Abnormal Amplification of Sound Waves Refracted by an Oblique Shock Wave

Igor MEN'SHOV and Yoshiaki NAKAMURA
Nagoya University, Department of Aerospace Engineering

ABSTRACT

Reflection and refraction of linear disturbance waves by an oblique shock wave is studied by a linear analysis. Several different cases are considered, when the incident plane wave is a fast acoustic, slow acoustic, entropy, or vorticity wave. Results show that (i) a critical angle of the wave incidence exists, beyond which the regular wave solution of the linear problem can not be realized, (ii) strongest sound waves are generated behind the shock wave, if the incident wave strikes the shock wave at the critical angle, and (iii) at large Mach numbers of the upstream flow, the sound amplification factor asymptotically increases as the Mach number squared, while at the critical incidence, it is increased as the Mach number cubed.

Introduction

The present paper addresses the problem of the interaction of small flow disturbances in the form of a monochromatic plane wave with plane, stationary, oblique shock waves. There exist three types of small disturbances that propagate in a uniformly moving compressible fluid. One represents isentropic pressure and density fluctuations, which propagate with the speed of sound relative to the fluid. The other two are fluctuations of vorticity and entropy that are carried with the fluid velocity. The impingement of such disturbance on the shock generates the waves that are composed of all three types.

The problem of the interaction of small disturbances with shock waves has been studied by several researches. Blokhintsev [1] solved the problem for the case of an acoustic wave normally striking a plane, normal shock wave in a perfect gas. Brillouin [2] studied oblique incidence of an acoustic wave on a normal shock wave, but obtained incorrect results. These results were later corrected by Kontorovich [3]. Kontorovich also extended the solution to the case of an arbitrary compressible medium.

In the present paper we would like to analize the interaction of plane weak waves with an oblique shock wave. The purpose is to evaluate the waves generated in the compressed flow behind the shock wave. After this study was completed, we learned of the paper by McKenzie and Westphal [4] where the similar problem has been already treated. We found that most of the results obtained are in fact not new and were published earlier. Nevertheless, we decided to submit the present results for publication by two reasons: to confirm the McKenzie and Westphal's results by solving the problem with an alternative approach and to stress an interesting phenomenon that is an abrupt amplification of transmitted waves at certain incidence angles, so-called critical angles.

1. Problem statement

We consider the interference of a plane monochromatic wave with an oblique shock wave. The basic flow consists of two uniform flows separated by the shock wave. This flow is defined by two parameters: the Mach number of the rarefied flow M and the angle between the shock plane and the rarefied flow direction β. The compressed flow parameters are denoted by the subscript s. Unit normal and tangential vectors to the shock plane are denoted by $\mathbf{n}$ and $\boldsymbol{\tau}$, respectively. The normal directs downstream of the shock. A system of coordinates is taken, in which the shock is at rest, and one ort is aligned with the rarefied flow. Figure 1 shows the flow configuration and basic notations.

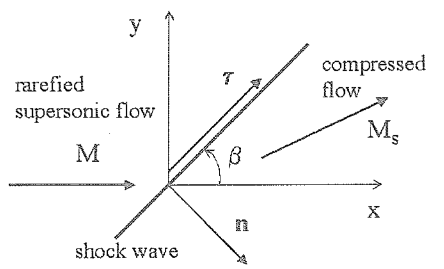

Figure 1: Flow configuration and basic notations.

The field of disturbances to be investigated is represented by the superposition of monochromatic plane waves of the following form:

$$\delta \mathbf{z} = \mathbf{z}' \exp\{i[(\mathbf{k}, \mathbf{x}) - \omega t]\} \quad (1)$$

where $\mathbf{z}'$ is the amplitude, ω ($\omega > 0$) is the circular frequency that is assumed to be positive, $\mathbf{k}$ is the wave vector. The propagation velocity of disturbances, i.e., the group velocity of the wave $\mathbf{c}$, is related to ω and $\mathbf{k}$ by means of $(\mathbf{k}, \mathbf{c}) = \omega$. The phase velocity of the wave propagation is defined as the projection of $\mathbf{c}$ onto $\mathbf{k}$.

Monochromatic plane disturbance waves that propagate in a homogeneous flow of velocity $\mathbf{u}$ are classified into 4 types: fast and slow sound waves, entropy waves, and vorticity waves. Dispersion relations between the wave vector and the frequency, group velocities, and amplitudes for these types of waves are as follows.

Fast sound waves

$$\omega = (\mathbf{u}, \mathbf{k}) + ak; \quad \mathbf{c} = \mathbf{u} + a\mathbf{k}/k \tag{2}$$
$$p' = \epsilon_p; \quad \rho' = \frac{\epsilon_p}{a^2}; \quad \mathbf{u}' = \frac{\epsilon_p}{\rho a}\frac{\mathbf{k}}{k}$$

Slow sound waves

$$\omega = (\mathbf{u}, \mathbf{k}) - ak; \quad \mathbf{c} = \mathbf{u} - a\mathbf{k}/k \tag{3}$$
$$p' = \epsilon_p; \quad \rho' = \frac{\epsilon_p}{a^2}; \quad \mathbf{u}' = -\frac{\epsilon_p}{\rho a}\frac{\mathbf{k}}{k}$$

Entropy waves

$$\omega = (\mathbf{u}, \mathbf{k}); \quad \mathbf{c} = \mathbf{u} \tag{4}$$
$$p' = 0; \quad \rho' = \epsilon_\rho; \quad \mathbf{u}' = 0$$

Vorticity waves

$$\omega = (\mathbf{u}, \mathbf{k}); \quad \mathbf{c} = \mathbf{u} \tag{5}$$
$$p' = 0; \quad \rho' = 0; \quad \mathbf{u}' = \epsilon_u \mathbf{m}$$

where a is the speed of sound, p and ρ denote pressure and density, ϵ denotes a characteristic amplitude of the wave, $\mathbf{m}$ is a unit vector normal to the wave vector $\mathbf{k}$.

It should be noted that because of $\omega > 0$, the wave vector must satisfy the inequality $(1, \mathbf{k}/k) > 1/M$ for slow acoustic waves, $(1, \mathbf{k}/k) > -1/M$ for fast acoustic waves, and $(1, \mathbf{k}/k) > 0$ for entropy and vorticity waves. Particularly, if the flow downstream of the shock is subsonic, no slow acoustic waves can exist behind the shock wave.

Depending on the group velocity, we distinguish incident and reflected or refracted waves, which are hereafter referred to as i-waves and r-waves, respectively. In i-waves, disturbances propagate towards the shock wave so that the group velocity vectors satisfy $(\mathbf{c}, \mathbf{n} > 0)$ for the waves upstream of the shock wave, and $(\mathbf{c}, \mathbf{n} < 0)$ for the downstream waves. On the other hand, disturbances in r-waves travel away from the shock wave so that the above-mentioned inequalities are opposite. As the rarefied flow is supersonic, no r-waves exist upstream of the shock.

The problem to be considered is to determine the resultant disturbance field when an i-wave strikes an oblique shock wave. Specifically, this problem may be classified under two types, namely reflection and refraction. In the refraction problem, a given i-wave, i.e., specific fast or slow acoustic, entropy, or vorticity wave, impinges on the plane of the shock wave from the side of the rarefied gas. This generates a set of r-waves downstream of the shock. In the reflection problem the incidence of the i-wave occurs from the side of the compressed flow, which also causes the appearance of several r-waves. All r-waves that are generated downstream of the shock have the same circular frequency as the i-wave. Therefore, to solve the problem, one has to determine which waves constitute the $r-$field and define their wave vectors and characteristic amplitudes.

2. Determination of wave vectors

The wave vector of a $r-$wave is determined on the base of the law of reflection and refraction. This law states a relation between wave vectors and propagation velocities of the i-wave and the generated r-wave. Using subscripts i and r to denote parameters of the $i-$ and $r-$wave, the law can be written in the following form:

$$\frac{(\mathbf{c}_i, \mathbf{n}_i)}{(\mathbf{n}_i, \boldsymbol{\tau})} = \frac{(\mathbf{c}_r, \mathbf{n}_r)}{(\mathbf{n}_r, \boldsymbol{\tau})} \tag{6}$$

where $\mathbf{n}_{i,r} = \mathbf{k}_{i,r}/k_{i,r}$ denotes the normalized wave vectors.

Eq. (6) can be recast as

$$(\mathbf{k}_r - \mathbf{k}_i, \boldsymbol{\tau}) = 0 \tag{7}$$

which means that the projections of the wave vectors of the i- and r-waves onto the plane of the shock wave must be the same. Therefore, the r-wave vector can be found as

$$\mathbf{k}_r = \mathbf{k}_i + \alpha\mathbf{n} \tag{8}$$

where α is a scalar parameter which has to be defined.

Assuming that the length scale is taken so that the i-wave has a wave vector of the unit length, i.e., $\mathbf{k}_i = \mathbf{n}_i$, the dispersion relation of the r-wave is then written in the following form:

$$\omega = (\mathbf{u}_s, \mathbf{n}_i) + \alpha u_{sn} + a_r k_r \tag{9}$$

where a_r should be taken as $-a_s$, $+a_s$, or 0, depending on whether the r-wave is a slow acoustic, fast acoustic, or etropy (or vorticity) wave. The subscript n in eq. (9) indicates the projection of the velocity onto the normal to the shock, $u_{sn} = (\mathbf{u}_s, \mathbf{n})$.

Eq. (9) states a linear dependence between α and k_r. Another relation between these two parameters results from (8):

$$k_r^2 = 1 + \alpha^2 + 2\alpha(\mathbf{n}, \mathbf{n}_i) \tag{10}$$

Eqs. (9) and (10) serve to determine α and specify the wave vector $\mathbf{k}_r$ in eq. (8). In the case of non-acoustic r-wave, $a_r = 0$ and the wave vector is simply defined as

$$\mathbf{k}_r = \mathbf{n}_i + \frac{\omega - (\mathbf{u}_s, \mathbf{n}_i)}{u_{sn}}\mathbf{n} \tag{11}$$

In the case of an acoustic r-wave, the solution to equations (9) and (10) exists under a certain condition on the incident angle. By introducing the vector $\boldsymbol{\phi}$ as

$$\boldsymbol{\phi} = \mathbf{n}_i + \frac{\omega - (\mathbf{u}_s, \mathbf{n}_i)}{u_{sn}}\mathbf{n} \tag{12}$$

the bound condition for the incident angle can be written as

$$\pm\phi_n \geq \sqrt{(\mu_{sn}^2 - 1)(1 - z^2)} \tag{13}$$

where $\mu_{sn} = a_s/u_{sn} = 1/M_{sn}$ and $z = (\mathbf{n}, \mathbf{n}_i)$. The upper sign in this equation should be used for fast acoustic $r-$waves, while the lower one for slow acoustic $r-$waves.

Once eq. (13) is satisfied, the parameter α is obtained:

$$\alpha = -z - \frac{\phi_n \mp \mu_{sn}^2\sqrt{\phi_n^2 - (\mu_{sn}^2 - 1)(1 - z^2)}}{\mu_{sn}^2 - 1} \tag{14}$$

and the wave vector $\mathbf{k}_r$ can be determined with eq. (8).

2.1. Refraction problem

In the refraction problem, the dispersion relation of an incident wave is given by

$$\omega = (\mathbf{u}, \mathbf{n}_i) - \varepsilon a \tag{15}$$

where ε should be taken as -1, $+1$, or 0 depending on whether the incident wave is a fast acoustic, slow acoustic, or entropy or vorticity wave. For this case, eq. (13) leads to the following conclusion: In addition to entropy and vorticity r-waves that have a wave vector defined by (11), only one acoustic wave, fast or slow, can exist in the r-field generated behind the shock. The type of the acoustic $r-$wave depends on the incidence angle. If this angle is so that $z_*^+ \le z \le 1$, the wave is fast. If $-1 \le z \le z_*^-$, the wave is slow. The limit values $z_*^\pm$ are functions of the normal Mach number upstream of the shock wave, only:

$$z_*^\pm = \frac{M_n \varepsilon \pm f\sqrt{M_n^2 + f^2 - \varepsilon}}{M_n^2 + f^2} \tag{16}$$

where $f = f(M_n) = a_s \sqrt{1 - M_{sn}^2}/a$.

If the incidence angle is so that z lies within the range z_*^- to z_*^+, neither fast, no slow acoustic wave can exist. In this case, only entropy and vorticity waves can form the r-field. However, this is not sufficient to resolve matching conditions for disturbances at the shock wave. Therefore, the regular refraction does not occur, if the incident wave strike the shock wave in the sector $z_*^- < z < z_*^+$.

Thus, the structure of the generated perturbation field in the refracted problem depends on the angle of incidence $\psi_i = \arccos z$. By introducing limit angles $\psi_*^\pm = \arccos z_*^\pm$, which we will refer to as *critical angles of incidence*, the structure can be described as follows.

$$If \quad \psi_*^- < \psi_i < \pi \quad or \quad -\pi < \psi_i < \psi_*^- \tag{17}$$

then the $r-$field consists of one entropy wave, one vorticity wave and one slow acoustic wave;

$$if \quad -\psi_*^+ < \psi_i < \psi_*^+ \tag{18}$$

then the $r-$field consists of one entropy wave, one vorticity wave and one fast acoustic wave;

$$if \quad \psi_*^- < \psi_i < \psi_*^+ \quad or \quad -\psi_*^- < \psi_i < -\psi_*^+ \tag{19}$$

no acoustic waves can exist in the $r-$field .

As previously noted, the latter situation means that the regular structure of the $r-$field, which consists of a set of monochromatic plane waves, does not exist. In this case, an additional surface wave appears that penetrates a finite distance the compressed flow region.

The wave vector of the incident wave must also satisfy the condition $(\mathbf{n}_i, \mathbf{l}) > \varepsilon/M$ to ensure $\omega > 0$. This imposes an additional restriction on the angle of incidence in the refraction problem that is given by

$$\arcsin\left(\frac{\varepsilon}{M}\right) - \beta < \psi_i < \pi - \beta - \arcsin\left(\frac{\varepsilon}{M}\right) \tag{20}$$

Figures 2 to 4 show the limits for the incidence angle given by (20) vs the rarefied flow normal Mach number at different shock wave angles. The markered curves correspond to the critical incidence angles (17): "triangle-up" marker indicates $\pm\psi_*^+$ angles and "triangle-down" marker indicates $\pm\psi_*^-$ angles. For nearly normal shock waves, i.e. $\beta \approx 90°$, the refraction of all types of $i-$waves is realized with a fast acoustic wave in the $r-$field. This is quite reasonable because the compressed flow is subsonic, and slow acoustic waves are not admitted. For moderate and small shock angles, $\beta \le 50°$, the refraction can be realized with both fast and slow acoustic waves depending on the incidence angle.

The critical angle depends on the parameters of the base flow, the upstream flow mach number M and the shock angle β. In Fig. 5 the critical value for the angle between $\mathbf{k}_i-$direction and the upstream flow direction is given versus the shock angle for two values of the upstream Mach number. As seen from these figure the critical angle is slightly less than the shock angle in almost all range of shock angles from normal to the Mach angle. That is, the critical incidence occurs when the incident wave vector is almost collinear with the shock plane.

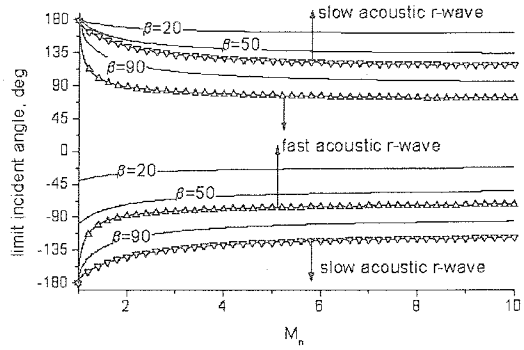

Figure 2: Critical angles vs normal Mach number for fast acoustic i-waves.

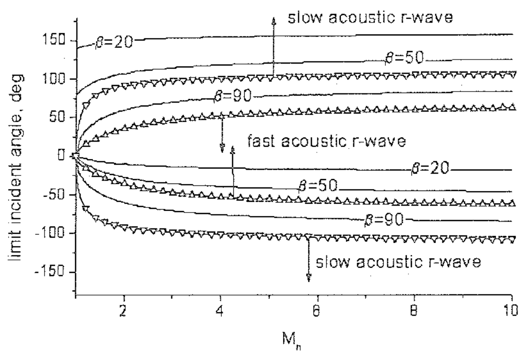

Figure 3: Critical angles vs normal Mach number for slow acoustic i-waves.

2.1. Reflection problem

In the reflection problem, the i-wave strikes the oblique shock wave from the side of the compressed flow. Therefore this wave is allowed to be either a fast or a slow acoustic wave, only; entropy and vorticity i-waves are not admitted. The incidence angle is restricted in this case by the following two inequalities:

$$(\mathbf{n}_i, \mathbf{l}) > \varepsilon/M_s, \quad (\mathbf{n}, \mathbf{n}_i) < \varepsilon/M_{sn} \tag{21}$$

where $\varepsilon = -1$ is assumed for fast waves, and $\varepsilon = 1$ for slow waves.

These two conditions define the limits of the incident angle in the reflection problem, which can be obtained from the diagram shown in Fig. 6 and written in the following form.

slow acoustic $i-$wave ($M_s > 1$):

$$\begin{aligned} arcsin\left(\frac{1}{M_s}\right) - arcsin\left(\frac{M_{sn}}{M_s}\right) &\le \psi_i \\ &\le arccos(M_{sn}) \end{aligned} \tag{22}$$

fast acoustic $i-$wave: if $M_s > 1$,

$$\pi - arccos(M_{sn}) \le \psi_i$$

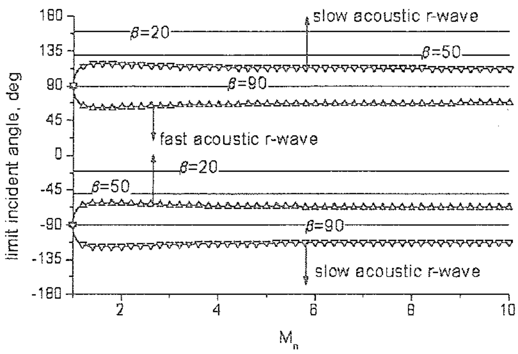

Figure 4: Critical angles vs normal Mach number for entropy and vorticity i-waves.

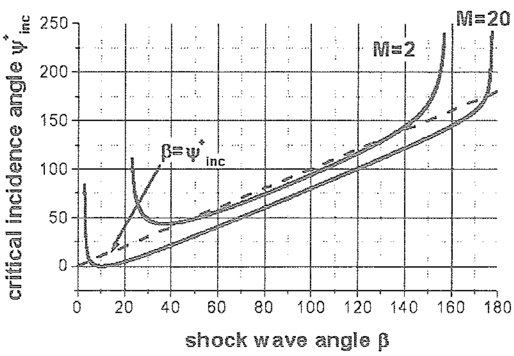

Figure 5: Critical angle versus shock angle.

$$\leq \quad \pi + arcsin\left(\frac{1}{M_s}\right) - arcsin\left(\frac{M_{sn}}{M_s}\right)$$

otherwise

$$\begin{aligned} \pi - arccos(M_{sn}) \quad &\leq \quad \psi_i \\ &\leq \quad \pi + arccos(M_{sn}) \end{aligned}$$

The bound condition on the angle of incidence given by eq. (13) for the reflection problem yields

$$\pm(M_{sn}z - \varepsilon) \geq \sqrt{1 - M_{sn}^2}\sqrt{1 - z^2} \qquad (23)$$

In the case of incidence of a slow acoustic wave, i.e., when $\varepsilon = 1$, the inequality of eq. (23) is met with the lower sign, only, which corresponds a slow acoustic $r-$wave; with the upper sign it is never satisfied. If $\varepsilon = -1$, i.e., in the case of a fast acoustic $i-$wave, the opposite is true: The condition of eq. (23) is met with the upper sign and never satisfied with the lower one. In other words, the reflection of a slow acoustic $i-$wave is always realized with the generation of a triple-wave $r-$field, which consists of one entropy wave, one vorticity wave and one slow acoustic wave. On the other hand, the reflection of a fast acoustic wave is always accompanied by the formation of a fast acoustic $r-$wave along with an entropy wave and a vorticity wave. The wave vector of the acoustic $r-$wave is determined by eqs. (10) and (14), where the upper sign should be used for fast waves, while the lower one for slow waves.

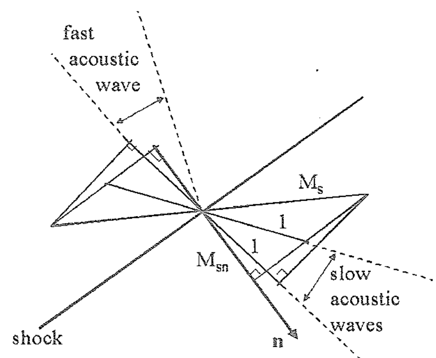

Figure 6: Diagram for admissible incident angles in the reflection problem.

3. Determination of wave amplitudes

The foregoing analysis allows us to determine the structure of the $r-$field generated by the incidence of an $i-$wave on the oblique shock wave and the wave vectors of all three $r-$waves that compose this field. Once this has been done, the wave amplitudes are next defined on the base of the linearized Rankine-Hugoniot relations which can be written in the following form:

$$\left(\frac{\partial \mathrm{f}}{\partial \mathrm{z}}\right)_s \tilde{\mathrm{z}}_s' - \frac{\partial \mathrm{f}}{\partial \mathrm{z}}\tilde{\mathrm{z}}' = C_s'\left(\mathrm{Q}_s - \mathrm{Q}\right) \qquad (24)$$

where z denotes the primitive vector, $\mathrm{z} = (\rho, u_n, u_\tau, p)$, $\mathrm{Q} = \mathrm{Q}(\mathrm{z})$ is the vector of corresponding conservative variables, and $\mathrm{f} = \mathrm{f}(\mathrm{z})$ is the flux vector in the direction normal to the shock wave. The super prime denotes disturbance parameters: $\tilde{\mathrm{z}}_s'$, $\tilde{\mathrm{z}}'$ represent the disturbance primitive vector downstream and upstream of the shock wave, and C_s' the perturbation of the shock normal velocity.

It should be noted that u_n' and u_τ' in the disturbance vectors $\tilde{\mathrm{z}}'$ and $\tilde{\mathrm{z}}_s'$ are the normal and tangential components with respect to the perturbed shock wave surface. They must take into account the perturbation of the normal and tangential unit vectors, i.e.,

$$\begin{aligned} u_n' &= (\mathrm{u}', \mathrm{n}) + (\mathrm{u}, \mathrm{n}') \\ u_\tau' &= (\mathrm{u}', \tau) + (\mathrm{u}, \tau') \end{aligned} \qquad (25)$$

The latter can be expressed in terms of the perturbation of the shock wave velocity as follows:

$$\begin{aligned} \mathrm{n}' &= \frac{(\mathrm{k}_i, \tau)}{\omega}\tau C_s' \\ \tau' &= -\frac{(\mathrm{k}_i, \tau)}{\omega}\mathrm{n} C_s' \end{aligned} \qquad (26)$$

The substitution of eqs. (25) and (26) into eq. (24) yields an equation in terms of the perturbation vector z', which is similar to $\tilde{\mathrm{z}}'$ with the exception of the velocity components that are also normal and tangential, but with respect to the non-perturbed shock wave surface. This equation can be written in the following form:

$$\left(\frac{\partial \mathrm{f}}{\partial \mathrm{z}}\right)_s \mathrm{z}_s' - \frac{\partial \mathrm{f}}{\partial \mathrm{z}}\mathrm{z}' = C_s'\left([\mathrm{Q}] - \frac{(\mathrm{k}_i, \tau)}{\omega}[\mathrm{b}]\right) \qquad (27)$$

where $\mathrm{b} = \partial \mathrm{f}/\partial \mathrm{z}\{0, u_\tau, -u_n, 0\}^T$ and the square brackets denotes the change at the shock wave, $[\cdot] = (\cdot)_s - (\cdot)$.

In the problem on wave reflection, the disturbance field is presented downstream of the shock, only, which is composed of the field of the incident wave and the field of the r−waves, i. e., $\mathbf{z}' = 0$, and $\mathbf{z}'_s = \mathbf{z}'_{s,i} + \mathbf{z}'_{s,r}$. In the refraction problem, the disturbance field upstream of the shock wave is represented by the i−wave, i.e., $\mathbf{z}' = \mathbf{z}'_i$, while downstream of the shock wave it consists of only the generated r−waves, $\mathbf{z}'_s = \mathbf{z}'_{s,r}$. The i−wave is assumed to be given, i. e., its type, wave vector and amplitude are known. The r−waves to be determined are one entropy wave, one vorticity wave and one acoustic, fast or slow, wave. Therefore, the disturbance vector of the r−field can be written as

$$\mathbf{z}'_{s,r} = \epsilon_\rho \mathbf{e}_{en} + \epsilon_p \mathbf{e}_{ac} + \epsilon_u \mathbf{e}_{vrt} \tag{28}$$

where $\mathbf{e}_{en}$, $\mathbf{e}_{ac}$, and $\mathbf{e}_{vrt}$ are normalized amplitude vectors that can be recognized from eqs. (2)-(5), and ϵ_ρ, ϵ_p, and ϵ_u are characteristic wave amplitudes that have to be defined. Substituting eq. (28) into eq. (27) leads to a linear system of 4 equations to determine 3 characteristic amplitudes and the perturbation of the shock velocity.

4. Results

Figures from 7 to 10 illustrate some results of solving this system of equations for the refraction of incident from the upstream fast and slow acoustic waves. In these figures, characteristic amplitudes of the r−waves made dimensionless by the incident wave amplitude and upstream flow parameters (density and speed of sound) are given versus the shock angle β for different values of the upstream Mach number M. The incident waves are considered, which have the wave vector collinear with the direction of the upstream flow. The shock angle is varied from normal to the limit Mach angle.

The refraction of the fast acoustic wave under these conditions is always realized with a fast acoustic r−wave; no regimes with the refraction in a slow acoustic wave were found. On the other hand, the slow acoustic wave is refracted with forming both fast and slow r−waves, with the latter being realized on weak shocks with β near to the Mach angle. Also, there is a range of shock angles, for which the regular reflection of slow acoustic waves does not exist, as seen in Fig. 8.

In Fig. 10 we show the ratio of pressure amplitudes in the incident and refracted waves for the case of a fast acoustic wave incident on the shock in the direction collinear to the upstream. This amplification factor is shown versus the shock angle. The maximal value of this factor is attained at normal shocks and then rapidly decreases as the shock angle decreases. The amplification factor increases with the increase of the upstream Mach number taking asymptotically the order $O(M^2)$. This is an expected result because the amplification of the base pressure itself behind the shock has the same order.

Fig. 11 shows the ratio of pressure amplitudes versus the incidence angle for the normal shock wave. A small attenuation of the transmitted wave is observed as the incident angle increases. However, an interesting thing occurs when this angle approaches the critical value: The amplification factor abruptly increases much exceeding the normal incidence factor. The ratio of pressure amplitudes in transmitted waves at critical and normal incidence angles is shown in Fig. 12 versus the shock angle. The amplitude at critical

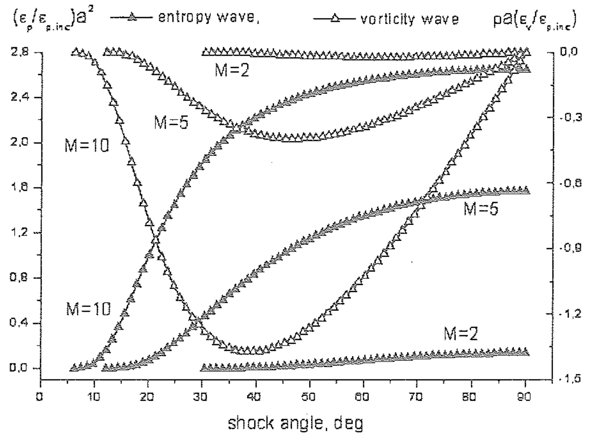

Figure 7: Amplitudes of entropy and vorticity r−waves vs shock angle for the fast acoustic wave refraction.

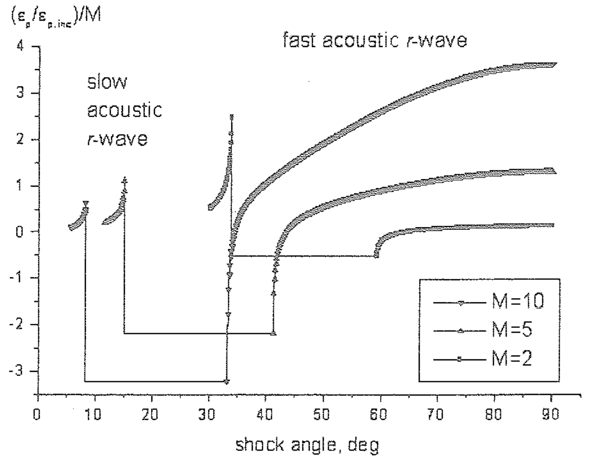

Figure 8: Amplitude of the acoustic r−wave vs shock angle for the slow acoustic wave refraction.

angles exceeds that at normal incidence more than one order. The difference between the amplitudes becomes much stronger as upstream Mach number increases. This can be seen in Fig. 13, which shows the dependence of the amplitudes on Mach number. The critical amplification factor behaves asymptotically as $O(M^3)$, while the normal that as $O(M^2)$.

Conclusions

The interaction of plane monochromatic waves of small disturbances with a stationary oblique shock wave has been investigated in the framework of the linear analysis. Main results have been obtained from this study are as follows.

(i) The regular interaction exists provided that the angle of incidence does not exceed a critical angle; beyond this angle no solution exists in the form of plane waves.

(ii) When the normal incidence occurs from the side of the rarefied medium, the pressure amplitude of the transmitted wave is asymptotically $O(M^2)$ greater than that in

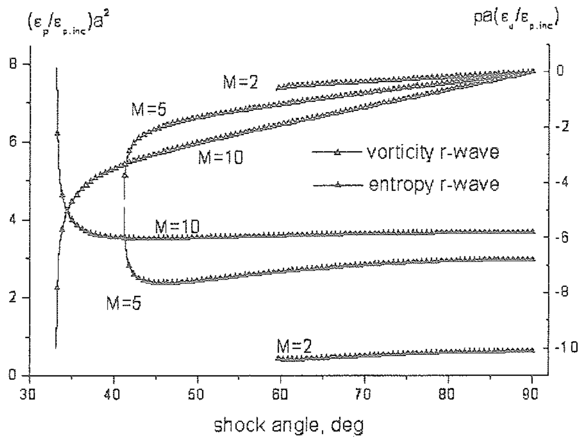

Figure 9: Amplitudes of entropy and vorticity $r-$waves vs shock angle for the slow acoustic wave refraction.

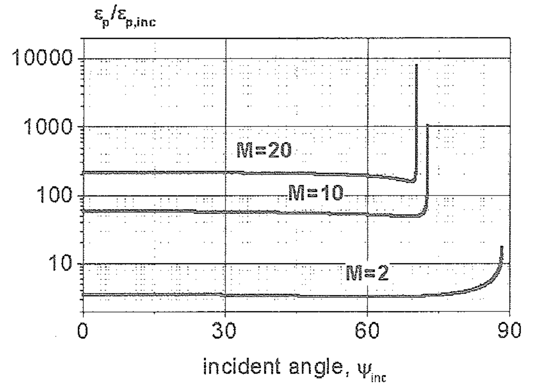

Figure 11: Ratio of pressure amplitudes in incident and refracted waves versus incident angle; the case of normal shock wave.

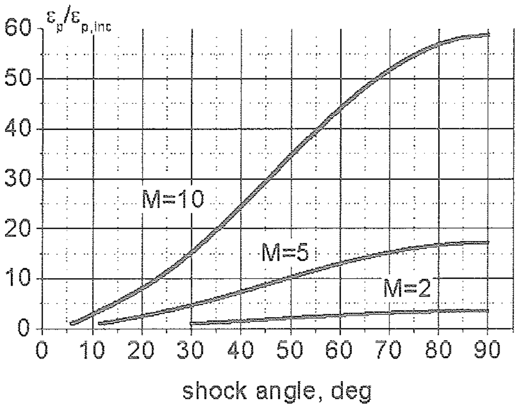

Figure 10: Ratio of pressure amplitudes in incident and refracted waves versus shock angle.

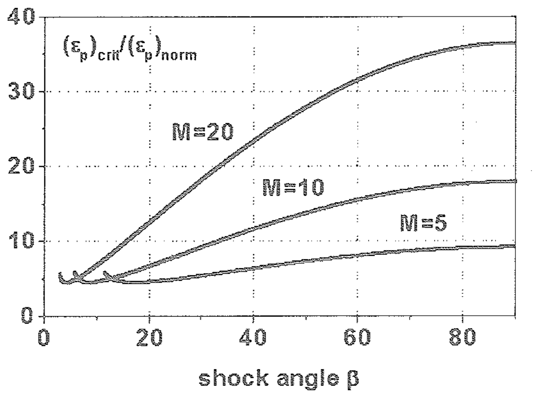

Figure 12: Ratio of pressure amplitudes in transmitted waves at critical and normal incidence angles versus shock angle.

the incident wave, where M is the Mach number upstream of the shock wave. As the angle of incidence increases tending to the critical one, the amplitude amplification factor is gradually decreased.

(iii) However, in a region very close to the critical angle the amplification of the transmitted waves is abruptly grows up; the amplitude amplification factor at the critical angle of incidence much exceed that at the normal incidence and has asymptotically the order $O(M^3)$ as M increases.

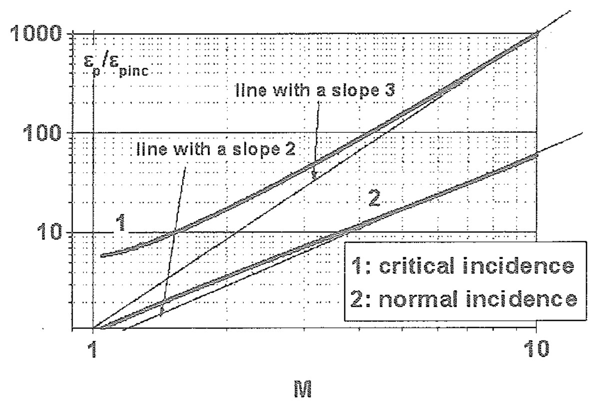

Figure 13: Amplification factor for critical and normal incidence versus Mach number.

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