Consideration on the increase of rocket attitude angle

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Research and development of small launch vehicles, such as SS520 launched from JAXA in 2019, is underway. Such a small rocket has the merit that the launch cost is low when trying to launch a small satellite on any orbit [1][2]. In such a small-sized cassette, in order to secure the size of the part which carries a payload, if the drive part for attitude control is removed, there is a demand of flying only by attitude control by a tail wing. In the SS520 rocket, although the attitude of the aircraft is controlled with only the tail wing from the first stage ignition, the phenomenon that the angle of attack increases while oscillating during the flight of the aircraft was observed. In this research, the cause, tendency, and countermeasure are considered about this increase phenomenon of angle of attack.

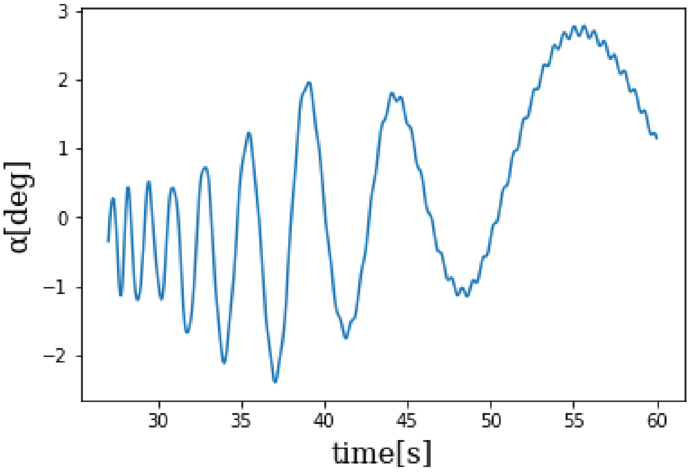

Fig.1 SS520-5 Yaw Direction Angle

Method

In the tailed rocket, the aerodynamic force acting on the tail of the rocket is the restoring force, and the static stability of the attitude is realized. Therefore, if the aerodynamic force as this restoring force becomes small, it is considered that the stability of the posture is lost.

Therefore, it is hypothesized that the aerodynamic force that is the restoring force of the rocket's attitude decreases as the altitude rises, and this may be the cause of this rocket's vibration increase phenomenon, and the following approximations will be used for verification. The modeling used was reproduced the behavior of the rising rocket, and it was confirmed whether the vibration increase occurred.

・Rocket is a rigid rod

・Ideal gas state equation

$$P = \rho RT \tag{1}$$

・Temperature, velocity and path angle are constant

$$T = T_0,\ \ V = V_0,\ \ \gamma = \gamma_0 \tag{2}$$

Because of the hierarchical structure of air

$$p(h + dh) + \rho dh = p(h) \tag{3}$$

From (1), (2), (3)

$$\rho = \frac{p_0}{RT_0} e^{-\frac{g}{RT_0}h} \tag{4}$$

If it does not stall at the wing tip,

$$C_L = C_{L\alpha} \times \alpha \tag{5}$$

From (3)

$$h = h_0 + V_0 t \sin\gamma_0 \tag{6}$$

Equation of motion representing rotation around the center of gravity of rigid rocket

$$I\frac{d^2\alpha}{dt^2} + M = 0 \tag{7}$$

Where,

$$M = L_{SM} \times \frac{1}{2}\rho V_0^2 S C_{L\alpha}\alpha \tag{8}$$

$$\ddot{\alpha} + l_0 e^{-\frac{g}{RT_0}V_0 t \sin\gamma_0}\alpha = 0 \tag{9}$$

$$(\mathrm{l}_0 = \frac{1}{2}\frac{1}{I}V_0^2 S C_{L\alpha} L_{SM}\ , \text{constant})$$

From (7), (8), (9), there is an analytical answer

$$\alpha = c_1 J_0\left(\frac{2RT_0}{gV_0}\sqrt{l_0 \exp\left(-\frac{gV_0}{RT_0}t\sin\gamma_0\right)}\right) + c_2 Y_0\left(\frac{2RT_0}{gV_0}\sqrt{l_0 \exp\left(-\frac{gV_0}{RT_0}t\sin\gamma_0\right)}\right) \tag{10}$$

$$c_1, c_2\ \cdots \text{constant}$$

Where, $\mathrm{J}_0(x)$ and $Y_0(x)$ are vessel functions.

$$J_0(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m!\,\Gamma(m+1)} \left(\frac{x}{2}\right)^{2m} \tag{11}$$

$$Y_0(x) = \lim_{\alpha \to 0} \frac{J_0(x)\cos(\alpha\pi) - J_0(x)}{\sin(\alpha\pi)} \tag{12}$$

Result

Using the conditions in Table 1 and plotting the analytical solution of (10) with the attitude angle and angular velocity at 27 s, the end of combustion of the first stage rocket, as initial conditions, the diagram is as shown in the figure below. From the results, it was confirmed that the vibration amplitude increased in the model. This indicates that the model can reproduce experimental results to some extent, as the decrease in atmospheric density leads to an increase in amplitude and the decrease in atmospheric density as the rocket's altitude rises. This indicates that the decrease in air density is one of the causes of the increase in rocket amplitude.

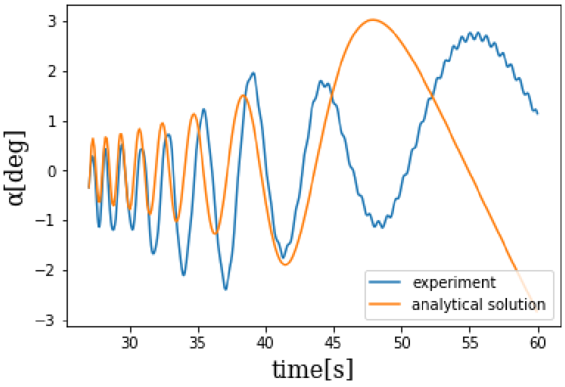

Fig.2: Analytical amplitude increase

l_0	56	
T	216.65	K
V	2000	m/s
γ	80	deg
g	9.8	m/s^2
R	287	$J/kg\ K$

Table.1: Calculation condition

References

[1] A.J.P. van Kleef, B.A. Oving, C.J. Verberne, B. Haemmerli, M. Kuhn, I. Müller and I. Petkov , “Innovative Small Launcher”, Reinventing Space Conference 2015 , P60-74, (2015)

[2] Kazuhiro Yagi, Seiji Matsuda, Jun Yokote, Takayoshi Fuji, Kenji Sasaki, Mitsuteru Kaneoka, Shinichiro Tokudome, Yohsuke Nambu and Masaaki Sugimoto, “A Concept of International Nano-Launcher”, 23nd Annual AIAA/USU Confarence on Small Satellites