Simulation of the Cluster Spacecraft Floating Probe Potential

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Abstract

In this study, a numerical model of the relationship between the double probe potential with respect to spacecraft potential and the plasma parameters (density, temperature...) has been developed. A fit of the data between 1 and 80 cm^{-3} has been performed which allows to calibrate the model with suitable parameters for the photoelectron emission. The model can then be alternatively used for either density or temperature estimate. Uncertainties and range of validity are discussed.

1. Introduction

Cluster wave measurements provide a very accurate method for density measurements below 80 cm^{-3} when the plasma frequency can be clearly identified. When the wave measurements are not clear enough or when the plasma density is above 80 cm^{-3} the electron density estimation along the Cluster orbit could in principle be assessed through the modelling of the spacecraft potential data as a function of the ambient plasma density. In this study, a numerical model of the relationship between the double probe potential with respect to spacecraft potential and the plasma parameters (density, temperature...) has been developed. The mathematical model has been validated by comparison with numerical simulation results relying on a narrower set of hypotheses. A fit of the data between 1 and 80 cm^{-3} has been performed which allowed to calibrate the model with suitable parameters for the photoelectron emission. It is shown that the spacecraft potential is only weakly depending of the ambient plasma temperature for a density up to ~100 cm^{-3}. Beyond this value the model will provide an assessment of the density which is subject to strong uncertainties and will critically depend on the ambient electron temperature.

2. Models:

To compute the floating potential of Cluster probe with respect to the spacecraft ground, one needs to compute the current balance of the coupled system. Therefore, one needs to compute the current fluxes of the various species.

For Langmuir probes it is often possible to use simplified current formulas (so-called Langmuir probe formulas) valid in principle in two limiting cases: (1) the orbit motion limited case (OML) studied by Mott-Smith and Langmuir [1926] and (2) the sheath limiting case studied by Langmuir and Blodgett [1924].

2.1 Current from ambient plasma

In the two limiting cases expressed above, the current collected on a spherical conductor from ambient particles of charge q can be expressed as:

$$\begin{cases} q\phi > 0 \Rightarrow I_q = n_q \times V_q \times q \times S \times \exp\left(-\dfrac{q\phi}{kT_q}\right) \\ q\phi < 0 \Rightarrow I_q = n_q \times V_q \times q \times S_{eff} \end{cases}$$

Where V_e and V_i are the electrons and ion thermal velocity: $V_e = \sqrt{\dfrac{kT_e}{2\pi m_e}}$ and $V_i = \sqrt{\dfrac{kT_i}{2\pi m_i}}$ and n_e and n_i are the electron and ion density. S is the surface of the collector and S_{eff} is the effective collection surface.

In the so-called sheath limiting case the effective collection surface is the electrostatic sheath [Langmuir and Blodgett, 1924]. These authors also described an approach to derive an estimate of S_{eff}. In the orbit motion limited regime Mott-Smith and Langmuir [1926] have shown that S_{eff} can be expressed as:

$$S_{eff} = S \times \left(1 - \frac{q\phi}{kT_q}\right)$$

2.2 Photoelectron emission current

In the following, the photoelectron space charge is neglected.

For negative spacecraft potential, the computation of the photoelectron currents is straightforward. Indeed, the photoelectron current of a negatively charged probe is equal to the photoelectron saturation current times the emitting surface, as all the photoelectrons emitted leave the surface.
However, for attractive potential, computing photoelectron currents requires a more refined modelling. According to Grard [1973], there are two extreme models of photoelectron emission, point and planar source, depending on the size of the spacecraft with respect to the shielding distance for the photoelectrons:

$$\chi = \frac{r_{SC}}{r_0 - r_{SC}} << 1 \Rightarrow \text{Point source}$$

$$\chi = \frac{r_{SC}}{r_0 - r_{SC}} >> 1 \Rightarrow \text{Planar source}$$

In the first approximation, the equipotential surfaces are spherical around a point source. If the photoelectrons are emitted radially out of the attractive probe (small

sample), the distance at which they are reflected is a function of their initial energy but is independent of the direction along which they have been emitted.
On the other hand, for the planar surface model, equipotential surfaces are planar and the distance at which an electron is reflected depends on the initial emission angle. An electron emitted perpendicular to the surface will reach a much larger distance than an electron with the same initial speed but emitted at a smaller angle.

Therefore, for an half Maxwellian distribution of the photo-electron at the source one can express the current of photoelectrons as follows:

$$\begin{cases} \phi \le 0 \Rightarrow I_{ph} = J_{ph}^{0} \times S_{ph} \\ \phi > 0 \Rightarrow \begin{cases} \chi = \dfrac{r_{SC}}{r_0 - r_{SC}} << 1 \Rightarrow I_{ph} = J_{ph}^{0} \times S_{ph} \times \left(1 + \dfrac{e\phi}{kT_{ph}}\right) \exp\left(-\dfrac{e\phi}{kT_{ph}}\right) \\ \chi = \dfrac{r_{SC}}{r_0 - r_{SC}} >> 1 \Rightarrow I_{ph} = J_{ph}^{0} \times S_{ph} \exp\left(-\dfrac{e\phi}{kT_{ph}}\right) \end{cases} \end{cases}$$

where S_{ph} is the photoelectron effective emission surface, and J_{ph}^{0} is the photo-electron current density at saturation.

Various types of photoelectrons distributions have been proposed in the literature. Grard [1973] suggested that good fits of the measured photo-electron distributions could be obtained with a single Maxwellian distribution with a temperature of 1.5 eV. However, several authors have shown that the photo-electron emission in space could be best represented by bi or tri-Maxwellian photoelectron distribution functions. For instance, the bi-Maxwellian distribution function used here was obtained by Escoubet et al. [1997] and based on ISEE-1 data, while the tri-Maxwellian distribution function was introduced by Nakagawa et al. [2000] and based on GEOTAIL data. The values of the parameters are given below.

Bi-Maxwellian [Escoubet et al.,1997]
$T_{ph}^{0} = 2.4eV$, $J_{ph}^{0} = 50.8\,\mu\text{A/m}^2$
$T_{ph}^{1} = 12.6eV$, $J_{ph}^{1} = 1.5\,\mu\text{A/m}^2$
Tri-Maxwellian [Nakagawa et al., 2000]
$T_{ph}^{0} = 1.6eV$, $J_{ph}^{0} = 53.0\,\mu\text{A/m}^2$
$T_{ph}^{1} = 3.0eV$, $J_{ph}^{1} = 21.0\,\mu\text{A/m}^2$
$T_{ph}^{2} = 8.9eV$, $J_{ph}^{2} = 4.0\,\mu\text{A/m}^2$

2.3 Current balance and floating potential

On the Cluster spacecraft, a bias current is sent to the probes to maintain their potential close to the ambient plasma potential. This current has to be taken into account in the current balance equation. The electrostatic equilibrium is reached on the spacecraft for a potential verifying:

$$I_i + I_{ph} - I_e + yI_{bias} = 0$$

where y is the number of probes on which the current bias is applied. On the probe, this equation becomes:

$$I_i + I_{ph} - I_e - I_{bias} = 0$$

Once the total current as a function of the potential is known, a simple dichotomy resolution leads to the computation of the floating potential.

3. Application to Cluster double probe

The parameters relevant to Cluster are listed in Table 1 below. The plasma parameters correspond to a plasmasphere environment except that much lower density values have also been explored (i.e. < 10 part/cm^3) in order to better analyze the trend of the current-potential relations as a function of the density.

Table 1:

Plasma temperature:	0.1 to 3 eV
Plasma density:	0.1 to 1000 part/cm^3
Cylindrical spacecraft radius	1.45 m
Cylindrical spacecraft height	1.3 m
Equivalent spherical spacecraft radius	1.41 m
Spherical probe radius	0.04 m
Probe bias current	140 nA
Photoelectron saturation current density	56 μA/m^2
Number of probes with bias current	4

Using the analytical expressions from the previous sections, it is possible to compute the floating potential of the Cluster probes in a Maxwellian plasma environment. One has shown by comparing the results based on the analytical expressions with results from numerical simulations that in the range of parameters relevant to Cluster, the best approximations are provided by the OML model and the source point model for the photoemission.

Unknown parameters in Cluster data are the ambient electron temperature and the shape of the photo-electron distributions. For the latter, the value of the photo-electron current at saturation is known from Pedersen et al. [2001].

In Figure 1, example of Cluster data are displayed together with an exponential fit derived from Moullard et al. [2002], a simulation using Escoubet et al. [1997] bi-Maxwellian photoelectron distribution function, an empirical law from Laakso et al. [1998] and new fitting curves based on bi-Maxwellian distribution functions obtained with various ambient plasma temperatures (ranging from 1 to 3eV).

The empirical density expression described by Laakso et al. [1998] is the following:

$$n = 6.6\times10^6 \frac{I_{bias}}{r_p^2\sqrt{T_{ph}}}\exp\left(-\frac{\Delta V}{0.9\times T_{ph}}\right)$$

where the density is expressed in m^{-3}, the bias current in nA, the probe radius r_p in cm, T_{ph} in eV and ΔV, the potential of the spacecraft with respect to the probe, in V.

Fits of the data by mono-Maxwellian distribution were also attempted but bi-Maxwellian always provided better results. The resulting parameters of the bi-Maxwellian distribution functions obtained via this method are displayed in Table 2. Three other fits were performed for ambient plasma temperatures below 1eV (not displayed on the Figure). The results of these fits are also provided in Table 2.

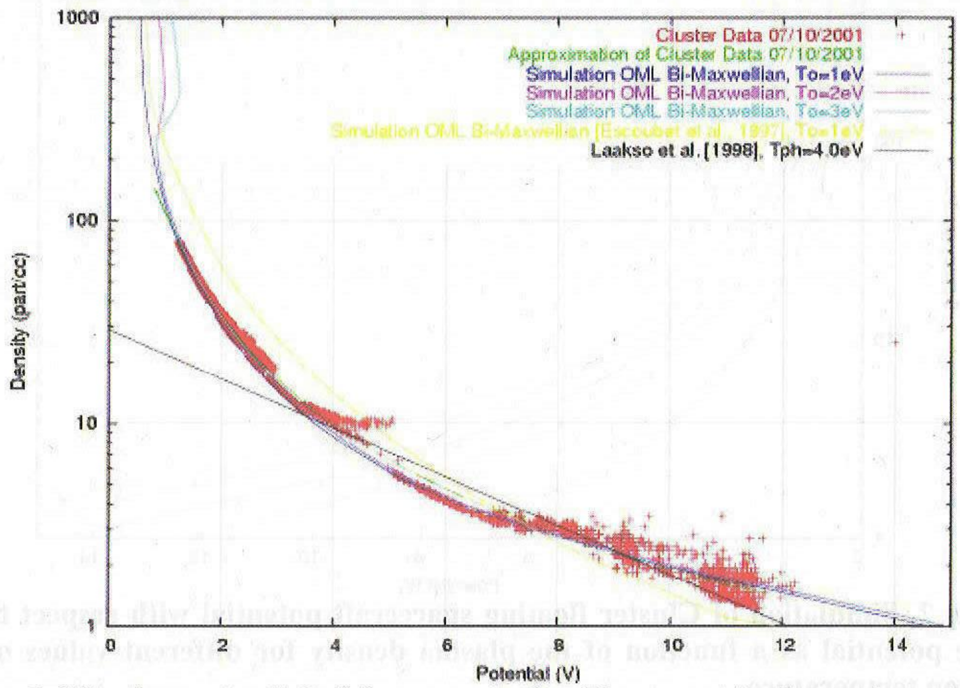

Figure 1. Floating potential of the spacecraft with respect to the probe potential versus plasma density with Cluster parameters; comparison with Cluster data 07/10/2001.

Table 2. Parameters of the fit of Cluster data by a bi-Maxwellian distribution function.

Ambient temperature (eV)	T^0_{ph} (eV)	J^0_{ph} (μA/m^2)	T^1_{ph} (eV)	J^1_{ph} (μA/m^2)	Total saturation current (μA/m^2)
0.5	1.3	40.0	8.0	6.5	46.5
0.8	1.3	45.0	8.0	5.5	51.5
0.9	1.3	48.0	8.0	5.3	53.3
1.0	1.3	55.0	8.0	5.0	60.0
2.0	1.3	65.0	8.0	4.0	69.0
3.0	1.3	75.0	8.0	3.5	78.5

It can be seen from Table 2 that the correct saturation photo-electron current is obtained for the fit corresponding to an ambient plasma parameter between 0.9 and 1 eV. To simplify we assumed in the following that the ambient electron temperature when the data were taken was equal to 1 eV and therefore we choose the parameter of the photo-electron distribution accordingly.

Consequently, simulations were performed assuming a bi-Maxwellian distribution function corresponding to this fit result for various ambient plasma temperatures.

Results are shown on Figure 2. It must be noted that there is still some uncertainty on the saturation current value to use since this parameter is changing during Cluster lifetime. For instance, A. Eriksson [private communication, 2004] reported observation of 49 $\mu A/m^2$.

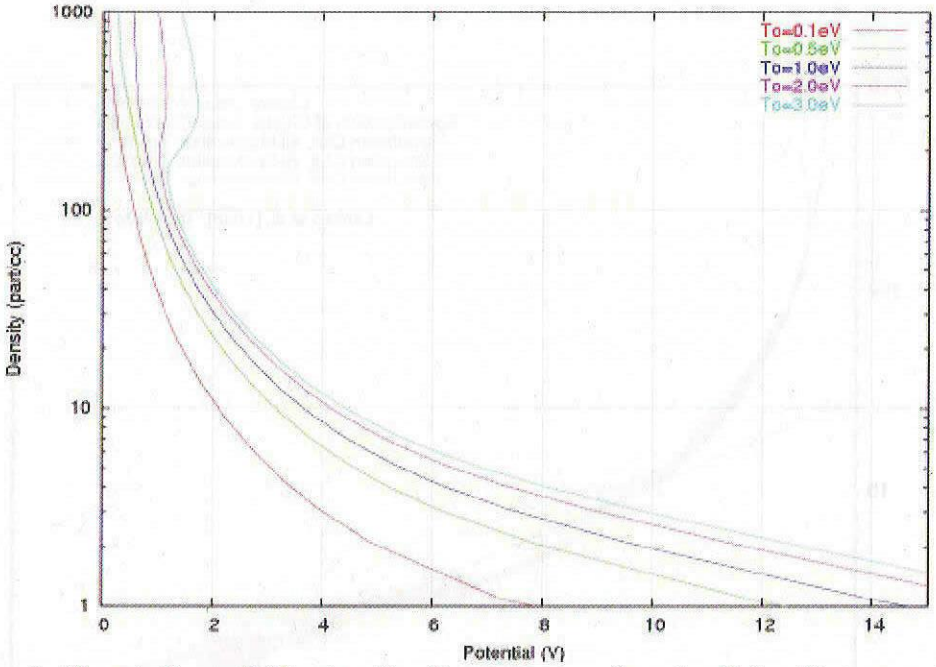

Figure 2. Simulation of Cluster floating spacecraft potential with respect to the probe potential as a function of the plasma density for different values of the electron temperature.

The ambient plasma temperature effect may be very important. In the plasma boundary layer the temperature varies from 1 to 3 in the lower density part (say when at high latitude or high altitude) and from 0.1 to 1 in the higher density part.
One can see that if one applies the theoretical model without any more precise information on the actual electron temperature it would results an uncertainty on the density of about 50 percent in the low density part (i.e., below 100 cm^{-3}) and of a factor 2 or 3 between 100 and 200 cm^{-3} and even higher above 200 cm^{-3}.

4. Conclusion

A simple model of the potential of the spacecraft with respect to the double probe floating potential has been developed. It fits very well Cluster measurements from 1 to 80 cm^{-3} used for this study and hereby provide useful information on the photo-electron distribution function around Cluster. It also allows to extrapolate the density when the ambient plasma temperature is known. If the ambient electron temperature is unknown it was found that the uncertainty on the density determination is about 50% below 100 cm^{-3} and about a factor 2 to 3 between 100 and 200 cm^{-3}. It must be noted that the influence of the actual geometry of the probe including the neighbouring boom and guards has been neglected.

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