Fuel-Free Station-Keeping of a Transformable Spacecraft around SEL2

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Abstract

Amplitude of natural halo orbits around the Sun-Earth L2 (SEL2), which is as much as 100,000 km or more, can be made smaller artificially by adding small amount of external force. This orbit is referred to as “the small-amplitude periodic orbit”. From its relatively small size, the geometric relationship among a spacecraft, the sun and the earth is practically fixed, which leads to the advantage of making the thermal condition of spacecrafts more stationary. The magnitude of the force required for the orbit maintenance is comparable to solar radiation pressure (SRP) applied on a surface of a spacecraft, which means the small amplitude periodic orbit can be achieved without fuel consumption if a spacecraft controls its attitude properly.

可変構造宇宙機の SEL2 まわりの推進剤不要軌道維持

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概要

太陽-地球第 2 ラグランジュ点 (SEL2) 周りのハロー軌道の大きさは一般的に 100,000 km 以上の非常に大きな値を取るが，微小外力を加えることにより，この大きさは飛躍的に小さくすることができる．この軌道は「小半径周期軌道」などと呼ばれる．軌道半径が小さいことにより，宇宙機と太陽・地球との幾何学関係はより固定されることになり，宇宙機に安定した熱環境を提供する．この軌道維持に必要な外力の大きさは太陽光圧の大きさ程度であり，宇宙機の姿勢を適切に制御すれば推進剤不要で実現できる．

1 Introduction

1.1 Background

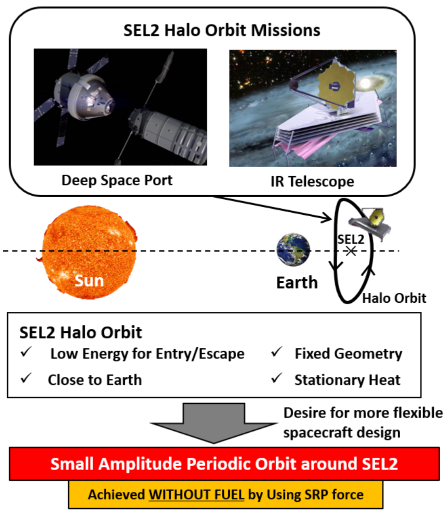

Figure 1: Summary of Background

5 equilibrium points are derived from a circular restricted three body problem which consists of a star, a planet and a spacecraft. Those equilibria are refered to as “Lagrange points”. As shown in the Figure 1, especially the 2nd Lagrange point in the Sun-Earth system (called as SEL2) has various advantages such as (1) fixed geometry among the Sun, Earth and a spacecraft, (2) relatively close position to Earth, (3) low energy cost for orbit entry/escape and (4) stationary input of heat. From the advantages (1), (2), (3), the orbit around SEL2 seems to be a good candidate of a space port for deep space explorers. Moreover, the feature (4) contributes to reduction of spacecraft thermal design cost, and thus fits to infrared observatory missions with strict thermal requirements.
Amplitude of natural halo orbits around the Sun-Earth L2 (SEL2) is usually as much as 1,000,000 km or more. If this orbital size is reduced to the size of Earth eclipse, mission design will become more flexible. Tarao proposed

that such a small-sized orbit is achieved only by adding minute continuous external force to a spacecraft[1]. This orbit is referred to as "the small-amplitude periodic orbit". Furthermore, Tanaka showed that the magnitude of the force required for the orbit maintenance is comparable to solar radiation pressure (SRP) applied on a surface of a spacecraft, which means the small amplitude periodic orbit can be achieved without fuel consumption if a spacecraft controls its attitude to properly[2].

1.2 Problems

Above-mentioned orbit and attitude dynamics is complicated because SRP induces attitude disturbance which subsequently leads to orbital perturbation. In addition, since SEL2 is an unstable equilibrium point, feed forward control proposed in the research such as [2] will cause divergence from a nominal orbit in long time propagation. Moreover, though the system of a three body problem exhibits strong nonlinearity, the previous researches conducted the simulations only in a linear system. Such a situation is insufficient for

1.3 Composition of This Paper

In this study, the control method which achieves

In this paper, basic theories of the dynamics are introduced first. Next, the control strategy is proposed, followed by its demonstration by numerical simulations. From its result, it is shown that this strategy is significantly effective for the transformable spacecraft.

2 Dynamics of the System

2.1 Circular Restricted Three Body Problem

A general three body problem is known as an unintegrable system, in other word, the equations cannot be solved analytically. Thus some approximation should be imposed to analyse the motion of particles in a three body system. One often used approximation is "circular restricted three body problem" (shortly, CR3BP), which satisfies (1) one particle has negligible mass which doesn't influence the other two massive bodies (these two main celestial bodies are refered to as "primaries"), (2) two primaries are on a planar circular orbit around their barycenter. In Sun-Earth-Spacecraft system, the particle corresponds to a spacecraft while the Sun is labeled as the 1st primary and Earth as the 2nd primary. For simplicity of notations and convenience in numerical calculations, following all formulations are normalized such that the unit length equals to distance between primaries (in the Sun-Earth case, 1AU), the unit mass to sum of two primaries' mass and the unit time to primaries' rotational period devided by 2π (that is, one year corresponds to 2π in the Sun-Earth case). The system is illustrated in Figure 2 and definition of parameters are shown in Table 1. This x and y in the rotating frame are on the orbital plane of the two primaries.

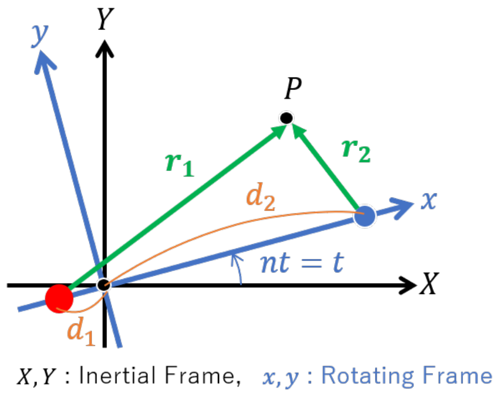

Figure 2: The System Illustration

Table 1: Definition of Parameters

name	character
mass of 1st primary	m_1
mass of 2nd primary	m_2
mass ratio of 1st primary	$\mu_1 = \dfrac{m_1}{m_1 + m_2}$
mass ratio of 2nd primary	$\mu_2 = \dfrac{m_2}{m_1 + m_2}$
distance of m_1 and particle	r_1
distance of m_2 and particle	r_2

According to the normalization, gravity constant G becomes 1, mean motion ($= 2\pi/T$, where T is the period of primaries) also equals to 1 and the distance between the barycenter and m_1 and m_2, that is, d_1 and d_2 respectively corresponds to μ_2 and μ_1.

First of all, in inertial frame lagrangian is described as

$$
\begin{aligned}
L &= E_{kinetic} - E_{potential} \\
&= \frac{1}{2}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right) - \left(-\frac{\mu_1}{r_1}-\frac{\mu_2}{r_2}-\frac{1}{2}\mu_1\mu_2\right)
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
\mathbf{r_1} &= \begin{bmatrix} X+\mu_2\cos t & Y+\mu_2\sin t & Z \end{bmatrix}^{\mathrm{T}} \\
\mathbf{r_2} &= \begin{bmatrix} X-\mu_1\cos t & Y-\mu_1\sin t & Z \end{bmatrix}^{\mathrm{T}}
\end{aligned}
\tag{2}
$$

Coordinate transformation from inertial frame to rotating frame is expressed as

$$
\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = A_t \begin{bmatrix} x \\ y \\ z \end{bmatrix}
\tag{3}
$$

By differentiating equation (3), the following equation is obtained

$$
\begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} = \dot{A}_t \begin{bmatrix} x \\ y \\ z \end{bmatrix} + A_t \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = A_t \begin{bmatrix} \dot{x}-y \\ \dot{y}+x \\ \dot{z} \end{bmatrix}
\tag{4}
$$

Using the relationship described in equation (4), equation (1) is transformed into

$$
\begin{aligned}
L &= \frac{1}{2}\left(\dot{X}^2+\dot{Y}^2+\dot{Z}^2\right) - \left(-\frac{\mu_1}{r_1}-\frac{\mu_2}{r_2}-\frac{1}{2}\mu_1\mu_2\right) \\
&= \frac{1}{2}\left[(\dot{x}-y)^2+(\dot{y}+x)^2+\dot{z}^2\right] \\
&\qquad - \left(-\frac{\mu_1}{r_1}-\frac{\mu_2}{r_2}-\frac{1}{2}\mu_1\mu_2\right)
\end{aligned}
\tag{5}
$$

where

$$
\begin{aligned}
\mathbf{r_1} &= \begin{bmatrix} x+\mu_2 & y & z \end{bmatrix}^{\mathrm{T}} \\
\mathbf{r_2} &= \begin{bmatrix} x-\mu_1 & y & z \end{bmatrix}^{\mathrm{T}}
\end{aligned}
\tag{6}
$$

Finally, the equation of motion is obtained from Euler-Lagrange equation $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}_i} - \frac{\partial L}{\partial x_i} = 0$,

$$
\begin{cases}
\ddot{x} = 2\dot{y} + x - \dfrac{\mu_1(x+\mu_2)}{r_1^3} - \dfrac{\mu_2(x-\mu_1)}{r_2^3} \\
\ddot{y} = -2\dot{x} + y - \dfrac{\mu_1 y}{r_1^3} - \dfrac{\mu_2 y}{r_2^3} \\
\ddot{z} = -\dfrac{\mu_1 z}{r_1^3} - \dfrac{\mu_2 z}{r_2^3}
\end{cases}
\tag{7}
$$

This set of equations can be linearlized into following form,

$$
\begin{cases}
\ddot{x} = 2\dot{y} + (1+2c_2)x \\
\ddot{y} = -2\dot{x} + (1-c_2)y \\
\ddot{z} = -c_2 z
\end{cases}
\tag{8}
$$

Equations (7) are still unintegrable, they are much simpler to analyse than that of a general three body problem. After some calculation for these equations, L2 point can be determined as the unstable equilibrium point (detail derivation is described in [3]).

2.2 Formulation of Solar Radiation Pressure

As stated in chapter 1, we focus on the orbital maintenance utilizing solar radiation pressure (SRP). Before explaining the control law, the SRP force applied on surface of a spacecraft must be formulated. SRP force is decomposed into 3 factors: specular, diffusive and absorption (See Figure 3).

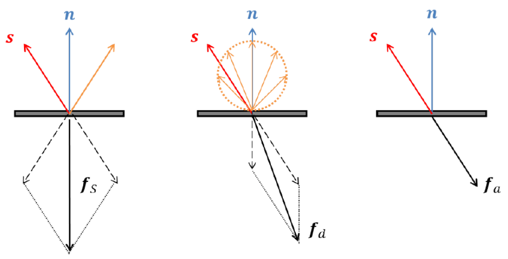

Figure 3: 3 factors of SRP force
(from left, Specular, Diffusive, Absorption)

Total acceleration by SRP force is expressed as

$$
\begin{aligned}
a_{SRP} = -\frac{PA}{m}(\mathbf{s}\cdot\mathbf{n}) \Bigg[& (C_{abs}+C_{dif})\mathbf{s} \\
& + \left(\frac{2}{3}C_{dif} + 2(\mathbf{s}\cdot\mathbf{n})C_{spe}\right)\mathbf{n} \Bigg]
\end{aligned}
\tag{9}
$$

where $C_{spe}, C_{dif}, C_{abs}$ is coefficients of specular, diffusive, absorption respectively and $\mathbf{s}, \mathbf{n}$ is the vector from the Sun and a normal vector of a surface respectively (both are unit vectors) and m is mass of a spacecraft. Typical optical properties are given in Table 2.

Table 2: Optical Properties List

Material	Spe.	Dif.	Abs.
Polyimido	0.375	0.255	0.370
Solar Cell	0.086	0.060	0.854
Mirror	1.0	0.0	0.0

3 Control Law

3.1 Attitude Control Law

In this section, the control method which enables the small amplitude periodic orbit around SEL2 to be kept using SRP. A spacecraft has to control its attitude to the Sun to obtain proper external force to keep the orbit. This control law is based on that proposed by Tanaka [2], which is developed in linearly approximated system. In this study, the spacecraft motion is calculated according to a CR3BP nonlinear equations (7). Validity of the linear control law in such a nonlinear system is confirmed in numerical simulations.
Attitude of the spacecraft against the Sun is expressed with 2 angles. These angle are named as "sun angle", which is depicted in Figure 4. In this figure, φ is the out-of-plane angle and ψ is the in-plane angle ("plane" means the orbital plane of the spacecraft here).

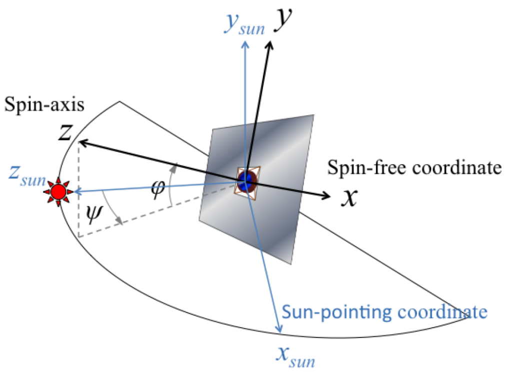

Figure 4: Definition of Sun Angle

The normal vector $\mathbf{n}$ and the sun vector $\mathbf{s}$ are approximated as following equations assuming sun angle to be small enough,

$$\mathbf{n} = \begin{bmatrix} -\cos\phi\cos\psi \\ -\cos\phi\sin\psi \\ \sin\phi \end{bmatrix} \simeq \begin{bmatrix} -1 \\ -\psi \\ \phi \end{bmatrix}$$
$$\mathbf{s} \simeq \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix} \tag{10}$$

Substituting these vectors into equation (9), we obtain

$$\begin{bmatrix} S_1 \\ S_2\psi \\ -S_2\phi \end{bmatrix} \tag{11}$$

where

$$S_1 = \frac{PA}{m}\left(C_{abs} + \frac{5}{3}C_{dif} + 2C_{spe}\right)$$
$$S_2 = \frac{PA}{m}\left(\frac{2}{3}C_{dif} + 2C_{spe}\right) \tag{12}$$

Now the target orbit shape is given in the following equations expressed in xyz coordinate,

$$\begin{aligned} x &= -A_x\cos(\omega t + \theta_{xy}) + x_e \\ y &= \alpha A_x \sin(\omega t + \theta_{xy}) \\ z &= A_z\cos(\omega t + \theta_z) \\ \theta_z &= \theta_{xy} + n\pi, \quad (n = 0, 1) \end{aligned} \tag{13}$$

substituting equations (11), (13) into linearized CR3BP equation (8), we obtain the sun angle control law as following equations (see detail in [2]).

$$\psi = \frac{1}{S_2}\left(-\omega^2 + \frac{2\omega}{\alpha} + c_2 - 1\right)\alpha A_x \sin\omega t + \theta_{xy}$$
$$\phi = -\frac{1}{S_2}\left(-\omega^2 + c_2\right)A_z\cos\omega t + \theta_z \tag{14}$$

3.2 Feedback Control

L2 is a unstable equilibrium point and thus tends to generate orbit divergence as propagation time grows longer. For this reason, some feedback control method must be conducted. In this paper, LQR feedback control is adopted to linearized CR3BP equations (similar orbit maintenance is showed in [4]).
Objective function is expressed as

$$J = \int_0^\infty (\tilde{\mathbf{x}}^{\mathrm{T}} R_1 \tilde{\mathbf{x}} + \tilde{\mathbf{u}}^{\mathrm{T}} R_2 \tilde{\mathbf{u}})dt \tag{15}$$

x and u in this equation are state vector and control input vector reppectively. Weight matrix R_1 and R_2 are defined as

$$R_1 = \begin{bmatrix} A_x^{-2} & & & & & \\ & A_y^{-2} & & & O & \\ & & A_z^{-2} & & & \\ & & & (A_x\omega)^{-2} & & \\ & O & & & (A_y\omega)^{-2} & \\ & & & & & (A_z\omega)^{-2} \end{bmatrix}$$
$$R_2 = \begin{bmatrix} R_{21} & 0 \\ 0 & R_{22} \end{bmatrix}$$
$$R_{21} = 10\left[\frac{1}{S_2}\left(-\omega^2 + \frac{2\omega}{\alpha} + c_2 - 1\right)\alpha A_x\right]^{-2}$$
$$R_{22} = 100\left[\frac{1}{S_2}\left(-\omega^2 + c_2\right)\alpha A_z\right]^{-2} \tag{16}$$

4 Simulation

In this study, validity of the control method is verified with numerical simulation. In this simulation, system dynamical equations are given in non-linear CR3BP system while the control laws are developed in the linearized system.
Spacecraft configuration is assumed as panel shaped spacecraft. Each panel has 1m×1m×0.1m square area an thickness, 10 kg wight and different optical properties. In this simulation, total panel numbers are set to be 17 where the surface of 8 of them are polyimido (MLI), 6 are solar cell and 3 are mirror panel. These optical properties are given in Table 2. Propagation time is 600 days (about 3.3 orbital period). As seen in Figure 5 and 6, the spacecraft seems not to maintain the nominal orbit without feedback control whereas feedback control keeps the orbit more stable. From this result, the control law derived in section 3 is valid in the non-linear CR3BP system.

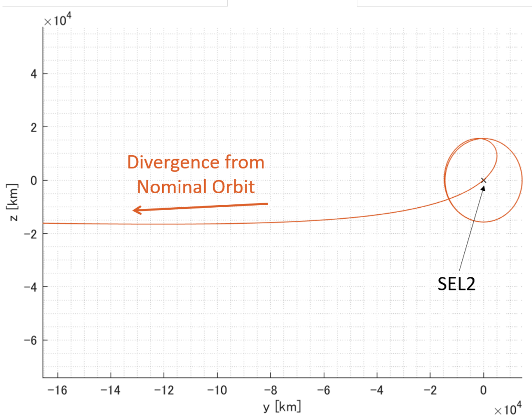

Figure 5: Orbit **WITHOUT** Feedback Control, Propagation Time: 600 days

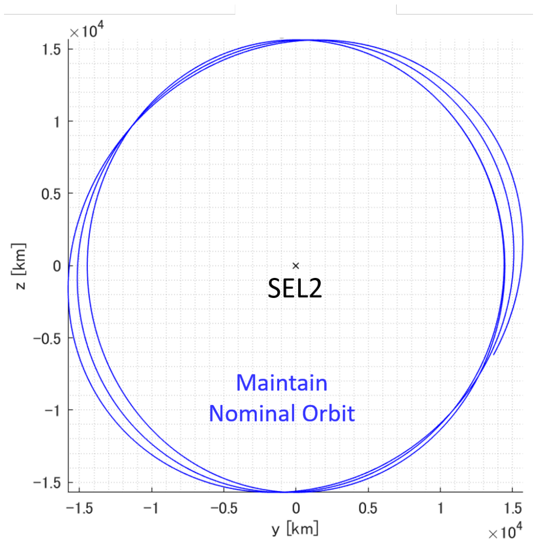

Figure 6: Orbit with Feedback Control, Propagation Time: 600 days

5 Summary

In this paper, the long-term station-keeping around SEL2 is developed. Using SRP force as the external force input, the orbital maintenance is achieved without fuel consumption. Furthermore, combinating this control law and the LQR feedback control, the spacecraft can keep on this orbit even around the unstable equilibrium point such as SEL2.
As future works, this control should be revised into discrete form because a spacecraft cannot perform completely continuously in actual operations.

References

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