Secondary arc description on satellite solar generators

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Abstract – In this paper, we propose a quasi-neutral model with non-vanishing current describing the expansion of a plasma in an inter-cellular gap on a satellite solar array. Moreover, an electric arc cathode spot model is proposed in order to give suitable boundary conditions for the expansion model.

1. Introduction

1.1 General framework

We are interested in the modeling of secondary arcs formation on satellite solar generators. Power on satellites is generally provided by solar generators which use semiconductor solar cells to convert solar radiation energy into electrical power. Solar generators consist of individual solar cells of about a few cm^2 in surface, which are connected in series into 'strings' to provide the required potential difference (usually ranging from 50 to 150 Volts). Strings are then connected in parallel to deliver the requested power to the satellite equipments. Solar cells are made of high quality semiconductor materials (usually Silicium) and are very expensive to fabricate. It happens too often that after a certain operation time, an entire portion of the solar generator undergoes a permanent failure. The reason for it is the occurrence of an electrical arc which shortcuts one or several strings.

At the beginning of the scenario which leads to the secondary arc, the satellite charging in the earth environment plasma triggers a primary discharge between a cell interconnect and the dielectric which is used in the protective layer [1]. The so-created plasma plume expands and eventually connects and shortcuts two neighboring solar cells. The potential difference between the two cells (which is generated and maintained by the operation of the cells themselves) induces the transition of the primary discharge into a secondary arc [2],[3], [1]. Once this arc is established, it pyrolyzes the insulating kapton substrate and may transform it into a conductor, which provides a permanent solid-state shortcut between the two cells, thereby irremediably deteriorating this part of the solar generator. The second phase of the discharge scenario, namely the expansion of the plasma plume and the transition from the primary discharge into the secondary arc can be modeled.

In this paper, we present a modeling of a secondary arc. Two phenomena are treated which are respectively the plasma expansion in the inter-cellular gap and the arc cathode spot.

1.2 Description of the plasma expansion

In the first part of this paper, we are interested in the modeling of a quasi-neutral plasma with non-vanishing current. The plasma is considered as fully ionized and constituted of only one ion species. Such a plasma can be described by an isentropic Euler system for each species (ions and electrons) coupled with the Poisson equation. Due to the very short length scale associated with the Poisson equation (Debye length), the discretization of this model requires very fine meshes. Therefore, this model leads to expensive simulations in practical situations. In the present work, we propose a quasi-neutral model which avoids the resolution of the Poisson equation. In spite of being quasi-neutral, the model allows the current and the electric field to be non-zero.

In order to test the numerical efficiency of the quasi-neutral model, two one-dimensional situations are implemented. The first one is a periodic perturbation of a quasi-neutral uniform steady state with non-zero current. This configuration is described in [4] and an analytical solution of the linearized system is known. The second situation is a case of plasma expansion between two electrodes in vacuum. A high density plasma is emitted from the cathode and undergoes a thermal expansion. An electron current is emitted at the plasma-vacuum interface. This current obeys a Child-Langmuir law and generates a non-zero current inside the plasma. This phenomenon has been studied in [5], [6] in relation with physical applications [2], [1], [7].

1.3 Modeling of the arc cathode spot

In fact, the electric arc has a complex structure which can be reduced to two regions. The first one is the quasi-neutral plasma plume which has been described previously, and the other one is the cathode spot which is himself constituted of three sub-regions : the cathode, the sheath and the presheath.

The sheath zone is a space charge layer zone measuring few Debye lengths characterized by the electric potential drop near the cathode. In this zone, the electric field is strong and attracts the ions to the cathode. Then, the combined effects of the electric field and of the ion bombardment lead to the cathode metal vaporization and a thermo-field electron emission at the cathode. A condition for the sheath stability is that the ions have a sufficient velocity to enter the sheath (Bohm criterion). At the point where this condition is supplied begins the presheath zone which is a quasi-neutral plasma zone where the ions are created by the ionization of the vaporized cathode atoms and accelerated by a residual electric field toward the Bohm velocity.

The cathode spot has two basic functions. It provides for the discharge medium by emission of matter in the gap and for current continuity at the cathode by emission of electrons. From a numerical point of view, the cathode spot is a time evolutive boundary condition for the plasma expansion model which is coupled to the plasma himself by the circuit. In this paper, we present the main ideas for a modeling of this sheath zone and some numerical results obtained for a silver cathode spot.

2. Expansion of a quasi-neutral plasma

2.1 Euler-Poisson model

We denote by $m_{i,e}$ the masses of ions and electrons, $n_{i,e}$ their densities, $u_{i,e}$ their velocities and $q_{i,e} = \pm q$ their charges where $q > 0$ is the elementary charge. As a first approximation, the particle pressure laws are assumed isentropic and are given by $p_{i,e} = c_{i,e} n_{i,e}^{\gamma_{i,e}}$, where $\gamma_{i,e} > 1$ are the ratio of specific heats and $c_{i,e}$ are given constants. The temperatures are given by $T_{i,e} = p_{i,e}/n_{i,e}$. Moreover, V is the electric potential.

On a domain $\Omega \subset \mathbb{R}^d$, $d = 1,\ 2,$ or 3, the particle conservation laws are given by $\forall x \in \Omega,\ \forall t \in \mathbb{R}_+^*$,

$$(1) \quad \left\{ \begin{array}{ll} (n_i)_t + \nabla.(n_i u_i) = 0, & m_i\left[(n_i u_i)_t + \nabla(n_i u_i \otimes u_i)\right] + \nabla p_i(n_i) = q n_i\, E, \\ (n_e)_t + \nabla.(n_e u_e) = 0, & m_e\left[(n_e u_e)_t + \nabla(n_e u_e \otimes u_e)\right] + \nabla p_e(n_e) = -q n_e\, E, \end{array} \right.$$

and the electric field $E = -\nabla V$ is given by the Poisson equation

$$(2) \qquad -\varepsilon_0\, \Delta V = q\,(n_i - n_e)\,,$$

where ε_0 is the vacuum permittivity. If we denote by n_0 the scale of the plasma density and by T_0 the scale of the plasma temperature (in units of energy), we recall that the Debye length λ_D which is the length scale where electrostatic interactions occur in the plasma, is given by

$$(3) \qquad \lambda_D^2 = \varepsilon_0 T_0/(q^2 n_0).$$

The numerical resolution of the 2-fluid Euler-Poisson model (1)-(2) presents a very restrictive constraint due to the coupling with the Poisson equation. Indeed, the Debye length λ_D must be resolved by the space discretization to guarantee the stability of the scheme (i.e. $\Delta x < \lambda_D$ where Δx is the mesh spacing). In practical situations where quasi-neutrality is established, the Debye length is very small. This implies very large computational costs in multi-dimensional cases. This is the reason why studying the quasi-neutral limit is necessary to remove the time and length scale constraints related with the electrostatic ion-electron interactions.

2.2 Quasi-neutral model

For the study of the formal quasi-neutral asymptotical limit in Euler-Poisson model we refer to [5]. This limit leads to a quasi-neutral model given by $\forall x \in \Omega$ et $t > 0$,

$$(4) \quad \left\{ \begin{array}{ll} (n_i)_t + \nabla.(n_i u_i) = 0, & m_i\left[(n_i u_i)_t + \nabla(n_i u_i \otimes u_i)\right] + \nabla p_i(n_i) = -q n_i \nabla V, \\ (n_e)_t + \nabla.(n_e u_e) = 0, & m_e\left[(n_e u_e)_t + \nabla(n_e u_e \otimes u_e)\right] + \nabla p_e(n_e) = q n_e \nabla V, \end{array} \right.$$

with a divergence free constraint for the current :

$$(5) \qquad \nabla.(n_i u_i - n_e u_e) = 0.$$

We note that the potential V is now a kind of Lagrange multiplier of the quasi-neutrality constraint reformulated by (5). However, equation (5) allows to compute V. Indeed, taking the time-derivative of (5) and using the momentum conservation laws of (4) leads to an elliptic equation for the potential:

(6)

$$-q\,\nabla.\left(\left(\frac{n_i}{m_i}+\frac{n_e}{m_e}\right)\nabla V\right)=\nabla\cdot\nabla\left(n_iu_i\otimes u_i-n_eu_e\otimes u_e\right)+\frac{1}{m_i}\Delta p_i(n_i)-\frac{1}{m_e}\Delta p_e(n_e).$$

2.3 Numerical study for two mono-dimensional applications

The numerical method is based on a time splitting. The transport part of the systems is resolved by a Godunov scheme (cf. [8]). Then the source term part is taken into account with an Euler scheme. We refer to [5] for the details of the numerical method.

2.3.1 Test-Case: Perturbation of a uniform steady plasma

In this section we perturbate a uniform steady solution $n_i = n_e = n^0$, $u_i = u_i^0$, $u_e = u_e^0 \neq u_i^0$, and $V^0 = 0$ of the Euler-Poisson and quasi-neutral model on $[0, L]$. For a weak amplitude initial perturbation that is quasi-neutral and of uniform current, the perturbed solution is closed to the solution of the linearized models. Following the parameters, the perturbed solution is stable or instable. We do the comparison of the numerical solutions in a stable case.

Numerical values for the simulation are almost typical of the secondary arc establishment on a solar array generator (cf. [2], [1]). We take $m_i = 10^4\, m_e$, $L = 10^{-3}\, m$, $\gamma_i = \gamma_e = 3/2$ and $k_i = k_e = 1.6\,10^{-39}\, J.m^{3(\gamma-1)}$. The steady state is the following $u_i^0 = 4190\, m.s^{-1}$, $u_e^0 = 8380\, m.s^{-1}$, $n^0 = 5.53\,10^{19}\, m^{-3}$ and $V^0 = 0\, V$. At the initial state, the perturbation on velocities is sinusoidal of amplitude $\delta = u_i^0\, 10^{-2}$ and of wave length L. The boundary condition are set periodicals. We note that the density is lower than what it should be in order to realize Euler-Poisson simulations in reasonable computational times.

The figure 1 (left) illustrates the numerical convergence of the Euler-Poisson and quasi-neutral models. As we choose the linearized analytical solution as a reference, it is normal to observe that the relative error of the numerical methods tends to a threshold. However, this approximation validate the quasi-neutral approach for the description of the plasma.

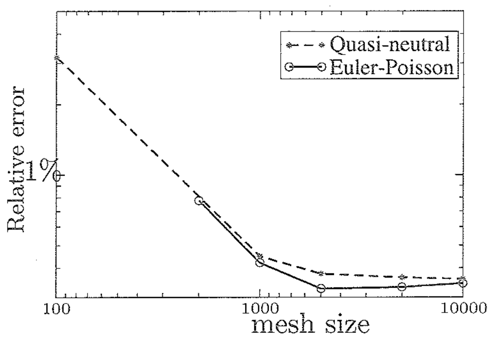
2.3.2 Expansion of a plasma between two electrodes

A dense plasma is injected at the cathode and undergoes a thermal expansion towards the anode. For this problem, the inter-cellular gap $[0, L]$ is divided in two sub-regions: the region $[0, X(t)]$ is the quasi-neutral plasma zone (described by the quasi-neutral model), and the region $]X(t), L]$ is an electronic beam zone (described by an analytical Child-Langmuir law). The current in the plasma is equal to the current created by the electronic emission at the plasma/beam interface [6].

For the numerical simulations, we use the same masses and pressure laws as in the previous section. The boundary conditions are $n(0, t) = 5.53\,10^{19} m^{-3}$, $u_i(0, t) = u_e(0, t) = 4190\, m.s^{-1}$, $V(0, t) = 0\, V$ and $V(L, t) = 100\, V$. The gap is free of plasma at the initial time.

As the quasi-neutral region $[0, X(t)]$ is evoluting in time, an interface tracking method is implemented where the fluxes at the interface are given by the resolution of a Riemann problem between the plasma and the beam given in [6].

The figure 1 (right) illustrates the progression of the plasma in the gap at different times for the Euler-Poisson model (reference solution) and the quasi-neutral model. The good behavior of the quasi-neutral approximation is shown for the density excepting close to the cathode where a thin non-quasi-neutral layer is observed. For the other variables, analog results are observed. We refer to [5] for a detailed analysis of these results.



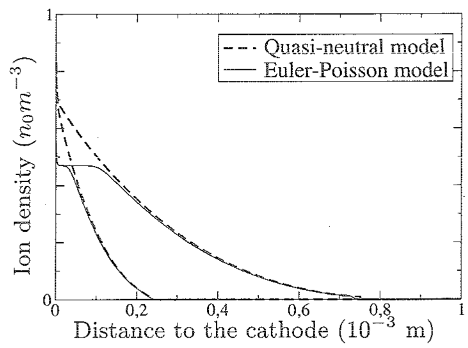


Figure 1: **Left figure - perturbation test-case:** relative error between the analytical solution of the linearized Euler-Poisson system and the numerical solution of non-linearized model. **Right figure - expansion of a plasma:** ion density as a function of x at times $t = 23.8\,ns$ and $71.6\,ns$ computed by the quasi-neutral model (dashed line) et by the Euler-Poisson model (full line). Mesh size: $N = 1000$.

3. Modeling of an arc cathode spot

The global modeling of the arc cathode spot is based on the following hypotheses. At first, we suppose that the sheath and presheath zones can be described by a quasi-stationary model, the cathode spot has a cylindric geometry. For thermal effects, we assume that the metal cathode is at liquid state and that pressure at the surface is given by a Clapeyron/Langmuir law. In the sheath, we consider that the charged species are the electrons emitted from the cathode in a "thermo-field" regime, the ions generated in the presheath by ionization and accelerated towards the cathode, and the electrons retro-diffused towards the cathode. Moreover we assume that the ion and electron currents are uniforms. In the presheath, we suppose that the total current only is uniform and that ion and electron temperatures are equal and uniform.

3.1 Cathode energy balance

The description of the power exchanges at the cathode has been widely studied in [9] and our modeling is based on it. The sources and sinks of energy at the cathode surface which have been retained in our model are: the energy carried by the ions strongly accelerated in the sheath by the electric potential drop, the energy lost by the thermo-field emission of electrons (detailed in [9], [10]), the energy lost by the thermal conduction of the material, and the energy lost by the cathode vaporization. We note that the Joule effect and the energy given by the retro-diffused electrons have not been taken into account by the model for simplicity but should be added for a precise computation.

3.2 Resolution of the Poisson equation in the sheath

One decisive parameter determining the electron emission at the cathode is the surface field strength. We recall that the sheath zone is a space charge layer generated by a surplus of ions. It obeys the Poisson equation which can be integrated with the hypotheses made in section 3.1. One integration leads to an approximation of the electric field at the cathode surface E_s given by:

$$E_s^2 = \frac{4\sqrt{V_{sh}}}{\varepsilon_0\sqrt{2q}}\left(J_i\sqrt{m_i} - J_e\sqrt{m_e}\right) - \frac{n_{sh}k_bT_e}{\varepsilon_0}\left[1 - \exp\left(-\frac{qV_{sh}}{k_bT_e}\right)\right]. \tag{7}$$

where J_i and J_e are respectively the ion and electron currents, T_e is the electron temperature and V_{sh} is the drop of potential in the sheath. The quantity n_{sh} is the quasi-neutral density at the sheath edge which is determined by the Bohm criterion such as:

$$n_{sh} = J_i\ \left(k_b(T_i + T_e)/m_i\right)^{-\frac{1}{2}}, \tag{8}$$

where T_i is the temperature of ions.

3.3 Presheath energy balance

In the presheath, the surface sources and sinks of energy are given by the entering and outing energy fluxes. Moreover, volume effects such as ionization or electron acceleration in the presheath have to be taken into account. Then we have

$$\bar{E}_{e,in} - \bar{E}_{e,out} + \bar{E}_{i,in} - \bar{E}_{i,out} - \bar{E}_{coll} + \bar{E}_{el} = 0. \tag{9}$$

where the different terms of this balance equation are detailed in the following sections.

3.3.1 Surface fluxes

At the sheath edge, the electrons emitted by the cathode have a thermal energy k_bT_s (k_b is the Boltzmann constant) and have been accelerated by the potential drop V_{sh} while the ions are attracted to the cathode and leave the presheath with their thermal energy k_bT_i. Then,

$$\bar{E}_{e,in} = (J_e/q)\,(2k_bT_s + qV_{sh})\,\pi a^2, \qquad \bar{E}_{i,out} = (J_i/Zq)k_bT_i\,\pi\,a^2. \tag{10}$$

where a is the spot radius ($q > 0$ is the elementary charge and Zq is the ion charge). At the spot edge (plasma expansion zone/presheath zone interface), the electrons leave the presheath with their thermal flux while the flux of entering ions from the plasma can be neglected. Then

$$\bar{E}_{e,out} = Jk_bT_e\,\pi\,a^2/q, \qquad \bar{E}_{i,in} = 0. \tag{11}$$

3.3.1 Ionization in the presheath

In the presheath, only the electron-neutral collisions which lead to ionization are taken into account. Moreover, we suppose that collisions generate an ion and two electrons such as:

$$dJ_e(z)/dz = n_n\sigma_I J_e(z). \tag{12}$$

where $n_n\sigma_I$ is an approximation of the free mean path (n_n is the neutral density and σ_I is the collision cross section). Then we have an exponential law for the electron current increasment. Considering that the ion current is null at the spot edge gives an equation for the presheath length z_0 and for the energy sinks:

$$z_0 = (n_n\sigma_I)^{-1}\left[\log(J_i + J_e) - \log J_e\right], \qquad \bar{E}_{coll} = V_i(Z)\, J_i\pi\, a^2, \tag{13}$$

where $V_i(Z)$ is the ionization potential of a neutral atom.

3.3.2 Electric field in the presheath

In the presheath, a residual electric field accelerates the electrons. This gain of energy for the electron is given by:

$$\bar{E}_{el} = \eta(T_e) z_0 J^2 \pi\, a^2, \tag{14}$$

where $\eta(T_e)$ is the Spitzer resistivity for the electrons in a plasma [11].

3.4 Numerical results

In this spot model, three parameters must be fixed to resolve the system of equations. A natural parameter is the arc boundary applied voltage. Another one, which is experimentally known, is the ratio of ion and electron current at the cathode. Then the last one is the total current which is given either by the Child-Langmuir emission when the plasma is not connected to the anode (coupling with the plasma expansion model), or by the limitation of the generator when the connection has occurred. The results presented on table 1 and 2 are computed for a silver cathode with an applied voltage of 15 V, the ratio $J_i/J_e = 0.25$ and for several values of the total current I. In the first table we represent internal spot variables while in the second one we represent injection boundary conditions for the expansion plasma model.

We remark that while the current I is rather large, the magnitude of these results is correct excepting for the plasma temperature which seems a bit high. When current decreases, the mathematical solution get non-physical as the potential sheath value goes deacreses under the ionization potential.Then, under a threshold value for the current no physical solution can be found, which is in accordance with experiments [3].

4. Conclusion

The numerical results obtained for the cathode spot model show that this cathode spot can exist only once the current has reached a threshold value. A simple calculation shows that the electron Child-Langmuir emission at the begining of the plasma expansion does not give a sufficient current. Then, one possibility is that the primary discharge during a small but sufficient duration furnishes the necessary energy to vaporize the metal and allows matter injection into the inter-cellular gap. A model of primary discharge is in progress at this time.

	$E_s\ (V.m^{-1})$	$T_s\ (K)$	$V_{sh}\ (V)$	$a\ (m)$
$I = 5.10^{-2}A$	$6.80\,10^9$	$4.74\,10^3$	6.18	$4.23\,10^{-8}$
$I = 1A$	$3.57\,10^9$	$4.50\,10^3$	14.3	$9.78\,10^{-7}$
$I = 10A$	$1.62\,10^9$	$4.81\,10^3$	14.9	$6.35\,10^{-6}$

Table 1: Numerical results obtained for an applied voltage of 15 V, and the ratio $J_i/J_e = 0.25$. The cathode surface electric field and temperature E_s and T_s, the potential drop in the sheath V_{sh} and the cathode spot radius a are calculated as a function of I.

	$J\ (A.m^{-2})$	$T\ (eV)$	$n\ (m{-}3)$
$I = 5.10^{-2}A$	$8.88\,10^{12}$	10.8	$1.12\,10^{28}$
$I = 1A$	$3.78\,10^{11}$	9.36	$5.13\,10^{26}$
$I = 10A$	$7.9\,10^{10}$	9.30	$1.08\,10^{26}$

Table 2: Numerical results obtained for an applied voltage of 15 V, and the ratio $J_i/J_e = 0.25$. The injection condition of the quasi-neutral plasma in the gap which are electron current J, temperature T and density n are calculated as a function of the current I.

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