Escape Trajectory for Martian Moon Sample Return Mission Using Tube Dynamics Associated with Quasi-periodic Orbit

Kazutoshi Takemura (Waseda University, Graduate School), Nicola Baresi, Yasuhiro Kawakatsu (ISAS/JAXA), Hiroaki Yoshimura (Waseda University)

Abstract: In spacecraft trajectory design based on dynamical theory, research extended to the spatial problem is insufficient. However, there are cases that three dimensions must also be taken into consideration in actual missions. In this work, we discuss on designing Mars escape trajectory in the three-body system via invariant manifolds. In contrast with the conventional methods, we propose to use invariant manifolds from a quasi-halo orbit and then we investigate quasi-halo orbits and their associated tubes. Finally, we show how the invariant manifolds of quasi-periodic orbits can provide new escape trajectories from the Martian system in the circular restricted three-body problem.

準周期軌道に付随する不変多様体を用いた火星離脱軌道の検討

竹村和俊（早大院），Nicola Baresi，川勝康弘（ISAS/JAXA），吉村浩明（早大）

摘要 : 力学理論に基づく宇宙探査機の軌道設計においては，空間問題に拡張した研究が十分に行われていない．しかし現実に検討されている実ミッションにおいては，3 次元を考慮して軌道設計しなくてはならないケースも存在する．本研究では，3 体問題下におけるチューブと呼ばれる不変多様体を用いた火星離脱軌道の設計を行う．従来の方法と異なり，準周期軌道に付随する不変多様体を解析し，そのチューブが火星からの離脱を結論づけられるかどうかを調査する．

I. Introduction

With the progress of planetary science and space engineering, planetary exploration has been in full swing. In 2024, Japan Aerospace Exploration Agency (JAXA) will launch the Martian Moons eXploration (MMX) to retrieve the world first sample return from Phobos. From the viewpoint of trajectory design, MMX has a unique character. In terms of the exploration of planetary satellites, there is a Saturn space probe, Cassini-Huygens mission, developed by NASA and ESA. On the other hand, regarding the sample return mission, Hayabusa, Hayabusa2 and OSILIS-REx are enumerated. However, there is no example mission about sample return and escape from a gravitational body. In the MMX mission, there is no moons or asteroid around Mars to perform the gravity assist to increase v-infinity of a spacecraft since the mass of Phobos and Deimos is too small to make a swingby. Therefore a spacecraft must escape from Mars' sphere of influence and return to the Earth by its own propellants. The balance of scientific observation time, transfer time, total mission time and the mass of consumption fuel becomes very important and hence it must be considered. In the limited mission time, in order to maximize the observation time, it is necessary to reduce the transfer time. When thinking about escape from the gravitational object, here Mars, by its own propellants, it is important how quickly a spacecraft escape from Mars's sphere of influence (SOI). Therefore, a hybrid usage of chemical propulsion and electric propulsion was proposed on the return leg [1], in which a spacecraft can escape immediately from Mars' SOI by chemical propulsion so that it can increase the mission period. On the other hand, electric propulsion enables efficient acceleration in interplanetary flight and then can reduce fuel mass. In this proposal, since the electric propulsion is used in interplanetary transfer, how to quickly escape at low energy by the chemical propulsion becomes important.

In this context, we propose to use tube dynamics [2,3,4,5] as a different method from the conventional approach and also to extend the analysis by the tube dynamics in the planar problem to that in the spatial problem. Tubes are regarded as the invariant manifolds that depart from unstable periodic orbits around Lagrangian points of restricted three-body problems. For the planar cases, it is known that when a spacecraft is placed inside a tube, it can escape from a planet even if its Keplerian energy is less than 0. That is, a trajectory to escape with smaller energy than the two-body problem becomes possible. We take a Poincaré section of the tube to determine the state quantity. Under the system of PCR3BP, it is easy to describe the dynamics on two-dimensional Poincaré maps because we can reduce two variables due to an energy and the position of the section. However, in the spatial problem, the invariant manifolds from just a halo orbit cannot be used as a escape trajectory in the spatial problem [6] because a spatial problem is highly dimensional and an analysis of only the periodic orbit might be insufficient. Thus we propose to use the invariant manifolds from the quasi-halo orbit around L_1 of Sun-Mars system to apply the tube dynamics into the spatial problem.

II. Fundamental Settings

A. Dynamical Model

First let us consider the dynamics of the three-body system and the two-body systems. The Sun-Mars-Spacecraft (S-M-S/C) system may be regarded as a three-body system, in particular, the Circular Restricted Three-Body System (CR3BS). In this setting, the Sun and Mars have circular orbits about their common mass center. Phobos is assumed to have circular orbit about Mars on Mars's equatorial plane. Each of the spacecraft and Phobos is assumed to have a negligible mass. The Sun-Spacecraft and Mars-Spacecraft system is regarded as a two-body system, i.e., the restricted two-body problem. In the Sun-Spacecraft system, Mars and the Earth have respectively circular orbits about the Sun on the ecliptic plane with negligible masses.

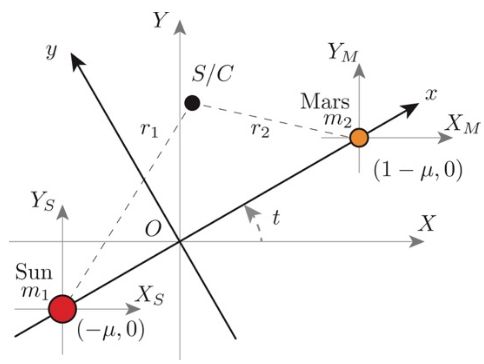


Fig. 1 Circular Restricted Three-Body Problem shown with four different coordinate systems.

B. CR3BP and Tube Dynamics

The spacecraft with a negligible mass is moving under the gravitational influence of the Sun and Mars with each having a mass of m_1 and m_2 respectively. For normalization, we take the unit of length by the distance between the Sun and Mars, and we choose the unit of time such that the angular velocity of the Sun and Mars is equal to 1. The unit of mass is taken by the sum of the Sun and Mars. Using the mass parameter $\mu = m_2/(m_1 + m_2)$, we can derive the equation of motion in the rotating frame as follows:

$$\ddot{x} - 2\dot{y} = -\frac{\partial \bar{U}}{\partial x},$$
$$\ddot{y} + 2\dot{x} = -\frac{\partial \bar{U}}{\partial y},$$
$$\ddot{z} = -\frac{\partial \bar{U}}{\partial z},$$

where

$$\bar{U}(x, y, z) = -\frac{1}{2}(x^2 + y^2) - \frac{1-\mu}{r_1} - \frac{\mu}{r_2} - \frac{1}{2}\mu(1-\mu)$$

is the effective potential. The energy of the system, E is defined as a function of position and velocities by

$$E(x, y, z, \dot{x}, \dot{y}, \dot{z}) = \frac{1}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) + \bar{U}(x, y, z),$$

which preserves constant along the solution curve. When the spacecraft is coasting, the energy of the system is fixed, and we could define the zero velocity curve. Note that the region surrounded by the zero velocity curve is called the forbidden region, in which region a spacecraft can not enter.

In the CR3BP, there are five equilibrium points known as the Lagrange points, three collinear equilibrium points (L_1, L_2, L_3) and two equilateral points (L_4, L_5). In particular, we focus on the L_1 and L_2, each of which has a local topology with the saddle×center structure.

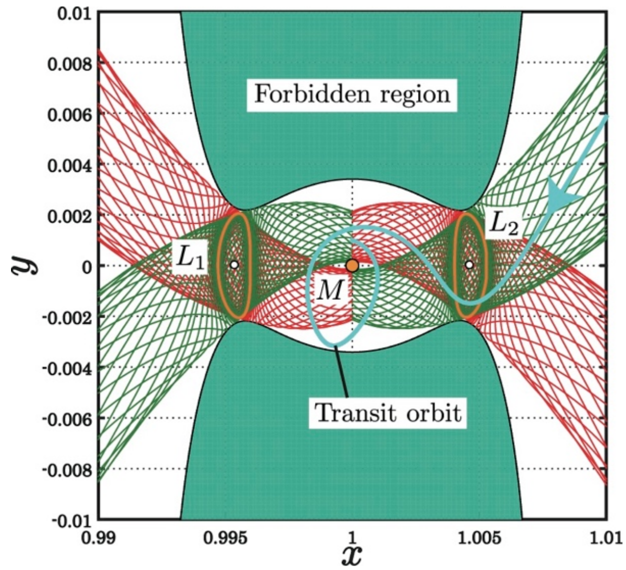


Fig. 2 Manifolds near Mars in CR3BP

There are unstable periodic orbits near the collinear Lagrange points, which associate with stable and unstable invariant manifolds called tubes. When the spacecraft is inside of an invariant manifold, the spacecraft moves through regions by passing through the collinear Lagrange points. If the spacecraft is outside of the tube, the spacecraft remains in the same region. In Fig. 2 we show the region near Mars. The orange orbits around two Lagrange points, L_1 and L_2 are the Lyapunov orbit. Associated with each Lyapunov orbit, there are two unstable manifolds that are repelling depicted in the color of red and two stable manifolds that are attracting in the color of green. A trajectory depicted in the color of blue, whose initial condition is located inside the green stable manifold, comes from the exterior region and passes through L_2, gate to get inside the Mars region. The trajectory locates inside the red unstable manifold derived from L_2, but it is located outside the green stable manifold derived from L_1, which traps the spacecraft in the Mars region.

III. Quasi-halo Orbit and its Invariant Manifolds

A. Quasi-halo Orbit

In order to compute the invariant manifolds from the quasi-halo orbit around Sun-Mars L_1 and design trajectories, we employ a numerical continuation procedure--hereby referred to as "GMOS (Gómez, Mondelo [7], Olikara, and Scheeres [8]) "--that computes families of quasi-periodic tori (QPT) via invariant curves of stroboscopic mappings [9]. Fig.3 shows a quasi-halo orbit around the original halo orbit at the vicinity of L_1.

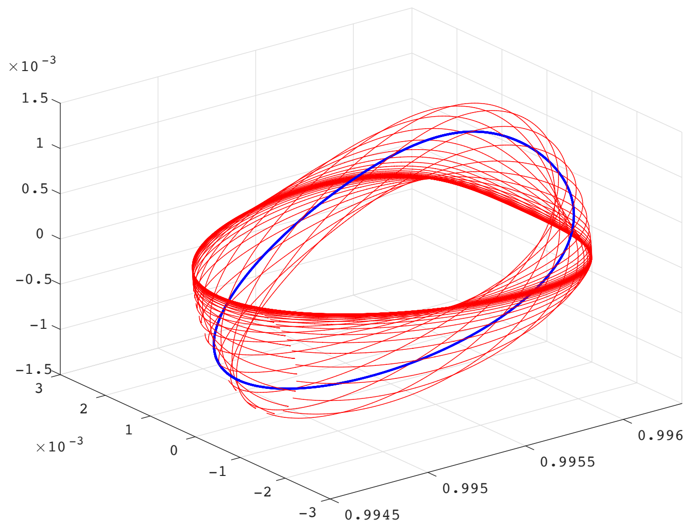


Fig. 3 Quasi-halo Orbit at L_1

B. Tube Dynamics

Next, we will compute the invariant manifolds associated with the above "quasi-halo" orbit. The Monodromy matrix is a state transition matrix for one period of the periodic orbit. By examining the eigenvalues and eigenvectors of the Monodromy matrix, the stability of the orbit can be clarified. The dominant eigenvalues and their eigenvectors increase exponentially and become an unstable manifolds diverging from the halo orbit by forward propagation. The backward propagation of micro displacement under eigenvalues that are smaller than 1 and the corresponding eigenvector results in a stable manifold, which asymptotically converges into the halo orbit. As we get the stability of the halo orbit from the Monodromy matrix, we can analyze the stability of the quasi-halo orbit. However, a quasi-halo orbit doesn't have monodromy matrix in a strict sense because the trajectory after 1 period doesn't return to the initial point. Also, one might try to infer the stability of a quasi-periodic torus from the differential map of $S_{-\rho} \equiv R_{-\rho} \circ \varphi_T$, where $R_{-\rho}$ is a rotation operator and φ_T represents a state after time T. However, if we use N solution points to approximate an invariant curve of the stroboscopic map φ_T, the Jacobian of $R_{-\rho}$ becomes a $nN \times nN$ matrix with nN eigenvalues (here n is six). In order to identify which of the eigenvalues of $DS_{-\rho}$ represents the stability of torus, we choose a method explained in [10]. That is to reduce the variable of the torus using a coordinate transformation such that the derivative of the stroboscopic map does not depend on θ. In this case, we can find that the nN eigenvalues μ_{jk} of $DS_{-\rho}$ can be immediately related to the n eigenvalues λ_j via

$$\mu_{jk} = \lambda_j e^{-ik\rho}$$

where $j = 1,2, \dots, N-1$. Consequently, the eigenvalues of $DS_{-\rho}$ lie on concentric circles of the complex plane, and each of these circles is representative of one eigenvalue.

By using the eigenvectors corresponding to these eigenvalues, we can compute an invariant manifold. One eigenvalue has $6 \times N$ eigenvector and each 6-value vector is a perturbation state vector $\delta\boldsymbol{U}$ of each point. Then, we calculate each perturbation state vector $\delta\widetilde{\boldsymbol{U}}$ during one period. This is very simple because each perturbation vector follows the state transition matrix (STM) :

$$\delta\widetilde{\boldsymbol{U}} = \mathrm{STM} \times \delta\boldsymbol{U}$$

After all, we propagate these displacements forward and backward, and then get invariant manifolds as Fig.4. A bunch of green trajectories is the "stable" manifold tube which is asymptotic to the quasi-halo orbit in forward time, and that of red ones is the "unstable" manifold tube which is asymptotic in backward time.

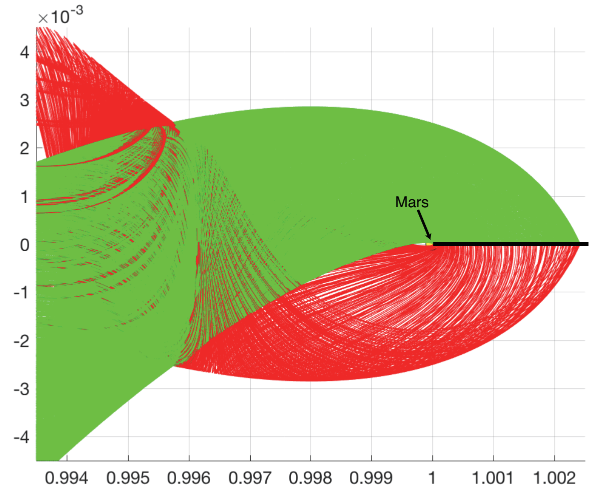
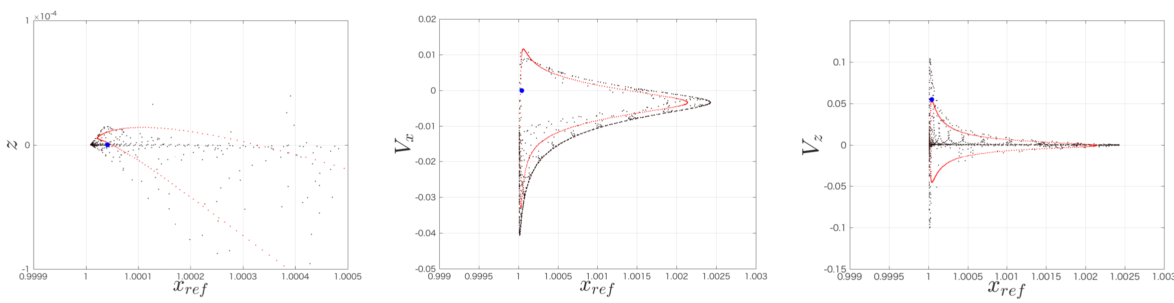


Fig.4 Invariant Manifolds near L_1 Associated with Quasi-halo Orbit

Since it is impossible to map four variables to a single Poincaré section, we take three Poincaré sections with x on the horizontal axis and z, V_x, and V_z on the vertical axis respectively. Here we show the Poincaré sections when $\varphi = 0$.



(a) (b) (c)

Fig5. Poincaré section with QP ((a)x-z (b)x-V_x (c)x-V_z)

Thus, in the case of the tube from the halo orbit, the state quantity of the spacecraft is not inside the tube, and it is impossible to show the effectiveness in the spatial problem. However, when the tube associated with the quasi-halo orbit is used, the state quantity actually is inside the tube, and agreed with the results of trajectory propagation with numerical integration

IV. Conclusion

In this paper, we have developed whether we can apply the tube dynamics in three dimension case. In our case, especially Sun-Mars system, and under a certain energy case, we can conclude whether a spacecraft can escape by using the tube dynamics. In particular, by using tubes, namely, the invariant manifolds associated with the halo orbit as well as the "quasi-halo" orbit, we can determine how the trajectory of the spacecraft can escape or not in the context of the SR3BP.

Acknowledgments

H.Y. is partially supported by JSPS Grant-in-Aid for Scientific Research (16KT0024, 24224004), the MEXT "Top Global University Project" and Waseda University (SR 2018K-195).

References

[1] M. Horikawa, Y. Kawakatsu, and H. Yoshimura, "Low Energy Escape Trajectory for the Mars Moon Sample Return Mission," 26th AAS/AIAA Space Flight Mechanics Meeting , Napa, CA, USA, 2016.

[2] Koon, W. S., Lo, M. W., Marsden, J. E., and Ross, S. D., Dynamical systems, the three-body problem and space mission design, Marsden Books, 2011.
[3] C. C. Conley, "Low Energy Transit Orbit in the Restricted Three-body Problem," SIAM J.Appl. Math. , Vol. 16, No. 4, 1968, pp. 732–746.
[4] R. P. McGehee, "Some Homoclinic Orbits for the Restricted Three-Body Problem," Master's thesis, University of Wisconsin, Madison, 1969.
[5] J. Llibre, R. Martinez, and C. Sim´o, "Transversality of the Invariant Manifolds Associated to the Lyapunov Family of Periodic Orbits near L_2 in the Restricted Three-Body Problem," J. Differential Equations, Vol.58, 1985, pp. 104-156
[6] K. Takemura, M. Horikawa,Y. Kawakatsu and H. Yoshimura, "Escape Trajectory for the Martian Moon Sample Return Mission Using Tube Dynamics (in Japanese)," *Proceedings of the 61st Space Sciences and Technology Conference*, 2017.
[7] G. Gómez and J. Mondelo, "The dynamics around the collinear equilibrium points of the RTBP," Physica D: Nonlinear Phenomena, Vol. 157, No. 4, 2001, pp. 283–321.
[8] Z. P. Olikara and D. J. Scheeres, "Numerical method for computing quasi-periodic orbits and their stability in the restricted three-body problem," *paper AAS 12-361 presented at the 2012 AIAA/AAS Astrodynamics Specialist Conference*, Minneapolis, MN, USA, 2012.
[9] N. Baresi, Z. P. Olikara, and D. J. Scheeres, "Fully Numerical Methods for Continuing Families of Quasi-Periodic Invariant Tori in Astrodynamics," The Journal of Astronautical Sciences , Vol. 65, No. 2, 2018, pp. 157-182
[10] A. Jorba, "Numerical computation of the normal behaviour of invariant curves of n-dimensional maps," Nonlinearity, No. 14, 2001, pp. 943–976.