On-board Orbit Propagation Methods for Micro Spacecrafts

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The increase of the number of satellites in LEO can lead to the space debris problem. There is clearly a great need of the satellite's deorbiting capability to realize sustainable development of the outer space. To make a radical solution to this problem, our group is now studying a new autonomous propulsive de-orbiting device for high LEO (higher than 600 km) satellite. As an initial work, a simple onboard orbit propagator which is the key element to achieve autonomous deorbiting operation was investigated. This paper will present an overview of mission sequence of the proposed device and the comparison study of simple orbit propagation methods, including the experiment of the implementation on a micro computer. For numerical integration, both the single-step Runge-Kutta and multi-step Adams-Bashforth-Moulton integrator methods are implemented. This study can provide knowledge of onboard orbit propagator for micro spacecrafts that is expected to make use of satellite constellations with autonomous operation.

Key Words: Onboard orbit propagator, Space debris, Deorbit device

Nomenclature

a	:	semi-major axis, km
e	:	eccentricity
i	:	inclination, deg
ω	:	argument of perigee, deg
Ω	:	right ascension of ascending node, deg
M	:	mean anomaly, deg
J_n	:	zonal gravitational coefficient ($n = 2, 3, 4$)
R_e	:	Earth equator radius, km
μ	:	Earth gravitational constant, km^3/s^2
r	:	spacecraft distance from Earth center, km
$\boldsymbol{r}$	:	spacecraft position vector, km
z	:	z component of s/c position vector
$\boldsymbol{i}_z$	:	unit vector of z-direction
h	:	$e\sin(\omega + \Omega)$
k	:	$e\cos(\omega + \Omega)$
p	:	$\tan(i/2)\sin\Omega$
q	:	$\tan(i/2)\cos\Omega$
λ	:	$\omega + \Omega + M$
Subscripts		
0	:	initial
f	:	final
m	:	mean

1. Introduction

The number of small satellites have recently risen because of their low cost and short development time. Moreover, space enterprises have planed satellites' constellation to make more beneficial services. Considering this situation, especially LEO satellites which will be launched in the future are desired to have de-orbiting ability to limit the increase of space debris. A wide variety of deorbiting devices have researched in recent years. Drag sail device is a method which has been studied the most in recent years.[[4)]] This method can enhance the drag force acted on the satellite by expanding large sail. However, this method cannot be used in high altitude low Earth orbit (higher than approximately 600 km) because the atmospheric drag is extremely low. On the other hand, de-orbiting with propulsion system, such as chemical thruster and cold gas jet, can be used even at high altitude. This type of device has also an advantages of quickness of lowering orbit. Considering a large number of satellites, operational cost increase will be inevitable. This study focuses on the propulsive deorbit device which can autonomously lower the satellite orbit. In the next section, I will provide mission overview of the device. Third section shows formulations of orbit propagation methods. Then, the results of numerical simulations will be presented. In the fifth section, the implementation results will be shown. Finally, I will summarize this paper.

2. Mission sequence

There are two phases of the device operation. One is the stand-by phase and the other is the mission phase. In the stand-by phase, the device does not perform critical action, but it regularly receives orbit and attitude information and electrical power supply from the satellite. After its power lost, the satellite become uncontrollable. Hence, the device is needed to know the satellite attitude untill deorbiting operation is done. The satellite power lost switch the device to the mission phase. Firstly, it starts to propagate satellite orbit with referring recent orbit data provided by the satellite as the initial position values. The device can know the attitude from orbit calculation described above. Moreover, the satellite attitude is observed by some sensors such as sun sensor and magnetometer. Thirdly, de-orbit at proper condition. Fig.1 illustrates the whole mission sequence.

3. Orbit propagation method

3.1. Simplified numerical method

Hardware burden and calculation cost are critical problems especially in onboard application. Hence, a simplified numerical method which includes gravitational zonal harmonics

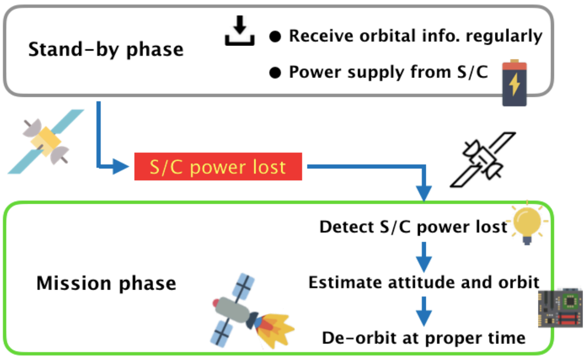

Fig. 1. Mission sequence

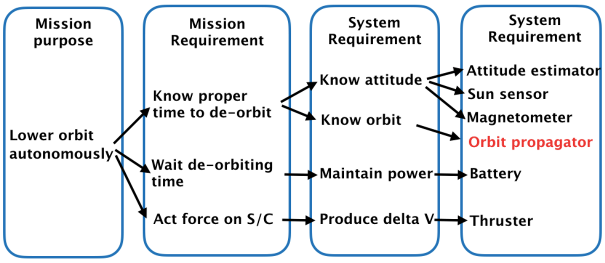

Fig. 2. Mission and system requirements

J_2, J_3, J_4[1] was formulated. I call this formulation SNJ4.

$$
\begin{aligned}
\ddot{\boldsymbol{r}} = -\frac{\mu}{r^3}\Bigg[1 &+ J_2\left(\frac{R_e}{r}\right)^3 \frac{3}{2}\left(1-5\frac{z^2}{r^2}\right) \\
&+ J_3\left(\frac{R_e}{r}\right)^3 \frac{5}{2}\left(3-7\frac{z^2}{r^2}\right)\frac{z}{r} \\
&- J_4\left(\frac{R_e}{r}\right)^4 \frac{15}{8}\left(1-14\frac{z^2}{r^2}+21\frac{z^4}{r^4}\right)\Bigg]\boldsymbol{r} \\
&+ \frac{3}{2}\frac{\mu}{r^2}J_3\left(\frac{R_e}{r}\right)^3 \boldsymbol{i}_z.
\end{aligned} \tag{1}
$$

In addition, I compared two integrators, fixed step 4th RungeKutta method (RK4), and the Adams-Bashforth-Moulton method (ABM). Fig.**??** shows that the difference between RK4 and ABM.

3.2. Semi-analytical method

Semi-analytical method is the combination of numerical integration and analytical method. This method was well formulated in Draper Semianalytical Satellite Theory DSST (Ref.[2]). In circular orbits, eccentricity is close to zero, therefore the singularities occurs in variation of Kepler elements. To avoid those singularities, the equinoctial elements are often used in analytical and semi-analytical propagation methods (Ref.,[3][4]). In semi-analytical methods, variation of orbital elements averaged by true anomaly or mean anomaly are numerically integrated. In this work, the unique formulation with equinoctial elements has done. In this formulation, only J_2 secular term and J_3 long term are considered referring Kozai's method (Ref.[5]). The averaged orbital variation equations are as follows.

$$
\left(\frac{da}{dt}\right)_m = 0 \tag{2}
$$

$$
\begin{aligned}
\left(\frac{dh}{dt}\right)_m &= \frac{3}{4}nJ_2\left(\frac{R_e}{p}\right)^2\left(-2\cos i + 4 - 5\sin^2 i\right) e\cos\zeta \\
&- \frac{3}{8}nJ_3\left(\frac{R_e}{p}\right)^3 \sin i\left(4-5\sin^2 i\right)\sin\Omega
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\left(\frac{dk}{dt}\right)_m &= -\frac{3}{4}nJ_2\left(\frac{R_e}{p}\right)^2\left(-2\cos i + 4 - 5\sin^2 i\right) e\sin\zeta \\
&- \frac{3}{8}nJ_3\left(\frac{R_e}{p}\right)^3 \sin i\left(4-5\sin^2 i\right)\cos\Omega
\end{aligned} \tag{4}
$$

$$
\begin{aligned}
\left(\frac{dp}{dt}\right)_m &= -\frac{3}{2}nJ_2\left(\frac{R_e}{p}\right)^2 \cos i \tan\left(i/2\right)\cos\Omega \\
&+ \frac{3}{8}nJ_3\left(\frac{R_e}{p}\right)^3 e\frac{\cos i}{1+\cos i}\left\{\sin\Omega\cos\omega\left(4-5\sin^2 i\right)\right. \\
&\left.- \cos\Omega\sin\omega\left(15\sin^2 i - 4\right)\right\}
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
\left(\frac{dq}{dt}\right)_m &= \frac{3}{2}nJ_2\left(\frac{R_e}{p}\right)^2 \cos i \tan\left(i/2\right)\sin\Omega \\
&+ \frac{3}{8}nJ_3\left(\frac{R_e}{p}\right)^3 e\frac{\cos i}{1+\cos i}\left\{\cos\Omega\cos\omega\left(4-5\sin^2 i\right)\right. \\
&\left.+ \sin\Omega\sin\omega\left(15\sin^2 i - 4\right)\right\}
\end{aligned} \tag{6}
$$

$$
\begin{aligned}
\left(\frac{d\lambda}{dt}\right)_m &= n + \frac{3}{4}nJ_2\left(\frac{R_e}{p}\right)^2\left\{\left(4-2\cos i - 5\sin^2 i\right)\right. \\
&\left.+ \left(2-3\sin^2 i\right)\sqrt{1-e^2}\right\} \\
&+ \frac{3}{8}nJ_3\left(\frac{R_e}{p}\right)^3 e\sin\omega\sin i\left\{4\sqrt{1-e^2}\left(4-5\sin^2 i\right)\right. \\
&+ \left(26-30\sin^2 i\right) \\
&\left.+ 5\cos^2 i + \cos i\left(4\tan\left(i/2\right)-15\right)\right\}
\end{aligned} \tag{7}
$$

4. Numerical simulation

In this section, the results of the orbit propagation simulations are shown. By way of comparison, HPOP in STK AGI inc. was used as the propagator which produces the reference orbit. This propagator has the advantage of enabling to add any perturbations and has the best accuracy. Simulation conditions are presented in Table. 1, Table. 2, and Table.3. Fig. 3 shows the comparison of phase error of two propagation methods that are SAJ4 (Semi-analytical) and SNJ4 (Numerical). This figure indicates that SNJ4 method performs better than SAJ4. SAJ4 method propagator reaches more than 80 deg error in 30 days. We can understand that SA-J4 method does not perform enough accuracy to calculate satellite orbits onboard.

5. Implementation on micro computer

In this section, I focus the implementation of the proposed estimators on Arduino Uno (Fig. 4). Tab. 5 shows the specification of Arduino Uno. The two type of orbit calculation algorithms are implemented on the micro computer and the hard-

Table 1. Conditions: HPOP

Coordinate system	J2000
Central body gravity	EGM2008 (degree:50, order:50)
Area/Mass	0.01 [m^2/kg]
Atmospheric drag	spherical, C_D = 2.20
Atmospheric density	Jacchia-Roberts
Solar flux	Daily, Average F10.7: 150.00000000
Geomagnetic index	3.00000000
Solar radiation pressure	spherical, C_r = 1.00
Third body attraction	Moon, Sun
Calculation step	variable. 60 sec
Integrator	7th order RungeKutta

Table 2. Conditions: SNJ4

Coordinate system	J2000
Calculation step	fixed, 1.0 sec
Integrator	4th order RungeKutta, Adams-Bashforth-Moulton

ware simulation was conducted. Fig. 5 shows the comparison of phase error of SNJ4 algorithm with RK4 and ABM. ABM integrator performed orbit propagation in good accuracy, less 10 degree in 90 days, while RK4 case reaches 60 deg in 60 days. In summary, We can understand that Adams Bashforth Moulton method is suitable for onboard orbit propagator. Fig.6 shows time to calculate orbital data on Arduino Uno with RK4 methods. It can provide one day orbit propagation in 5 min. This is enough speed for especially our deorbit device.

6. Conclusion

To conclude, simple onboard orbit propagators for the proposed deorbit device were investigated. The propagators were implemented on micro computer, Arduino Uno. As a result,

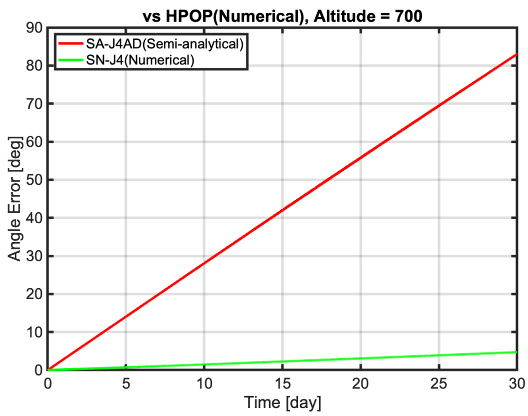

Fig. 3. Comparison of phase error at altitude of 700 km for 30 days (vs HPOP)

Fig. 4. Arduino Uno

Table 3. Conditions: SAJ4(semi-analytical)

Calculation step	fixed, 60.0 sec
Integrator	4th order RungeKutta

Table 4. Initial conditions

a_0[km]	6857.26, 6957.26, 7057.26
(Altitude [km])	(500, 600, 700)
e_0	0.001
i_0 [deg]	30.00
Ω_0 [deg]	0.00
ω_0 [deg]	0.00
M_0 [deg]	0.00

I confirmed that the simple numerical J4 method considered only the gravitational zonal harmonics up to J_4 can maintain the phase angle accuracy within less 15 deg for 60 days propagation in the altitude of 700 km. The calculation performance is also sufficient for the device. This study can contribute to the study of autonomous and onboard orbit propagation methods for micro satellites which have low performance computer. Further, the ground experiment for demonstration of the deorbiting operation with the device including orbit estimator and attitude sensors will be conducted.

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Table 5. Specification of Arduino Uno

Program memory	32 KB
RAM	2 KB
Clock speed	16 MHz

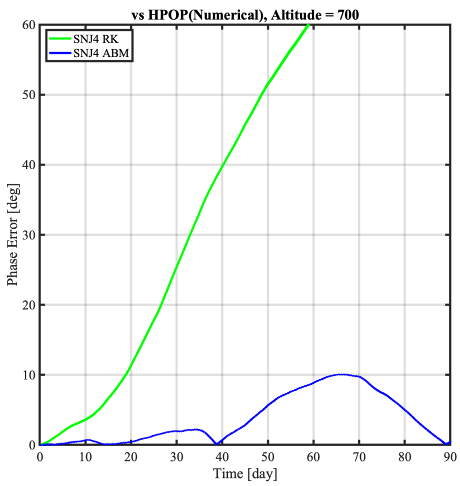

Fig. 5. Comparison of phase error: RK4 and ABM

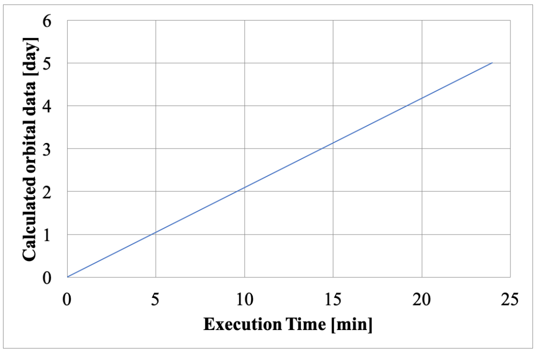

Fig. 6. Needed Time to calculate orbital data: RK4 with Arduino

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