The Role of Planar Unstable Periodic Orbits in Inclination Change

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Abstract

This paper investigates the effects of inclination excitations on quasi-satellite orbits in the Sun-Jupiter circular restricted three-body problem. Computations of invariant manifolds associated with vertical instability of planar periodic orbits enable quantifications of inclination excitations. We find that planar quasi-satellite orbits exhibit long-term stable, highly eccentric, periodic transitions between in-plane and out-of-plane states of inclination in tens of degrees. Analyses of phase-space structures reveal that stable, three-dimensional bifurcated families of periodic quasi-satellite orbits host long-term stable inclination oscillations. We point out that populations of undetected potentially hazardous asteroids of high eccentricity and inclination may reside in Jupiter's vertically unstable quasi-satellite orbits, which can intersect the orbits of the terrestrial planets, including the Earth by reducing their inclinations down to near zero via vertical instability. The existence of an asteroid 2004 AE_9 exhibiting a substantial inclination oscillation through a nearly coplanar state and a close approach to Mars during its Jupiter's quasi-satellite motion may support our conjecture.

軌道傾斜角の変化における平面不安定周期軌道の役割

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摘要

本論文では，quasi-satellite orbit の軌道傾斜角の励起現象を太陽-木星系の円制限三体問題において探求した．軌道傾斜角の励起を定量的に調べるため，平面周期軌道の軌道面外方向の不安定性に付随する不変多様体を計算した．その結果，平面 quasi-satellite orbit に付随する不変多様体は，長期安定かつ大きな離心率の状態で数十度の振幅をもつ軌道傾斜角の周期的な振動を示すことがわかった．相空間構造を調べることで，軌道傾斜角の振動の長期安定性は，三次元の安定な周期 quasi-satellite orbit によって引き起こされることを示した．木星の高い離心率の quasi-satellite orbit に捕捉されており，軌道傾斜角が小さい時期に発見された小惑星 2004 AE_9は，火星への接近や数十度からほぼゼロ度までの軌道傾斜角の振動を示すことに基づき，太陽-木星系の軌道面外方向の不安定性を持つ quasi-satellite orbit の高い軌道傾斜角の位置に，未発見の潜在的に危険な小惑星が存在する可能性を示唆した．

1. Introduction

Three types of co-orbital orbits are typical stable residences for large populations of asteroids revolving around the Sun for similar periods as planets. A tadpole orbit stays around one of the triangular Lagrange points L_4 and L_5, whereas a horseshoe orbit encompasses L_3, L_4 and L_5 [1]. A quasi-satellite orbit stays near a planet like a retrograde satellite [2]. From the perspective of Spaceguard, low-inclination objects have a greater likelihood of impacting the Earth [3], and it was estimated that the large part of potentially hazardous asteroids has not been found [4]. The incompleteness of detecting high-eccentricity and high-inclination objects [5] indicates that highly eccentric, highly inclined, stable orbits which will reduce their inclinations in the future could be candidates for hosting populations of undetected dangerous objects. Taking into account the strong ability of capturing objects of co-orbital orbits resulting from their stable nature, it could be interesting to explore inclination excitations or reductions of co-orbital orbits to prefigure possible orbits of hazardous asteroids.

Vertical instability of planar periodic orbits may induce inclination excitations, where a small out-of-plane deviation asymptotically grows. In the circular restricted three-body problem, Refs. [6, 7] showed that various planar periodic orbits have linear instabilities not only in in-plane directions but also in out-of-plane directions. We remark that Ref. [6] demonstrated one example of the significance of vertical instability in driving out vertically perturbed states from the vicinity of the original planar periodic orbit by propagating in the full nonlinear equations of motion in the case of equal masses of primaries. After the work of Refs. [6, 7], many studies have identified bifurcations from planar to three-dimensional families and vertical critical orbits, from which vertical instability emerges [8, 9]. Strongly related to the present paper, Ref. [10] found a vertical critical orbit and a three-dimensional family of a periodic quasi-satellite orbit in the circular restricted three-body problem, and Ref. [11] identified them not only in the circular model but also in the elliptic and general three-body problems.

However, only a few studies have addressed the fates or roles of vertical deviations excited by vertical instability. Ref. [12] showed that the vertical instability of a planar 3:1 resonant periodic orbit is weak and has little impact on the Interstellar Boundary EXplorer mission by propagating vertically perturbed states in the circular restricted three-body problem. Ref. [13] pointed out resonant gravity assists can increase inclinations of planar initial states to substantial levels based on the inspection of a three-dimensional Tisserand graph. Remarkably, recent studies for formations of planetary systems have shown that the vertical instability of planar resonant periodic orbits and bifurcated stable three-dimensional families can drive coplanar configurations to mutually inclined stable configurations in the general three-body problem Refs. [14, 15, 16].

This paper explores the effects of inclination excitations on quasi-satellite orbits in the Sun-Jupiter circular restricted three-body problem, which is a suitable model to focus on the fundamental dynamics of vertical instability. We investigate the vertical instability of planar quasi-satellite orbits and compute associated invariant manifolds of several values of the Jacobi constant to assess the fates of vertically perturbed states. As a result, inclination excitations occur and invariant manifolds associated with the vertical instability exhibit long-term stable, highly eccentric, periodic inclination oscillations between near zero and tens of degrees. Analyses of bifurcation and phase-space structures reveal that stable, three-dimensional bifurcated families host the stable inclination oscillations by trapping the invariant manifolds of the planar family around them, which appears similar to the mechanism of formations of the mutually inclined, resonant planetary systems mentioned above. The values of eccentricity of vertically unstable planar quasi-satellite orbits are so large that many of the orbits cross the orbits of the terrestrial planets, including the Earth. The ability of the inclination oscillations to capture objects due to their stable nature as well as the incompleteness in detecting high-eccentricity and high-inclination objects [5] indicate that Jupiter's vertically unstable quasi-satellite orbits may host undetected potentially hazardous asteroids at high-inclination locations. The existence of a Jupiter's high-eccentricity quasi-satellite 2004 AE_9 exhibiting a substantial oscillation of inclination through a nearly coplanar state and a close approach to Mars [17, 18] may support the conjecture.

The remainder of this paper is organized as follows. Section 2 introduces the background of this study. Section 3 presents the results of investigating inclination excitations of planar quasi-satellite orbits. Section 4 discusses a conjecture from the results as well as limitations and possible future prospects of this paper. Section 5 summarizes concluding remarks.

2. Backgrounds

This section introduces a mathematical model, the linear stability of a periodic orbit, and a computational method of invariant manifolds associated with the linear instability of a periodic orbit.

2.1 Circular restricted three-body problem

The circular restricted three-body problem is concerned with the motion of a massless particle, P_3, under the gravitational attractions of two massive bodies, P_1 and P_2, of masses m_1 and m_2 ($m_1 > m_2$), respectively. This model assumes that P_1 and P_2 revolve in circular orbits around their barycentre. The normalized equations of motion for P_3 in the P_1-P_2 rotating frame are (see Ref. [19] for details)

$$\ddot{x} - 2\dot{y} = -\partial\bar{U}/\partial x,\ \ddot{y} + 2\dot{x} = -\partial\bar{U}/\partial y,\ \ddot{z} = -\partial\bar{U}/\partial z,$$

where

$$\bar{U} = -\frac{1}{2}(x^2 + y^2) - \frac{1-\mu}{\sqrt{(x+\mu)^2+y^2+z^2}} - \frac{\mu}{\sqrt{(x-1+\mu)^2+y^2+z^2}},$$

and $\mu = m_2/(m_1 + m_2)$ is called the mass parameter.

This model is an autonomous system and possesses an integral of motion called the Jacobi constant:

$$\mathrm{C} = -(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - 2\bar{U}.$$

2.2 Linear stability of periodic orbits

The time evolution of a state transition matrix (STM) Φ in a dynamical system of the form $\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x})$ is

$$\dot{\Phi}(\boldsymbol{x}, t_i, t) = \mathrm{D_x f}\,\Phi(\boldsymbol{x}, t_i, t),\ \Phi(\boldsymbol{x}, t_i, t_i) = \mathbf{I},$$

where the subscripts i and f represent initial and final values, respectively, $\mathrm{D_x f}$ is a Jacobian matrix of the system, and $\mathbf{I}$ is an identity matrix.

Eigenvalues $\boldsymbol{D}$ of a monodromy matrix (STM over one full period of a periodic orbit) determine the linear stability of a periodic orbit, which come out in reciprocal pairs in Hamiltonian systems, such as the circular restricted three-body problem. This property results in the linear stability condition that a periodic orbit is linearly stable if and only if magnitudes of all eigenvalues are unity [9]. Linear instability arises from the condition $|D| > 1$, and a reciprocal pair ($1/D$, D) and associated eigenvectors indicate stable and unstable modes, respectively. Horizontal and vertical instabilities emerge if eigenvectors associated with stable and unstable eigenvalues of ($1/D$, D) with $|D| > 1$ have in-plane and out-of-plane components, respectively. A robust numerical continuation scheme, such as the pseudo-arclength continuation [20, 8] adopted in this study is able to track the linear stability of a family of periodic orbits to its full range of existence.

2.3 Computation of invariant manifolds

We follow Ref. [21] to summarize a method of computing invariant manifolds associated with linear instability of a periodic orbit. Let one point on a periodic orbit be $\boldsymbol{x_0}$ at the initial time $t = 0$ and the associated stable and unstable eigenvectors at that point be $\boldsymbol{Y}^s(\boldsymbol{x_0})$ and $\boldsymbol{Y}^u(\boldsymbol{x_0})$, respectively. Applications of an STM to the eigenvectors yield those for arbitrary points $\boldsymbol{x}(t)$ on the periodic orbit:

$\boldsymbol{Y}^s(\boldsymbol{x}(t)) = \Phi(t,0)\boldsymbol{Y}^s(\boldsymbol{x_0})$, $\boldsymbol{Y}^u(\boldsymbol{x}(t)) = \Phi(t,0)\boldsymbol{Y}^u(\boldsymbol{x_0})$.

Giving a small perturbation of $\pm\varepsilon$ ($\varepsilon = 10^{-4}$ in this study) to the stable and unstable eigenvectors yields states of stable manifolds and unstable manifolds, respectively:

$\boldsymbol{x}^{s\pm}(\boldsymbol{x}(t)) = \boldsymbol{x}(t) \pm \varepsilon\boldsymbol{Y}^s(\boldsymbol{x}(t))$,

$\boldsymbol{x}^{u\pm}(\boldsymbol{x}(t)) = \boldsymbol{x}(t) \pm \varepsilon\boldsymbol{Y}^u(\boldsymbol{x}(t))$.

One can globalize stable and unstable manifolds by numerically propagating $\boldsymbol{x}^{s\pm}(\boldsymbol{x}(t))$ backward in time and $\boldsymbol{x}^{u\pm}(\boldsymbol{x}(t))$ forward in time, respectively, for various $\boldsymbol{x}(t)$ along the periodic orbit. In this paper, we only propagate unstable manifolds due to the time reversal symmetries of the model [21].

3. Results

This section presents the fates of vertically perturbed states excited by vertical instability associated with planar periodic quasi-satellite orbits.

Fig. 1 shows reciprocal pairs of vertically stable (blue, |*D*|<1) and unstable (red, |*D*|>1) eigenvalues of monodromy matrices in terms of the Jacobi constant of the family of planar periodic quasi-satellite orbits. The strength of vertical instability is weak (|*D*| is near unity) but exists over the wide range of the Jacobi constant.

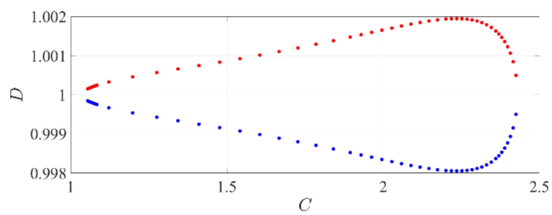

Fig. 1. Reciprocal pairs of vertically stable (blue, |*D*|<1) and unstable (red, |*D*|>1) eigenvalues of monodromy matrices in terms of the Jacobi constant *C* of the family of planar periodic quasi-satellite orbits.

We select 5 samples of vertically unstable, planar periodic quasi-satellite orbits (see Fig. 2) for subsequent computations of unstable manifolds associated with vertical instability. Note that we avoid choosing orbits too close to the Sun, but the selected orbits cross the orbits of the terrestrial planets, including the Earth. They also reach the vicinity of the Saturn's orbit.

We then compute 10 unstable manifolds associated with the vertically unstable eigenvalues for each of the 5 sample planar periodic quasi-satellite orbits, in total 100 unstable manifolds, by taking the signs of perturbation $\pm\varepsilon$ into account. We finish propagation if one of the following conditions is satisfied; propagation time exceeds 0.5 Myr; a trajectory collides with the surface of Sun or Jupiter; or a trajectory reaches |*x*|=2.5.

Fig. 3 shows the time evolutions of (a) the semi-major axis, (b) the eccentricity and (c) the inclination at perihelions of the unstable manifolds. The colour denotes the values of the Jacobi constant. The trajectories exhibit long-term stable collective behaviours of $a \approx 1$, highly eccentric, substantial inclination oscillations. The values of the inclination reach tens of degrees, whereas they spend relatively long durations at nearly coplanar states. We find that the vertically stable manifolds calculated based on the time reversal symmetries always exist in the vicinities of the propagated vertically unstable manifolds in the phase space, which can explain the asymptotic behaviours of the unstable manifolds, not only from but also to the planar periodic quasi-satellite orbits in a near-homoclinic manner.

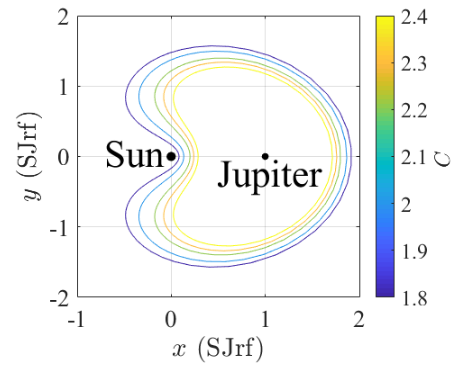

Fig. 2. 5 samples of vertically unstable planar periodic quasi-satellite orbits in the Sun-Jupiter rotating frame (SJrf). The colour denotes values of the Jacobi constant.

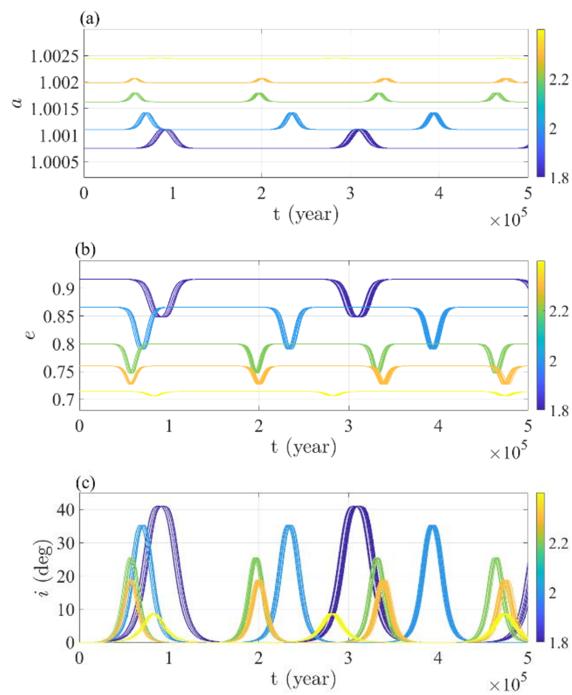

Fig. 3. Time evolutions of (a) the semi-major axis, (b) the eccentricity and (c) the inclination at perihelions of unstable manifolds emanating from the planar periodic quasi-satellite orbits shown in Fig. 2. The colour denotes values of the Jacobi constant.

Fig. 4 shows northern (blue) and southern (red) families of stable, three-dimensional, periodic quasi-satellite orbits bifurcate from the planar family at $C \approx 2.43$, where vertical instability of the planar family arises (see Fig. 1). Fig. 5 shows time evolutions over one period of (a) the semi-major axis, (b) the eccentricity and (c) the inclination of the northern family of stable three-dimensional periodic quasi-satellite orbits having the same values of the Jacobi constant as the unstable manifolds in Fig. 3. A comparison between this figure and Fig. 5 indicates two significances of investigating behaviours of invariant manifolds associated with vertical instability of planar periodic quasi-satellite orbits. First, the almost-constant values of inclination in Fig. 5(c) cannot predict the large inclination oscillations through the nearly coplanar states in Fig. 3 (c). The large inclination oscillations, which were not found from the information of the three-dimensional periodic quasi-satellite orbits in Refs. [10, 11], are crucial for reaching our conjecture, as will be discussed in Section 4. Second, the amplitudes of the inclination oscillations in Fig. 3(c) are larger than the inclinations of the three-dimensional periodic quasi-satellite orbits in Fig. 5(c), and non-negligible differences are observed, especially for small values of the Jacobi constant.

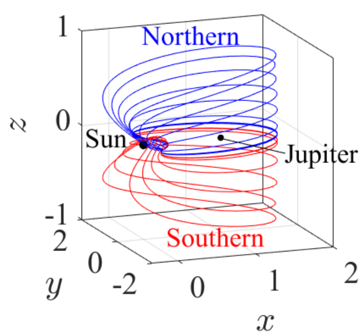

Fig. 4. Northern (blue) and southern (red) families of stable, three-dimensional, periodic quasi-satellite orbits in the Sun-Jupiter rotating frame.

Fig. 6 shows a typical example of an unstable manifold (grey) associated with the vertical instability of a planar periodic quasi-satellite orbit of C=2.2 propagated from a coplanar state to its most inclined state. The northern family of a stable, three-dimensional, periodic quasi-satellite orbit of the same Jacobi constant is superimposed (blue). Fig. 6 indicates that the stable, three-dimensional, periodic quasi-satellite orbit drives the inclination oscillation by trapping the unstable manifold around it.

In order to confirm this inference, we plot all of the perihelion states of the computed unstable manifolds superimposed on those of the northern and southern families of the stable, three-dimensional, periodic quasi-satellite orbits in Fig. 7. The figure shows that all of the unstable manifolds emanating from the planar periodic quasi-satellite orbits are trapped around the stable, three-dimensional, periodic quasi-satellite orbits not only (a) in the position space but also (b) in the velocity space. Though the unstable manifolds are trapped around the stable, three-dimensional, periodic quasi-satellite orbits, some of them change signs of *z* when passing through the planar vertically unstable periodic quasi-satellite orbits due to chaotic dynamics in the vicinity of unstable periodic orbits.

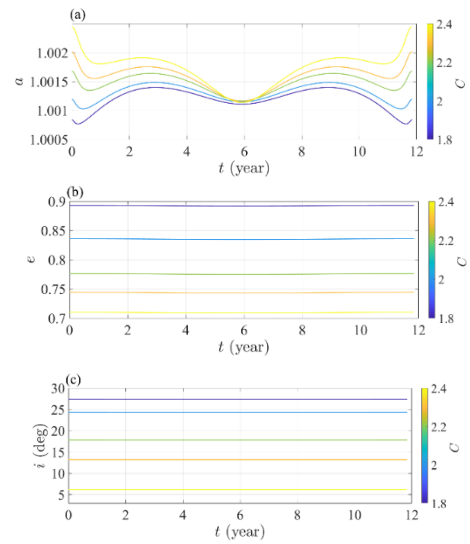

Fig. 5. Time evolutions of (a) the semi-major axis, (b) the eccentricity and (c) the inclination of the northern family of stable, three-dimensional, periodic quasi-satellite orbits over one period. The colour denotes the values of the Jacobi constant C=1.8, 2.0, 2.2, 2.3 and 2.4.

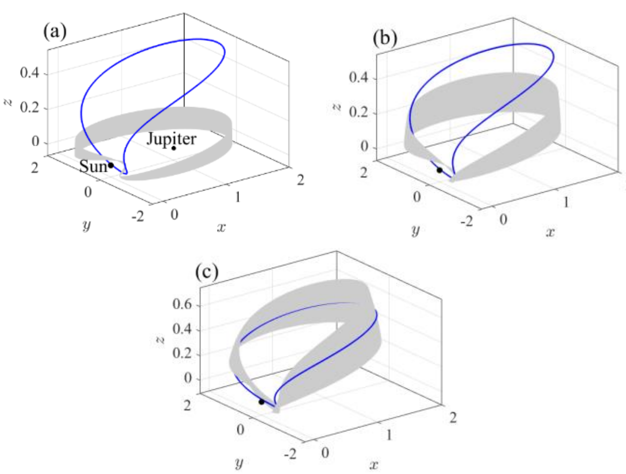

Fig. 6. A typical example of an unstable manifold (grey) associated with vertical instability of a planar periodic quasi-satellite orbit of C=2.2 propagated from a coplanar state to its most inclined state. The duration is divided into (a) 0-40102 (year), (b) 40102-46849 (year) and (c) 46849-55796 (year). The northern family of a stable, three-dimensional, periodic quasi-satellite orbit of the same Jacobi constant is superimposed (blue).

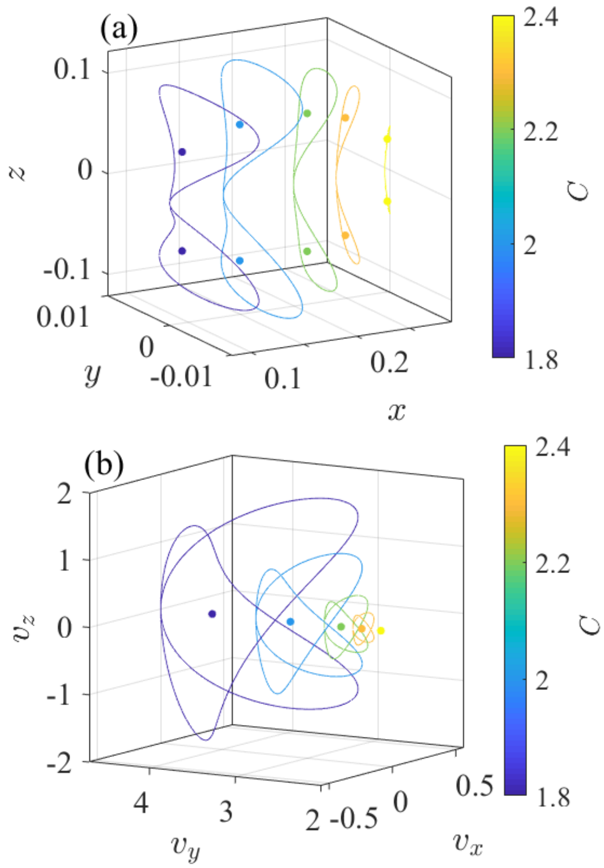

Fig. 7. All of the perihelion states of the computed unstable manifolds in Fig. 3 superimposed onto those of the northern and southern families of the stable, three-dimensional, periodic quasi-satellite orbits (the large dots) projected onto the (a) x-y-z and (b) v_x-v_y-v_z spaces. The colour denotes values of the Jacobi constant.

4. Discussions

The evolutions of eccentricity and inclination in Fig.3 indicate that vertical instability of quasi-satellite orbits can host high-eccentricity and high-inclination objects, which will reduce their inclinations down to near zero and cross the orbits of the terrestrial planets. Indeed, a Jupiter's high-eccentricity quasi-satellite 2004 AE_9, which was found at its low-inclination state, exhibits an inclination oscillation between near zero and tens of degrees (see fig. 5 in Ref. [17]). Moreover, this asteroid experiences close encounters with Mars [18]. Therefore, the existence of 2004 AE_9, the ability to capture objects due to the stable nature of the inclination oscillations, the incompleteness of detecting high-eccentricity and high-inclination objects, and the greater likelihood of low-inclination objects impacting the Earth (as mentioned in Section 1) suggest that extensive observation of high-inclination locations of vertically unstable quasi-satellite orbits may be helpful in detecting populations of potentially hazardous asteroids in advance *before* they reduce inclination.

The long-term stable inclination oscillations associated with quasi-satellite orbits obtained in the simplified model in this study may appear to be contradictory to the shorter lifetime of 2004 AE_9, of only tens of thousands years, on its quasi-satellite motion. We do not expect long lifetimes of such objects in the real world experiencing close approaches to planets at low-inclination states [18], but the stable nature of the inclination oscillations in the simplified model implies an ability to capture objects even in the real world. Certainly, 2004 AE_9 was trapped in the quasi-satellite motion exhibiting inclination oscillations from a chaotic orbit (see fig. 5 in Ref. [17]). The stable nature is significant in our conjecture, not because of the longevity *after* experiencing low-inclination states, but because of the ability to capture objects in the inclination oscillations at high inclinations *before* reducing their inclinations.

This paper focuses on the fundamental dynamics of vertical instability and inclination excitations of co-orbital orbits in the circular restricted three-body problem, but limitations exist due to the simplified assumptions of the model and additions of various perturbations, such as the eccentricity of Jupiter, and the gravitational effects of other planets could be promising directions. In such non-autonomous systems, computations of invariant manifolds associated with resonant periodic orbits [11] and quasi-periodic invariant tori [22] would be natural extensions of the present work.

5. Conclusions

This paper explored the effects of vertical instability on inclination excitations of quasi-satellite orbits in the Sun-Jupiter circular restricted three-body problem. Computations of invariant manifolds associated with vertical instability of the planar periodic orbits enabled quantitative analyses of inclination excitations. Quasi-satellite orbits exhibited long-term stable, highly eccentric, substantial inclination oscillations between near zero and tens of degrees without escaping from quasi-satellite states. We pointed out the possibility that populations of undetected potentially hazardous asteroids exist at high-inclination locations of Jupiter's vertically unstable quasi-satellite orbits. Analyses of bifurcation and phase-space structures revealed that stable, three-dimensional families of periodic quasi-satellite orbits host inclination oscillations in a stable manner.

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