A Proposal of Bicycles for Senior Ladies

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Astract: For senior ladies, car-driving is dangerous, and moreover they cannot ride bicycles, because of the loss reflexes with advance of age. These impose difficulty in their social life and give mal-effect in their health. The safe bicycles for senior ladies are strongly needed. Because the center of gravity of bicycles is much higher compared with cars, the bicycle must tilt at turn, although it is to be upright at zero speed. For the control of inverted pendulum, the Coefficient Diagram Method (CDM) gave a solution. With the same design, the bicycle for senior ladies may be realized, which tilts at turn, but keeps upright at zero-speed. Wright brothers discovered the importance of control technology in their activity of selling bicycles. They knew it is also essential in airplane, too. This led to their success in their first flight. The control of bicycle may be a good introduction of acquiring control technology in aerospace field.

Keywords: Control system design, Control theory, Inverted pendulum, Polynomial approach.

老婦人用自転車の提案

内容梗概： 老婦人は、自動車の運転が危険である上、反射神経を失っていて、自転車にも乗れない。社会生活に大きな不便があり、健康にも悪影響がある。老婦人用の安全な自転車の開発が強く望まれる。自転車が自動車と大きく異なる点は重心が非常に高い所にあることで、ゼロ速度では垂直に自立していても、曲がるときは適切に傾くことが必要になる。係数図法を用いた倒立振子の制御の原理で、傾くことができて転ばない、老婦人用自転車が実現できるのではないかと思われる。ライト兄弟は自転車の販売をしながら、制御の重要性に気付き、航空機でも制御が不可欠なことを知っていて、初飛行に成功した。自転車の制御は、航空宇宙での制御技術習得のよい入り口にもなるとも思われる。

1. INTRODUCTION

With age advance, people feel difficulty in riding bicycle, because of the loss of reflex in keeping proper upright posture. They cannot drive cars, because of latent danger. This is more prominent in senior ladies. They feel difficulty in daily shopping with the result of loss of proper social connection, which is detrimental to their physical and mental health. Senior people can ride bicycle, even they feel difficulty in walking. For walking, both legs have to be sufficiently strong, while in riding bicycle, one leg can be weak. Especially the handle bar is very helpful in keeping proper posture with weak waist muscle. It would be very helpful to senior people and as well as to the society as a whole, if bicycles, which do not fall down even at zero or very slow speed, are invented.

Because the author felt some difficulty in riding motor-assisted bicycle, and his wife is considering quitting driving (She cannot ride bicycle now, although she was able at younger age), he considered to buy 3-wheel motor assisted bicycle. To his astonishment, the bicycle shop owner strongly dissuaded him to buy it. He told him it will easily fall down.

4-wheel cars are very stable, because the center of gravity is at low position. But in bicycle, the center of gravity is at very high position. The human weight is about 60 kg, while the weight of the bicycle is 25 kg. For this reason, even 3-wheel motor-assisted bicycle has to tilt at the turn, while it has to keep the upright position in ordinary condition. When the seat of the rider is fixed, the rider feels difficulty in turn, and also he feels very uncomfortable when the road is tilted to one side, as usually is the case. When the seat can swing, the rider controls it like the 2-wheel bicycle and these difficulties can be overcome. But for this case the rider must retain the reflex to ride 2-wheel bicycle; difficult problem for senior ladies.

The 3-wheel motor assisted bicycle is heavy, and either stability or maneuverability is sacrificed. Thus, if intelligent side-wheels with the capacity that the arms will extend or detract as needed are attached to motor-assisted bicycles, stable and maneuverable light-weight bicycles will be realized. It is very helpful for shopping of senior ladies. It will contribute to their health, too.

The dynamics of bicycles is a difficult but scientifically interesting and challenging problem of classical dynamics. According to Prof. Åström （Lund Institute of Technology, Sweden） [2], German prominent physicist Prof. Sommerfeld has published "the theory of bicycle" in 1910 [3]. Four Nobel prize laurates came out from his laboratory (Heizenberg 1932, Debye 1936, Pauri 1945 and Bethe 1962). Even at present, dynamics of bicycle is actively studied in Europe and US; especially at Lund Institute of Technology (Sweden) and Delft University (Netherlands). Various Ph.D. dissertations are presented even at present.

In the study of bicycle dynamics, similar approaches are taken as in spacecraft and airplane. For this reason, reliable mathematical model is not presented even today for slow speed operation, where the effects of tire and friction are conspicuous.

For design of aircraft, the cooperation of specialists of four fields; namely fluid dynamics, structure, engine, and control. Similarly in bicycle design, the cooperation of specialists in four fields; namely dynamics, structure, power, and control.

Aircraft and bicycle are intrinsically unstable; aircraft longitudinally and bicycle laterally. In both, control is essential. The problems of aircraft are in take-off and landing, where speed is slow. Similarly the problem in bicycle is at stand-still and slow speed.

This paper is organized as follows. After introduction, in section 2, problems in the development are discussed. In section 3, brief introduction is made for CDM. Explanation of various terms used in CDM is made. In section 4, bicycle dynamics is presented. In section 5, the design example is shown. In section 6, realistic specifications for such bicycles are presented. In section 7, implementation of controller is discussed. In section 8, important results are summarized as conclusions.

2. PROBLEMS IN THE DVELOPNENT

Bicycles are unstable and parameter varying plants like aircrafts; the plant characteristics change dramatically with speed. For the design of controllers of such plants, the various control theories at present are not effective, except Coefficient Diagram Method (CDM) [1]. But CDM is not popularly accepted especially in Japan.

For the control of bicycles, the inverse response as evidenced in the inverted pendulum is essential [8]. When the rider of the bicycle wants to turn the left, he unconsciously turns the handle to the right. The bicycle tilts to the left due to centrifugal force. Then he intuitively turns the handle to the left to balance the effect of gravity by the centrifugal force.

The Wright brothers knew such inverse response already （[2] Wilbur Wright p. 34） . Their father was a pastor, and educated them in a carefree environment. They started a bicycle shop in Dayton, Ohio. Bicycle technology might be one of high technologies at the time (First flight in December 17, 1903). They understood the importance of control technology. This fact is understood as one cause of the success of the first flight. The other cause is the invention of the curved surface which gives larger lift for the same drag. They found the effectiveness of the curved surface through the experiment of small wind tunnel.

At that time, Prof. Langley was developing aircraft with the aid of U.S. Army. It is said that he was not successful in the flight of model aircraft, because he did not recognize the importance of control technology in aircraft. Wright brothers knew theory as well as the technology to realize it [4].

The difference of the useful common sense applicable to the system with control and without control is to be properly recognized. The center of gravity must be low for vehicles with no attitude control. But if the attitude is to be controlled, the vehicle with high center of gravity is easier to control, as the inverted pendulum with longer arm is easier to control.

In Japan, theory and practice is too much separated, and new technical development is difficult to take place. The bicycle shops are completely negative towards bicycles with intelligent side-wheels.

3. DEFINITIN OF TERMS USED IN CDM

Several important terms used in CDM will be briefly explained [1][9]. The characteristic polynomial is defined as follows.

$$P(s) = a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0. \qquad (1)$$

Stability index is defined as follows.

$$\gamma_i = a_i^2 / (a_{i+1} a_{i-1}). \qquad (2)$$

The equivalent time constant is defined as follows.

$$\tau = a_1 / a_0. \qquad (3)$$

Stability limit is defined as follows.

$$\gamma_i^* = 1/\gamma_{i+1} + 1/\gamma_{i-1}. \qquad (4)$$

Stability condition valid up to the fourth order is as follows.

$$\gamma_i = \gamma_i^*, \qquad 1 - \gamma_i^* / \gamma_i = 0. \qquad (5)$$

Stability condition, valid up to the fourth order, expressed in the coefficients of characteristic polynomial is as follows.

$$a_i = a_{i+2}\frac{a_{i-1}}{a_{i+1}} + a_{i-2}\frac{a_{i+1}}{a_{i-1}}. \qquad (6)$$

The CDM standard form is the case, when the stability indexes take the following form.

$$\gamma_{n-1} = \cdots = \gamma_3 = \gamma_2 = 2, \quad \gamma_1 = 2.5. \qquad (7)$$

4. BICYCLE DYNAMICS

The bicycle dynamics is well-explained by Åström [2]. The terms and equations are borrowed from the literature, when applicable.

General terms are defined as follows.

The point P_1 is the contact point of the rear wheel.

The point P_2 is the contact point of the front wheel.

The point P_3 is the intersection of the steer axis with the horizontal plain.

The coordinates are defined as follows.

The x-axis is the direction of motion of bicycle.
The y-axis is the left hand direction.
The z-axis is the upward direction.
The origin is at the point P_1.

Variables and parameters are defined as follows.

$\varphi =$ Tilt angle, right tilt plus ,
$\delta =$ Handlebar deflection, left turn plus ,
$h =$ Height of center of gravity ,
$V =$ Forward speed of bicycle ,
$a =$ Location of center of gravity at x-axis ,
$b =$ Contact point of front wheel at x-axis ,
$c =$ Trail, the distance between P_3 and P_2 ,
$\lambda =$ Head angle of steer axis, upright 90 deg .

The torque balance equation in x-axis is shown below.

$$J_{xx}\frac{d\omega_x}{dt} + J_{xz}\frac{d\omega_z}{dt} = T_{xgrav} + T_{xcent} + T_{xcgshiht} + T_{xsw}. \qquad (8)$$

The definitions for the terms used in the above equation are as follows.

Moments of inertia. $J_{xx} \simeq mh^2, \quad J_{xz} \simeq -mah.$

Angular velocities. $\omega_z = V\delta_f / b, \quad \omega_x = \dfrac{d\varphi}{dt}.$

Gravity torque. $T_{xgrav} = mgh\varphi$.

Centrifugal torque. $T_{xcent} = mVh\omega_z$.

Center of gravity shift torque. $T_{xcgshiht} = -macg\delta_f / b$.

Side wheel torque. T_{xsw}

The above equations are direct quote of [2]. However there is some doubt about the validity of the center of gravity shift torque. It seems to be correct for small deflection of handlebar, but not for larger deflection, as any riders feel in actual bicycles at slow speed. Further research is needed.

The front wheel touches the ground at point P_2. The ground force exerts feedback torque to the handlebar.

$$T_{hb} = (F_f + N_f\varphi_f)c\sin\lambda. \qquad (9)$$

The feedback torque T_{hb} is positive in clock-wise direction. The terms used in the equation are explained as follows.

The vertical component of the force acting on the front wheel at the ground contact. $N_f = amg / b$.

The horizontal component of the force acting on the front wheel at the ground contact. $F_f = (amV^2 / b^2)\delta_f$.

Front fork roll angle. $\varphi_f = \varphi - \delta\cos\lambda$.

Effective front fork angle. $\delta_f = \delta\sin\lambda$.

The torque balance equation Eq. (8) is further modified as follows.

$$mh^2\left[\frac{d^2\varphi}{dt^2} - \frac{g}{h}\varphi\right] = \frac{mahV\sin\lambda}{b}\frac{d\delta}{dt} + \frac{m(V^2h - acg)\sin\lambda}{b}\delta + T_{xsw}.$$

With $d / dt = s$, the final equation becomes as follows.

$$(s^2 - g/h)\varphi = [aVs + (V^2 - acg/h)]\frac{\sin\lambda}{bh}\delta + \frac{T_{xsw}}{mh^2}. \qquad (10)$$

Handlebar feedback torque equation is modified as follows.

$$\begin{aligned} T_{hb} &= \left[\frac{amV^2}{b^2}\delta\sin\lambda + \frac{amg}{b}(\varphi - \delta\cos\lambda)\right]c\sin\lambda \\ &= \frac{acmg\sin\lambda}{b}\left[\varphi + ((V^2/gb)\sin\lambda - \cos\lambda)\delta\right]. \end{aligned} \qquad (11)$$

For the side wheel controller, the result of the inverted pendulum [8] is used.

$$\begin{aligned} \frac{T_{xsw}}{mh^2} &= -G_c(s)\dot{\varphi}_s = -\frac{k_2s^2 + k_1s}{s^2 - l_1s - l_0}\dot{\varphi}_s, \\ \dot{\varphi}_s &= \dot{\varphi} + n. \end{aligned} \qquad (12)$$

In this controller, the input is $\dot{\varphi}_s$, which is the angular velocity of the tilt angle with the constant error n. This type of controller eliminates the effect of error n.

The riders control the handlebar in various ways according their skill. The senior ladies without balancing skill will completely on the sidewheel controller for balancing. They will control the handlebar only to change the direction of the bicycle.

$$\delta = \delta_r. \qquad (13)$$

The δ_r is the handlebar reference angle intended by the rider. The rider with balancing skill, without the side wheel, will control the handlebar torque. Then the handlebar responds by the following handlebar dynamics.

$$(M_{chb}s^2 + C_{chb}s)\,\delta = T_{hbr} - T_{hb}. \qquad (14)$$

T_{hbr} is the handlebar reference torque given by the rider. The counter-clock-wise is positive. It is an unknown and complicated function of δ_r, $\ddot{\varphi}$, δ, and V by human reflex. It is zero for hand-free case. M_{chb} is the inertia term and C_{chb} is the damping term.

5. DESIGN EXAMPLE

A design example will be given to clarify the design process. The basic design parameters used are shown below.

$$m = 80\,kg, \quad g = 9.8\,m/\sec^2, \quad h = 1\,m,$$
$$a = 0.35\,m, \quad b = 1.15\,m, \quad c = 0.05\,m,$$
$$\lambda = 70\,\text{deg}, \quad \sin\lambda = 0.940, \quad \cos\lambda = 0.342.$$

At this design the riders are senior ladies, and the following conditions are assumed.

(1) Handlebar angle is given by the rider $\delta = \delta_r$.

(2) The signal $\dot{\varphi}_s = \dot{\varphi} + n$ is available from the side wheel.

Design results are obtained as follows.

$$G(s) = G_c(s)G_p(s)$$
$$= \frac{(9.9612s^2 + 29.645s)s}{s^2 - 1.1068s - 0.24500}\,\frac{1}{s^2 - 9.8},$$
$$P(s) = s^3 + 8.8544s^2 + 19.6s + 10.847s + 2.401, \qquad (15)$$
$$\gamma_i = [4 \;\; 4 \;\; 2.5], \quad \tau = 4.5176,$$
$$s_i = -5.5716, \quad -2.4027, \quad -34005 \pm j0.2398,$$
$$\phi_m = -38.72\,\text{deg}, \quad g_m = 0.46826.$$

The CAD commands by CDMCAD [1] are as follows.

```
ap=[1 0 -9.8];bp=1;ac=[1 -1.1068 -0.24500];
bc=[9.9612 29.645 0 0];tm=0.5;ba=0;ba,bc,ac,
[aa,g,tau,gs,rr,pmgm,wpmgm]=c2g(ap,bp,ac,bc,ba,tm)
```

The block diagram is given in Fig. 1. The coefficient diagram is shown in Fig. 2. The frequency responses, the step response of the complementary function, and pole locations are shown in Fig. 3.

The design is made in the same manner as [8]. The characteristic polynomial is given as follows.

$$\begin{aligned} P(s) &= (s^2 - l_1 s - l_0)(s^2 - 9.8) + (k_2 s^2 + k_1 s)s \\ &= s^4 + (k_2 - l_1)s^3 + (k_1 - l_0 - 9.8)s^2 \\ &\quad + 9.8 l_1 s + 9.8 l_0 \qquad (16) \\ &= a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0. \end{aligned}$$

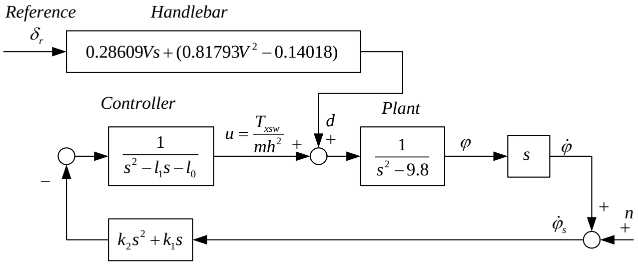

Fig.1. Block diagram of the control system

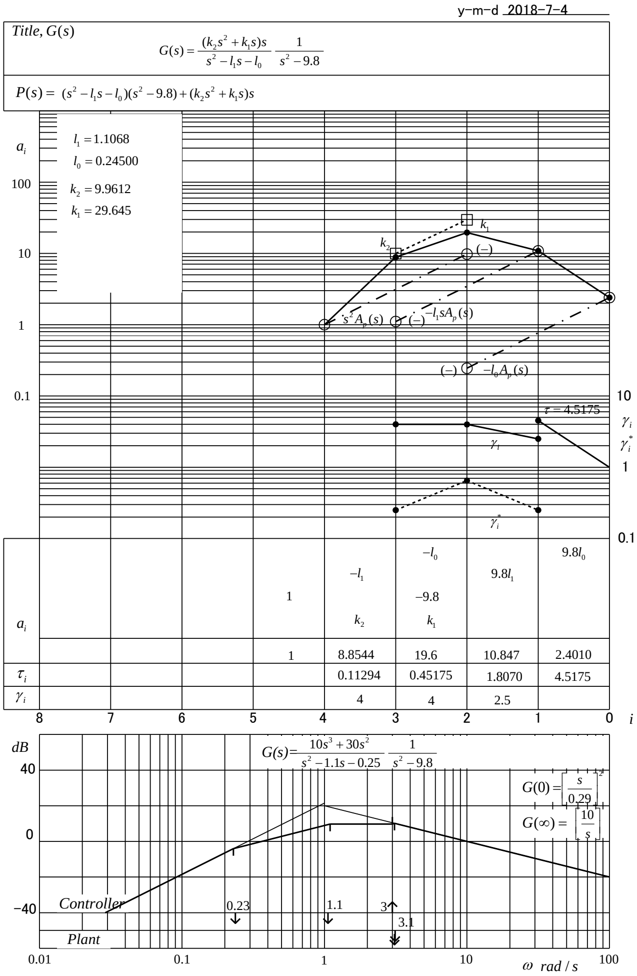

Fig. 2. Coefficient diagram

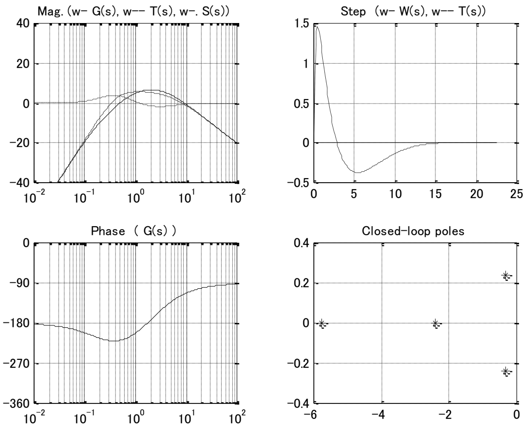

Fig. 3. Frequency/step responses and pole location

The first choice is to make $a_2 = 19.6$ or twice of 9.8 to guarantee the robustness for the gain variation. Then choice is made for stability indexes as $\gamma_3 = \gamma_2 = 4$ and $\gamma_1 = 2.5$ also for robustness reason. Then the controller parameters can be automatically calculated from the coefficient diagram. The "c2g" command of CDMCAD produces the design results.

The more detailed step responses for disturbance d are shown in Fig.4(a)(b)(c)(d). The unit step of $d = 1\, rad/\sec^2$ roughly corresponds to the 8 degree handlebar deflection at the bicycle speed $V = 3\ \mathrm{m/sec}$ as shown in the following equation, which comes from Eq. (10) and Fig. 1.

$$
\begin{aligned}
d &= [aVs + (V^2 - acg/h)]\frac{\sin\lambda}{bh}\delta_r \\
&= [0.28609Vs + (0.81793V^2 - 0.14018)]\delta_r \qquad (17) \\
&= [0.85826s + 7.2163]\times 8 \times \pi/180 \\
&= (0.11893s + 1)\times 1.0076 \simeq 1
\end{aligned}
$$

The speed, $V = 3\, m/\sec = 10.8\, km/hr$, corresponds to two and half times of the ordinary walking speed.

Explanations of the responses are as follows.

(1) The unit step disturbance d corresponds to plus x-axis torque or the right tilt of $mh^2 d = 80\ m\text{-}N$.

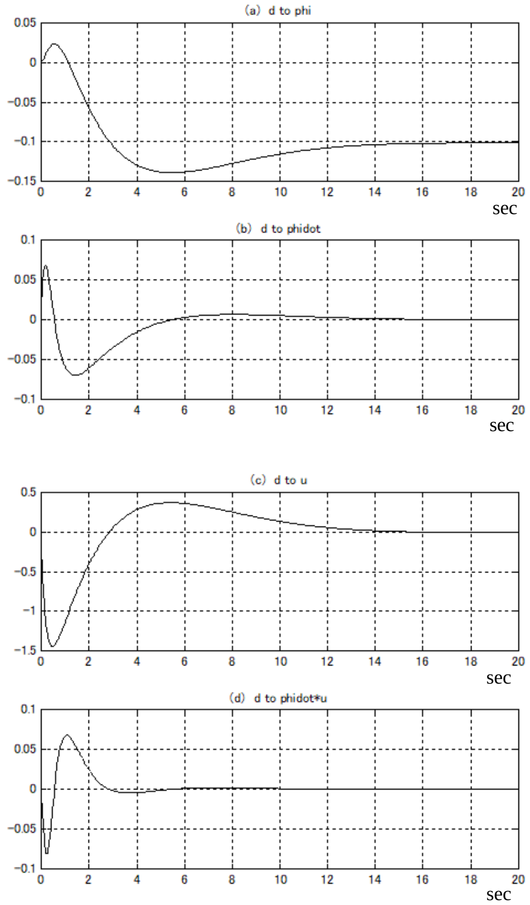

Fig. 4. Various responses to disturbance

(2) The response of u by d is given as follows.

$$u = -\frac{9.9612s^3 + 29.645s^2}{s^4 + 8.8544s^3 + 19.6s^2 + 10.847s + 2.401}d.$$

Or approximately,

$$u \simeq -(29.645s^2/19.6s^2)d \simeq -1.5d.$$

This is observed in Fig. 4(c).

(3) This strong negative torque tilts the bicycle to the left. This action continues even the tilt returns to zero due to the unstable nature of controller.

(4) Due to the left tilt, the strong left tilt torque of the gravity is produced.

(5) The torque gradually replaces the controller output u which finally settles to zero.

(6) The maximum torque of the controller is

$$T_{xsw} = 1.5mh^2 d = 120\ m\text{-}N.$$

(7) The maximum angular velocity is

$$\dot\varphi = 0.07\ rad/\sec = 4.0107\ \deg/\sec.$$

(8) The final tilt angle is

$$\varphi = -d/(g/l) = -1/9.8 = -0.010204\ rad$$
$$= -5.8456\ \deg \simeq -6\ \deg.$$

(9) The maximum motor power is observed in Fig. 4(d) at around 1.2 sec, where

$$\dot\varphi \times u = 0.07, \text{ or } \dot\varphi \times T_{xsw} = 5.6\,W.$$

(10) The maximum torque multiplied by the maximum angular velocity is

$$\dot\varphi \times T_{xsw} = 0.07 \times 120 = 8.4\,W.$$

6. REALISTIC SPECIFICATIONS

The realistic specifications for bicycles for senior ladies are listed as follows.

(1) The weight of the bicycle shall be less than 15 kg, preferably less than 10 kg, so that the bicycle can be handled by senior ladies.

(2) The capacity of the battery shall be about 5 ampere-hours. Light weight rather than long distance travel is more important.

(3) Commercial motor-assisted bicycle is to be used. The intelligent side-wheels are to be designed such that they are easily attached to such commercial bicycles.

(4) The length of side-wheel arm is around 0. 2 m. The maximum tilt angle is 0.1 rad. The vertical travel distance of the each side wheel is $\pm 0.02\,\mathrm{m}$. The maximum force at the side wheel is 120/0.2=600 N.

(5) The arms of intelligent side-wheels must have the characteristics of human legs; strong force actuation with bilateral action (moves easily by external force) and velocity sensing capability.

7. A PROPOSAL OF CONTROLLER IMPLEMENTATION

There are several proposals to implement such controller.

One of such proposal is explained below.

(1) The controller is reorganized as follows, so that instability is confined to a single unit.

$$G_c(s) = \frac{s^2k_2 + k_1 s}{s^2 - l_1 s - l_0}$$

$$= k_2 + \frac{1}{s-\alpha}[(k_1 + l_1 k_2) + \frac{l_0 k_2 - (k_1 + l_1 k_2)\beta}{s+\beta}],$$

$$s^2 - l_1 s - l_0 = (s-\alpha)(s+\beta).$$

$$G_c(s) = 9.9612 + \frac{1}{s-1.2959}\left[40.670 + \frac{-5.2487}{s+0.18906}\right].$$

(2) Proper saturation units are placed to prevent excessive noise n, ($\pm L_1$), and prevent the run-away of the unstable unit, ($\pm L_2$).

(3) The intelligent side-wheels consist of the left and right wheels with legs which can extend and detract.

(4) The leg consists of a motor, gearbox, rack-pinion, and lever. By these mechanisms, the force acting on the side wheel is controlled. Eventually the torque around x-axis, T_{xsw}, is controlled.

(5) The current amplifier supplies current to each motor, which will give a proper torque. The combination of two motors gives proper torque around x-axis. The circuits are provided such that two motor speeds are computed from the terminal voltages (V_{TL} and V_{TR}) and currents (i_L and i_R). The combination of two motor speeds gives signal $\dot{\varphi}_s = s\varphi + n$. This signal is the input to the controller. The output $u = T_{xsw}/(mh^2)$ is computed by the controller, and it produces proper current references to each current amplifiers.

Block diagram of the proposed implementation is shown in Fig. 5.

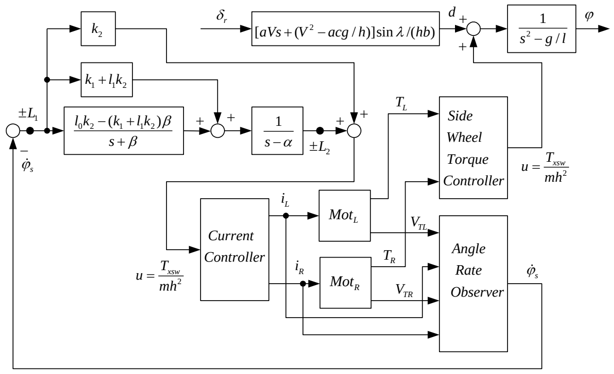

Fig. 5. Controller implementation

8. CONCLUSIONS

The important conclusions are summarized as follows.

(1) A proposal of the bicycles for senor ladies is made. The proposed bicycle with intelligent side-wheels will be stable and maneuverable. It will greatly contribute to the welfare of senior ladies. For this reason, legislative efforts to make the status of such bicycle equal to that of wheel chair for handicapped is strongly recommended.

(2) The bicycle dynamics is presented and its controller design by Coefficient Diagram Method is proposed. Stability is guaranteed by proper design of characteristic polynomial. Maneuverability is provided by the unstable controller, which tilts the bicycle to proper angle at the turn.

(3) The development of actuator like human muscle is found to be the key technology.

For this design, Japanese patent is applied to Japan Patent Office as of October 10, 2018, and the identification number, 518358088, was assigned. Any researchers may feel free to use the technology for their research purpose. Any company, who wish to commercialize this technology, may contact the author, who is intending to give most favorable condition for the patent use.

REFERENCES

[1] Manabe, S (2018). http://www.cityfujisawa.ne.jp/~manabes/index.htmCD MBook.

[2] Åström, K. J. (2005). “Bicycle Dynamics and Control,” *IEEE Control Systems Magazine,* vol. 25, no. 4, August 2005, pp. 26-47.

[3] Klein, F and A. Sommerfeld (1910).“Über die Theorie des Kreisels,” vol. Heft IV, Leibzig, Germany: Teubner, 1910, pp. 863-864.

[4] Wikipedia, Wright Brothers.

[5] Wikipedia, Bicycle and motorcycle dynamics.

[6] Meijaard, J. P. et al. (2007). “Linearized dynamics equations for the balance and steer of a bicycle: a benchmark and review,” Proceedings of the royal society A, Vol. 463, 2007, pp. 1955-1982.

[7] Kooijman, J. D. G. et al. (2011). “A bicycle can be self-stable without gyroscopic or caster effects,” Science, vol. 332, no. 6027, April 15, 2011, pp. 339-342.

[8] Manabe, S. (1994). “A low-cost inverted pendulum system for control system education,” IFAC 3rd Symposium on Advances in Control Education, Tokyo, August 1-2, 1994, pp.21-24.

[9] Manabe, S. (2002). “Brief tutorial and survey of coefficient diagram method,” *4th Asian Control Conference*, Singapore, September 25-27, 2002, pp. 1161-1166.