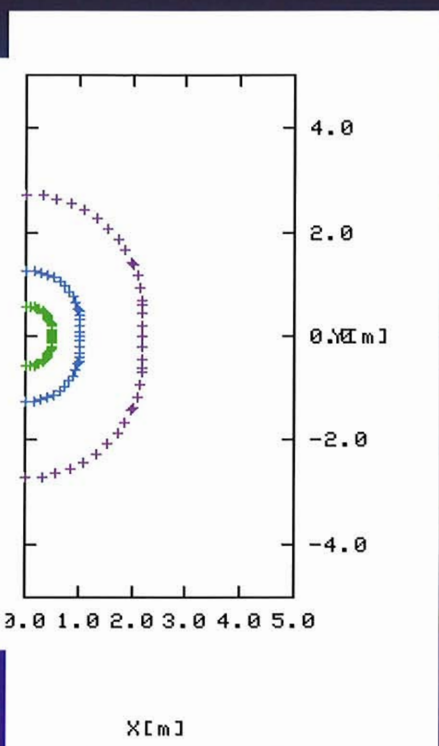
Injected Plasma : Ar

N_{in} = 3.0×10^{20} $[m^{-3}]$
v_{in} = 4.0×10^{3} [m/s]
T_i = 0.5 [eV]
T_e = 4.0 [eV]
$B_{r=rp}$ = 0.02 [T]
$R_{r=rp}$ = 0.084 [m]
β_{in} = 1

Magnetic inflation

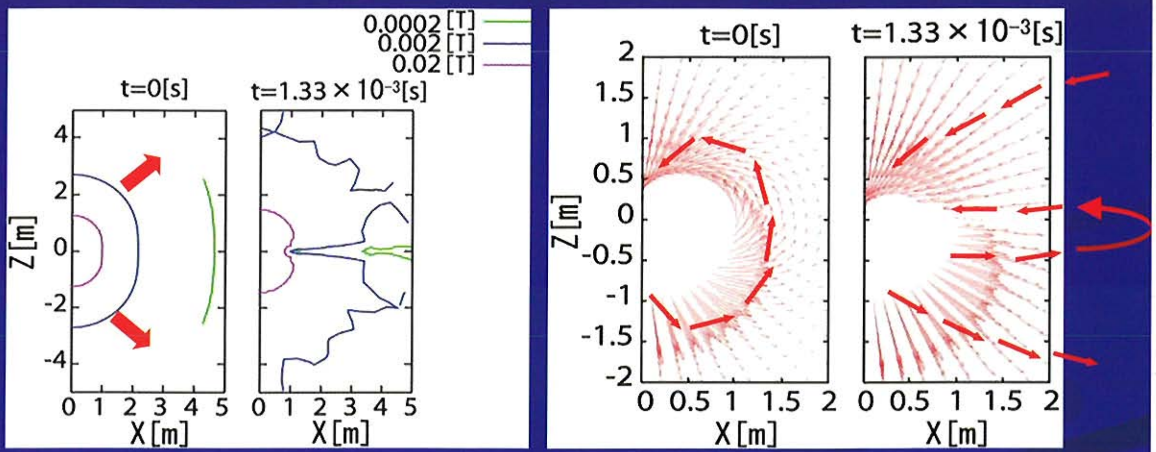
- Simulation results





Magnetic inflation

- Simulation results



Compared with the original dipole magnetic field of B∝r-3 (θ =0) after the plasma injected, it gradually decreases at a rate of B∝r-2.1.

Difficulty of simulation (Inflation)

- We need to perform simulations in a larger region as we want to confirm how far the magnetic field is inflated.

- However, in such large field, density of plasma will be low and calculations by using hybrid code will be more difficult.
  - This is because in our hybrid code, the plasma region and vacuum region are clearly separated and in each region. The calculation method of the electric field is quite different. In the vacuum region, the Laplace's Equation is used.

Our 3D Hybrid Code

- **Basic equations (e: electron, i: ion)**

Ion momentum equation

$$m_i \frac{d\mathbf{v}_i}{dt} = Ze(\mathbf{E} + \mathbf{v}_i \times \mathbf{B}) \qquad \frac{d\mathbf{x}_i}{dt} = \mathbf{v}_i$$

Electron momentum equation

$$n_e m_e \frac{d\mathbf{v}_e}{dt} = -en_e(\mathbf{E} + v_e \times \mathbf{B}) - \nabla P_e$$

Ampere's law

$$\nabla \times \mathbf{B}_p = \mu_0 (\mathbf{J}_e + \mathbf{J}_i)$$

Faraday's law

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Current density

$$\mathbf{J}_e = -en_e\mathbf{v}_e, \mathbf{J}_i = en_i\mathbf{v}_i$$

Position of ions $x_i^{n+1} = x_i^n + \mathbf{v}_i^{n+1/2}\Delta t$

Velocity of ions $\mathbf{v}_i^{n+1/2} = \mathbf{A}^{-1}\mathbf{S}$

Electric field (Plasma Region)

$$\mathbf{E}^n = \frac{1}{n_i}\left\{\frac{1}{\mu_0 Z_e}(\nabla \times \mathbf{B}_p^n) \times \mathbf{B}^n - \frac{1}{Ze}\mathbf{J}_i^n \times \mathbf{B}^n - \frac{T_e}{e}\nabla n_i^n\right\}$$

Electric field (vacuum region)

$$\nabla^2 \mathbf{E} = 0$$

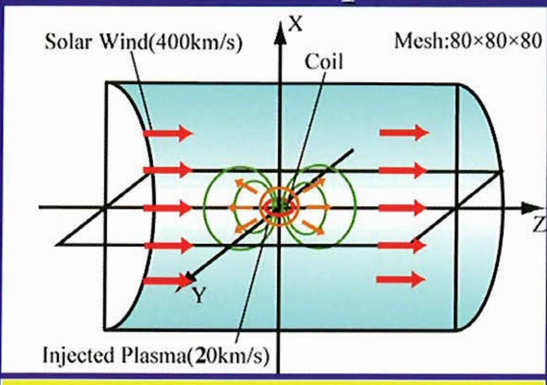
CFL condition $\Delta t < \frac{\Delta x}{V_A}$

Numerical stability condition $\omega_{ci}\Delta t < 0.2$

Electron energy equation is not used. (T_e = const)

Whole simulation

- Simulation model
- Simulation parameters



rL(solar wind)/L>>1

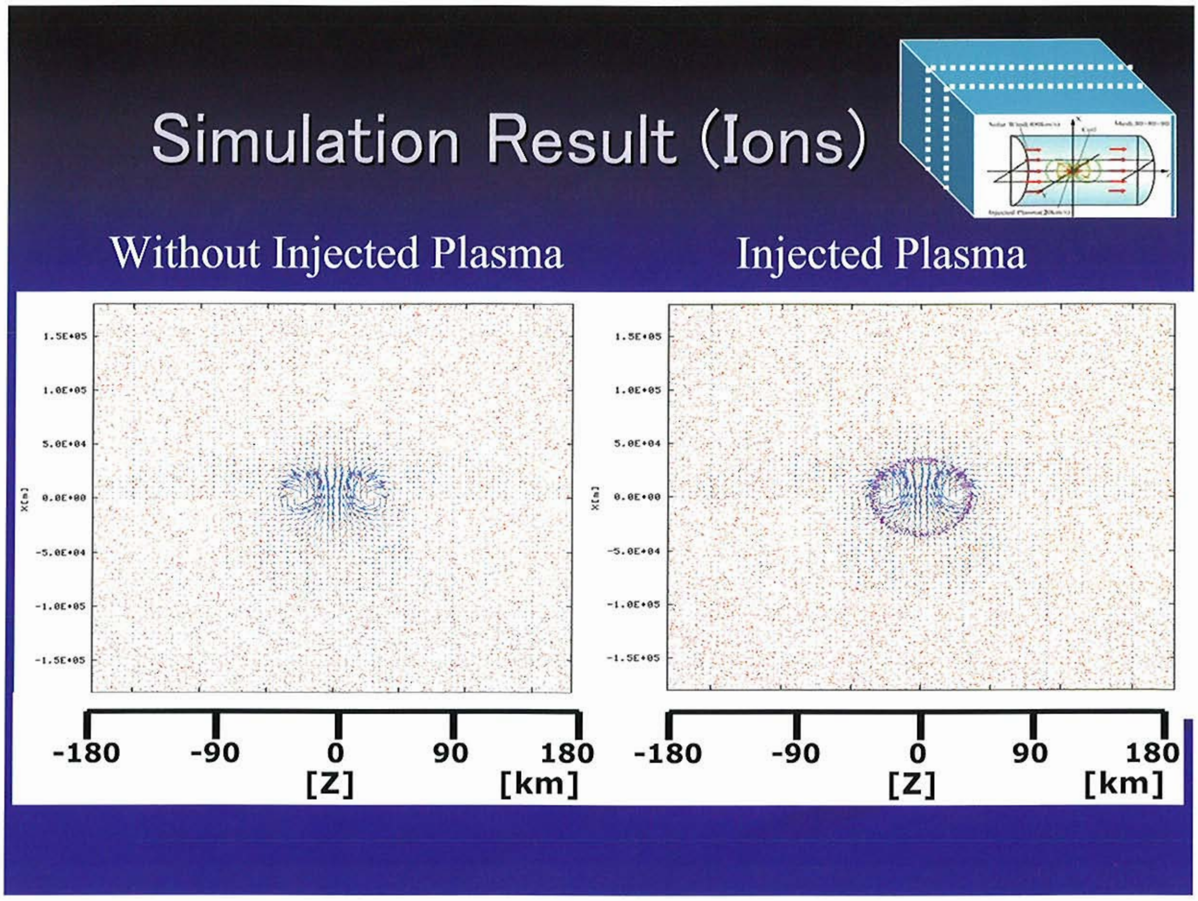
Injected Plasma β=1.0

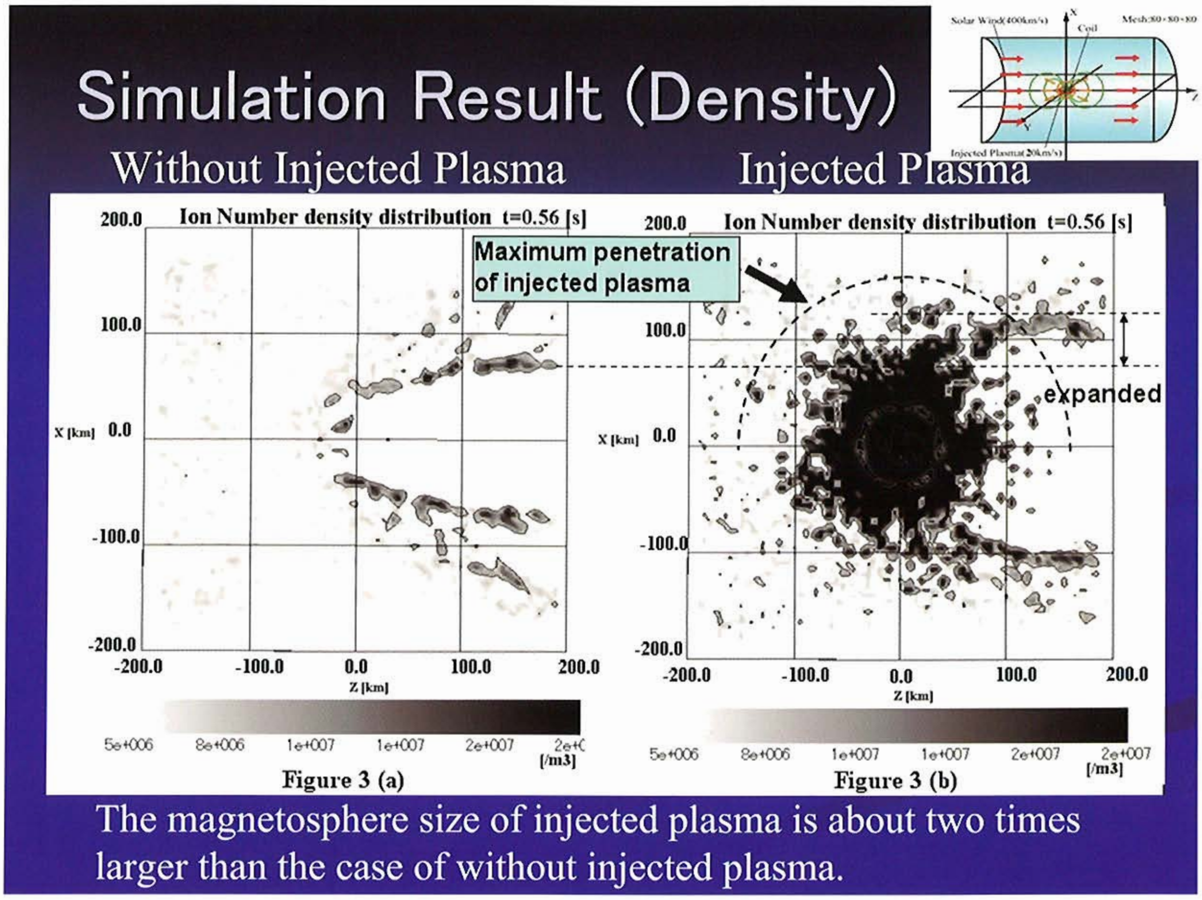
rL(Injected Plasma)/L~1.0

Ions (solar Wind) :hydrogen
Ions (Injected Plasma) : Ar
Mesh numbers :80 × 80 × 80
Mesh size [km] : Δx=5
Delta T [s] : Δt=1.2 × 10^{-5}
(near the coil) (0.2 ω_{ci}^{-1}t)

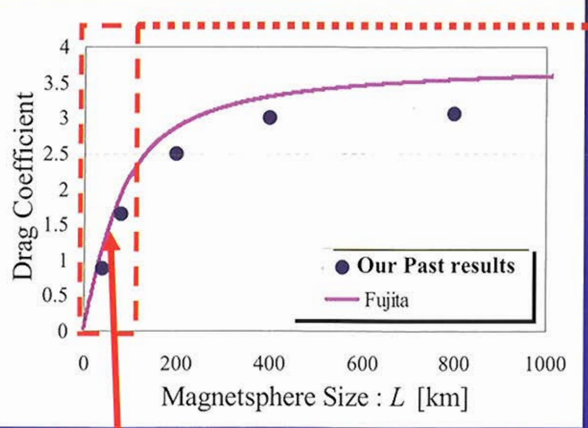
* ω_{ci} :ion cyclotron frequency

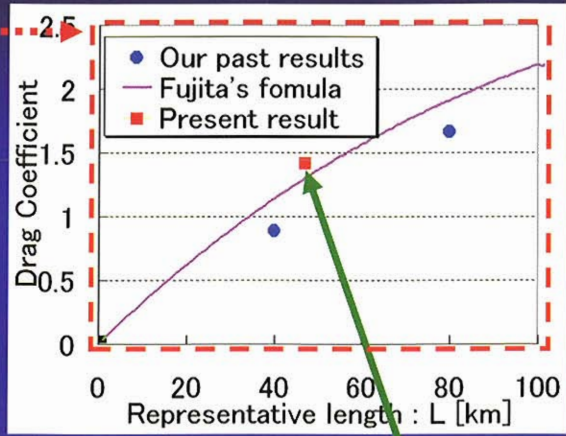
Time steps :50,000
: 0.6 [s]
Particle number :3,000,000
In each cell :6


Simulation Result (Ions)
Without Injected Plasma
Injected Plasma
-180 -90 0 90 180
[Z] [km]
-180 -90 0 90 180
[Z] [km]


Simulation Result (Density)
Without Injected Plasma
Injected Plasma
Ion Number density distribution t=0.56 [s]
Maximum penetration of injected plasma
expanded
Figure 3 (a)
Figure 3 (b)
The magnetosphere size of injected plasma is about two times larger than the case of without injected plasma.

Thrust Estimation (Without Injected Plasma)





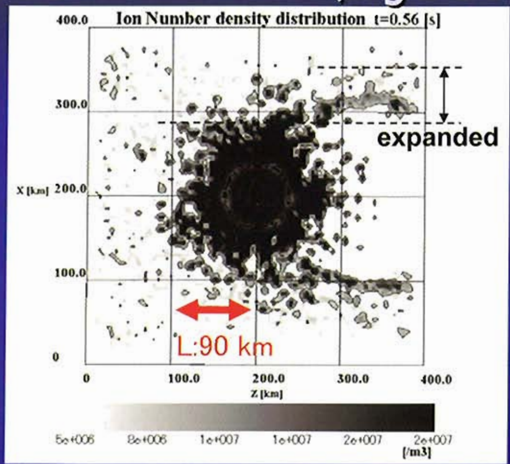
$$C_d = \frac{3.4}{R_L}\exp(-\frac{0.22}{R_L^{\ 2}})$$

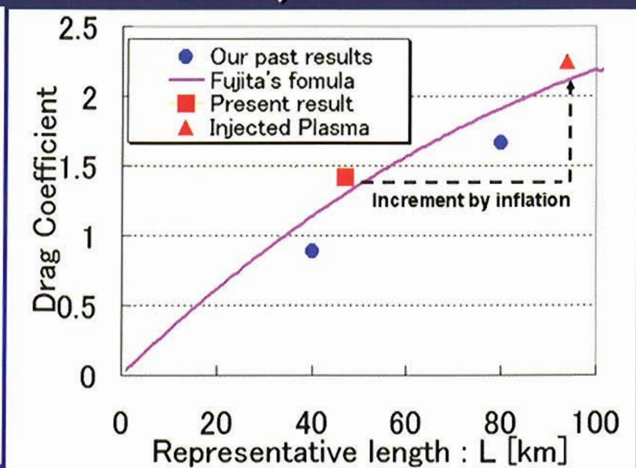
$$R_L \geq 1 \ : R_L = rL1/L$$

The magnetosphere size of without Injected plasma is 45 km.
And calculated drag coefficient is 1.4.
This is good agreement with the formula derived from Fujita.

Fujita, K., "Particle Simulation of Moderately-Sized Magnetic Sails," Journal of Space Technology and Science, Vol.20, No.2, pp.26-31, (2005).

Thrust Estimation (Injected Plasma)





The magnetosphere size with injected plasma is about twice the size than without injected plasma.
The magnetosphere size with Injected plasma is around 90 km.
The calculated drag coefficient obtained from the simulation is 2.3.
This also agrees with the results from Fujita's formula.

Discussion

- We had not been sure whether if MPS could really obtain the thrust through the interaction between the inflated magnetic field and solar wind.
- In this whole simulation, we confirmed that MPS could obtain the thrust from inflated magnetic field, and this thrust value also can be calculated by the Fujita's formula.
- We need to resolve the issue :how large can MPS inflate the magnetic field in the solar wind.

Difficulty of simulation (whole simulation)

- Coil radius (~m) and required magnetosphere (~km), so we have to treat the very wide region.
- Density 5[/cc] (solar wind) and 10^{14}[/cc] (injected plasma), so we have to treat the ions which have a big differences of density. (it needs some techniques)
- Velocity 400 [km/s] (solar wind) and 4 [km/s] (injected plasma) so we have to treat the ions which have a big differences of velocity. (small delta T)

→We need huge cost for simulation…..

Future Plan

- If we try to simulate under the actual scaling parameters, we have to use a much smaller mesh and time steps. It is necessary to develop a code suitable for parallel processing.
- Furthermore, in order to use a mesh variable in size of kilometers rather than meters, we are now developing code using spherical polar coordinates for whole simulation.

END